Are complex financial systems unstable ?

Analyzing cascading failures in networks of financial institutions

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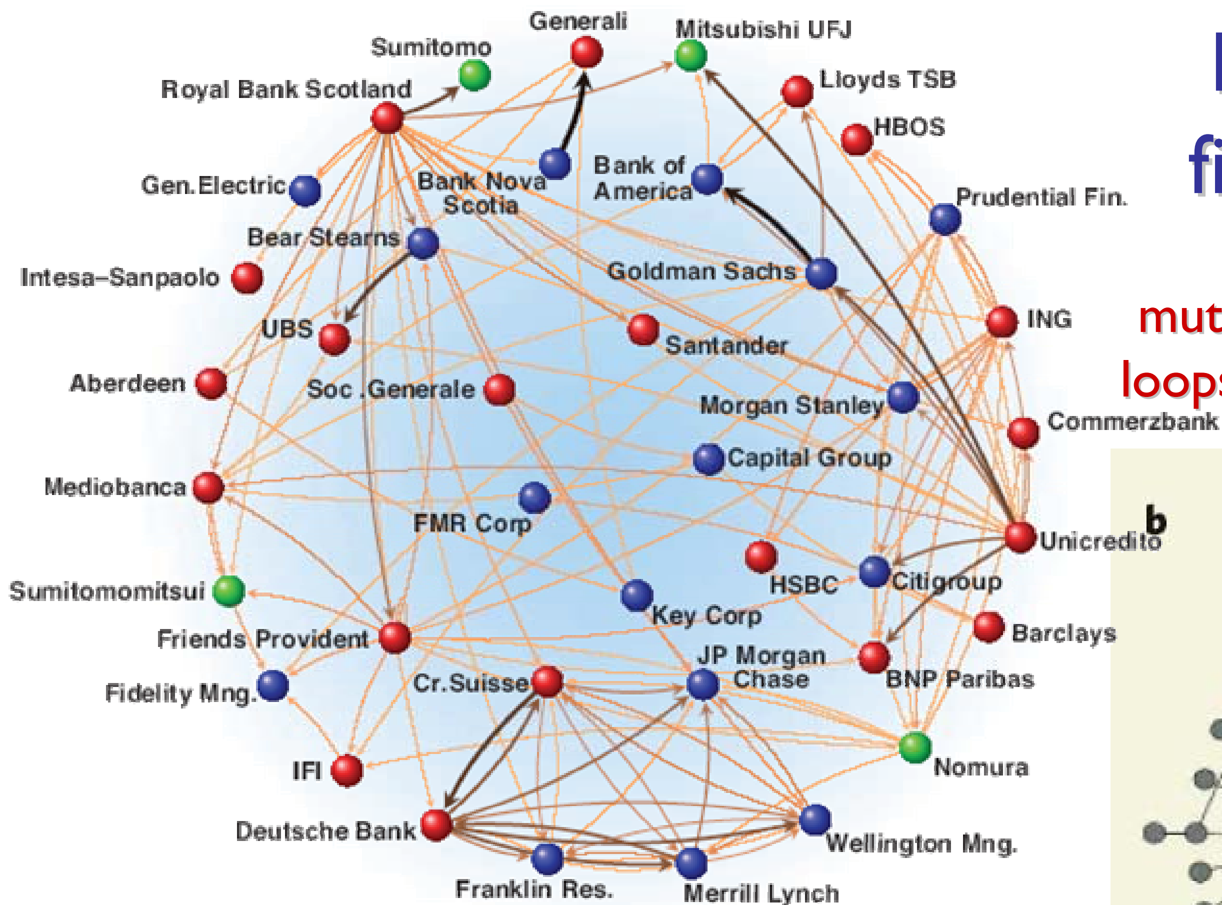
Outline

- Background: Systemic Risk and the dynamical systems approach to Ecosystem stability
- Characterization of Inter-bank network from bilateral exposure data of US and European banks
- Investigating heterogeneity & modularity of the network
- The dynamics of cascading failures: local & global stability
- From network topology to dynamics: using structural measures to identify critical nodes
- Global failure: possible role of liquidity crisis ?

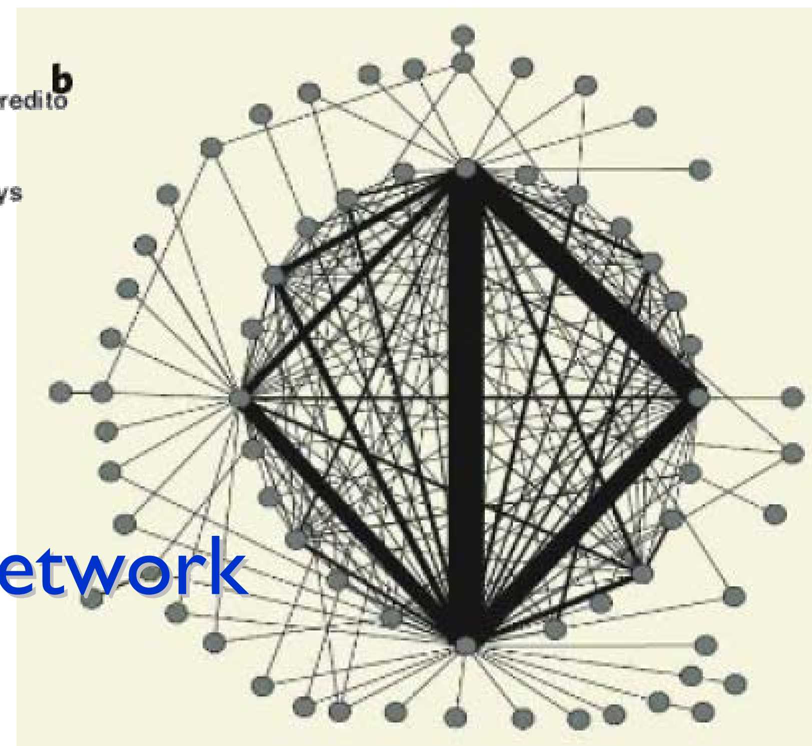
Financial Markets are Complex Systems !

Network of major financial Institutions

High connectivity in terms of mutual share-holdings and closed loops ⇒ strong inter-dependence



F Schweitzer et al, Science, 2009



Fedwire Interbank Payment Network

"Core" of 66 banks accounts for 75% of daily transactions in value (900 billion US$) – subset of 25 banks fully connected !

K Soramaki

Was the recent worldwide financial crisis a disaster just waiting to happen ?

Systemic Risk (of collapse of entire financial system) ≡ Network susceptibility to small perturbations resulting in a cascading process due to excessive connectivity ?

Complex markets are unstable

As the interaction between agents increase in complexity
- the connection density increases, and/or the
- interactions become stronger,

the system *almost certainly* becomes unstable.

follows from May-Wigner Theorem

Complexity → Instability in Networks

Stability of large networks:

State of the network of N nodes: $x = (x_1, x_2, \ldots, x_N)$,

x_i : state of the i^{th} node.

Time evolution of x is given by a set of equations $d\, x_i / d\, t = f_i\ (x)$ $(i = 1, 2, \ldots, N)$

Fixed point equilibrium: $x^0 = (x^0_1, x^0_2, \ldots, x^0_N)$ such that $f(x^0) = 0$

Local stability of x^0 : Linearizing about the eqlbm: $\delta x = x - x^0$

$d\, \delta x / d\, t = A\, \delta x$ where Jacobian A: $A_{ij} = \partial f_i / \partial x_j |_{x = x0}$

Long time behavior of δx dominated by λ_{max} (largest real part of the eigenvalues of A)
$|\, \delta x\, | \sim \exp\, (\lambda_{max}\, t)$

The equilibrium $x = x^0$ is stable if $\lambda_{max} < 0$.

What is the probability that for a network, $\lambda_{max} < 0$?

Each node is independently stable $\Rightarrow$ diagonal elements of A < 0 (choose $A_{ii} = -1$).

Let A = B - I where B is a matrix with diagonal elements 0 and I is $N \times N$ identity matrix.

For matrix B, the question: What is the probability that $\lambda'_{max} < 1$?

Applying Random Matrix Theory:

Simplest approximation: no particular structure in the matrix B, i.e., B is a random matrix.

B has connectance C, i.e., $B_{ij} = 0$ with probability 1 - C .

Non-zero elements: i.i.d. random variables from Normal $(0, \sigma^2)$ distribution.

For large N, Wigner's theorem for random matrices apply.

Largest real part of the eigenvalues of B is $\lambda'_{max} = \sqrt{(N C \sigma^2)}$.

For eigenvalues of A: $\lambda_{max} = \lambda'_{max} - 1$

For large N, probability of stability $\rightarrow 0$ if $\sqrt{(N C \sigma^2)} > 1$,
while, the system is almost surely stable if $\sqrt{(N C \sigma^2)} < 1$.

Large systems exhibit sharp transition from stable to unstable behavior when N or C or σ^2 exceeds a critical value.

$\Rightarrow$ Complexity $\rightarrow$ Instability

Criticism of May-Wigner theorem:
Complexity → Instability

- ❑ Assumes random network of interactions (although the most real-world networks clearly are structured)

Solution: Consider networks which have structures (patterns) in the arrangement of their interactions

- ❑ Based on linear stability (does not take into account periodic or chaotic dynamics of nodes)

Solution: Consider global stability in a system having nodes with a rich variety of dynamical behavior

A Fresh look at Complexity → Instability

- Consider networks which have structures in the arrangement of their interactions

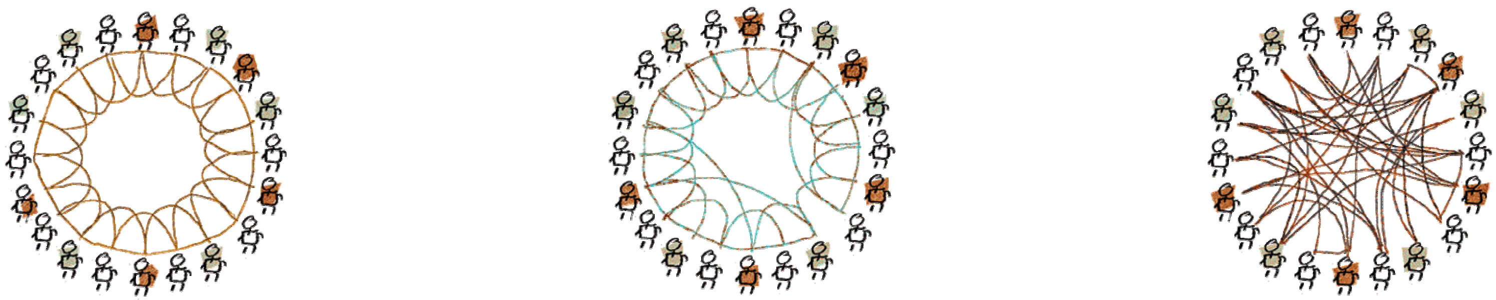
Small-world connectivity: SS, *Physica A*, 2005
Modular organization: R. K. Pan and SS, *PRE Rapid*, 2007
Hierarchical modular connectivity: R. K. Pan and SS, *Pramana*, 2008
Scale-free degree distribution: M. Brede and SS, *arxiv preprint*

- Consider networks with full dynamics (fixed point, oscillatory, chaotic) at each node

SS and Sudeshna Sinha, *Phys Rev E*, 2005
SS and Sudeshna Sinha, *Phys Rev E*, 2006

Introducing **complex structures** or **complex dynamics on networks** does **NOT** change basic result of May: increased complexity promotes instability.

Example: *small-world* networks



Regular Network
$p = 0$

"Small-world" Network
$0 < p < 1$

Random Network
$p = 1$

Increasing Randomness

Watts and Strogatz (1998): Many biological, technological and social networks have connection topologies that lie between the two extremes of completely regular and completely random.

Question:

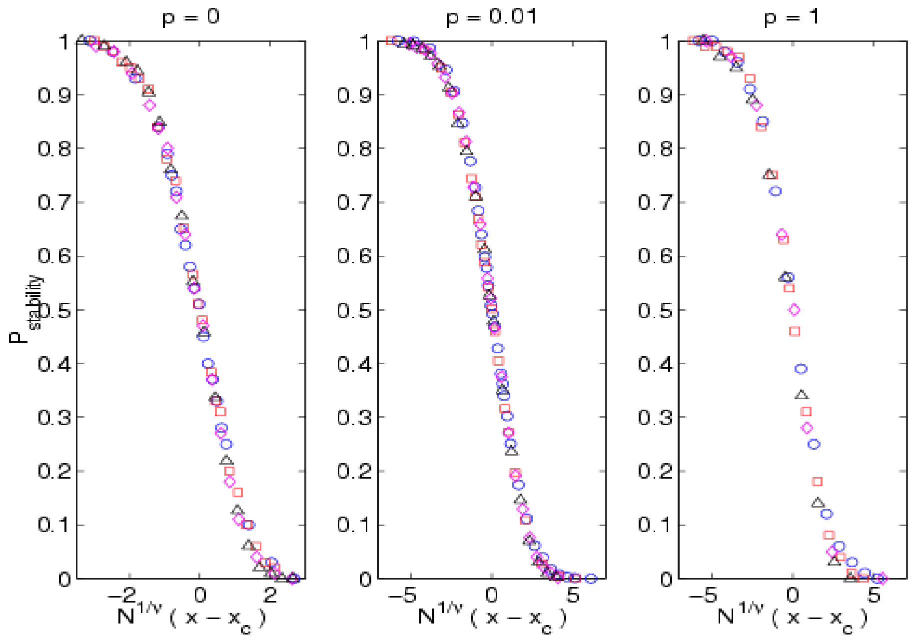
Does WS small-world topology affect stability of a network ?

Answer: NO! (SS 2005)

Stability-instability transition in Small-World Networks
(SS, 2005)

Probability of stability in a network
Finite size scaling: N = 200, 400, 800 and 1000.

$\nu \approx 2.0$ $\nu \approx 1.72$ $\nu \approx 1.5$



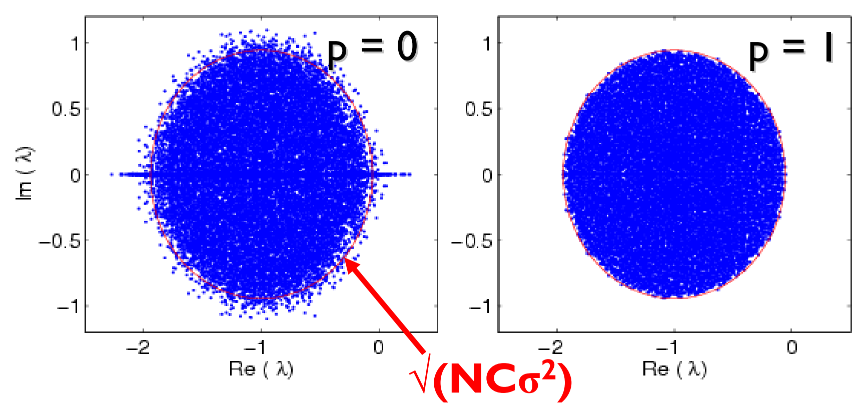
Regular vs Random Networks

$x = \sqrt{(N\,C\,\sigma^2)} - 1$,

$x_c \to 0$ as $N \to \infty$

The stability-instability transition occurs at the same critical value as random network …
but transition gets sharper with randomness

The eigenvalue plain



N = 1000, C = 0.021, σ = 0.206

Dynamics on Networks

Nodes may have non-trivial dynamics

Introduce explicit dynamics at the nodes :

$X(n+1) = F(X(n))$

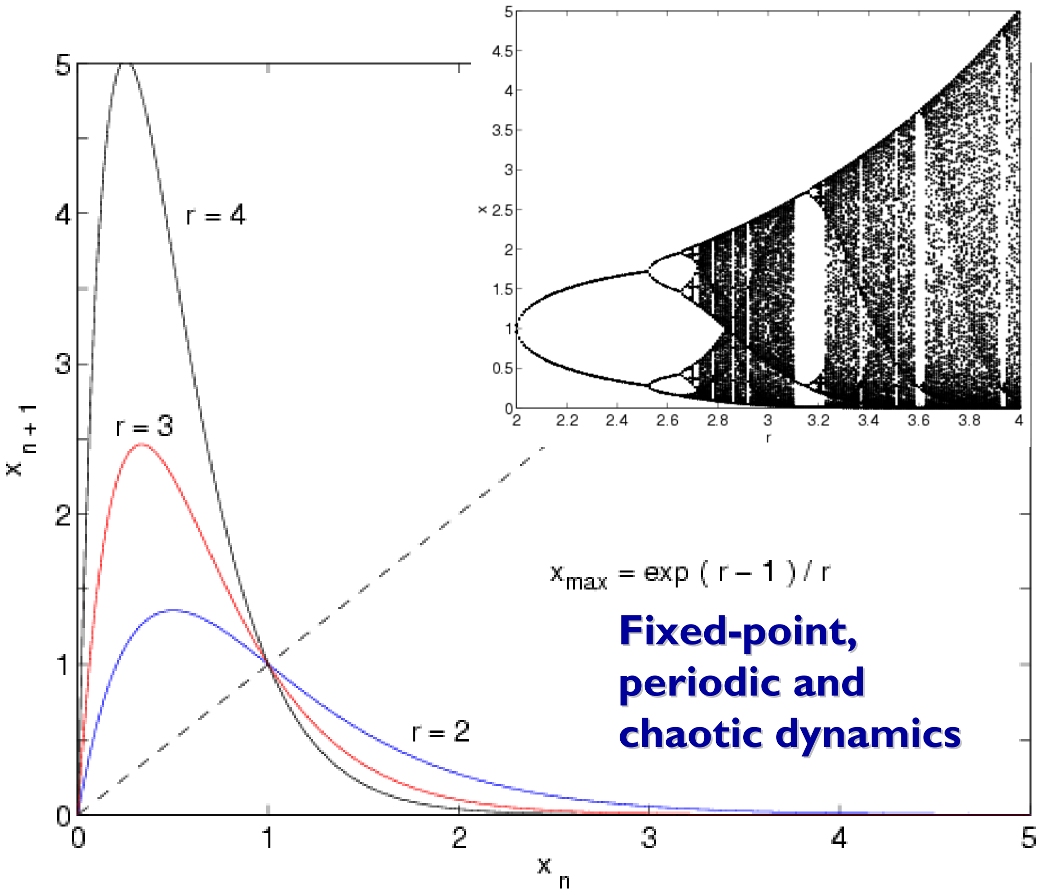
What happens when such nodes are coupled together to form a sparsely connected network ?

Activity at a node may stop as a result of interactions

Dynamics of network nodes : X (n+1) = F(X (n)) (Sinha & Sinha, 2005)

Example: Discrete exponential logistic growth model

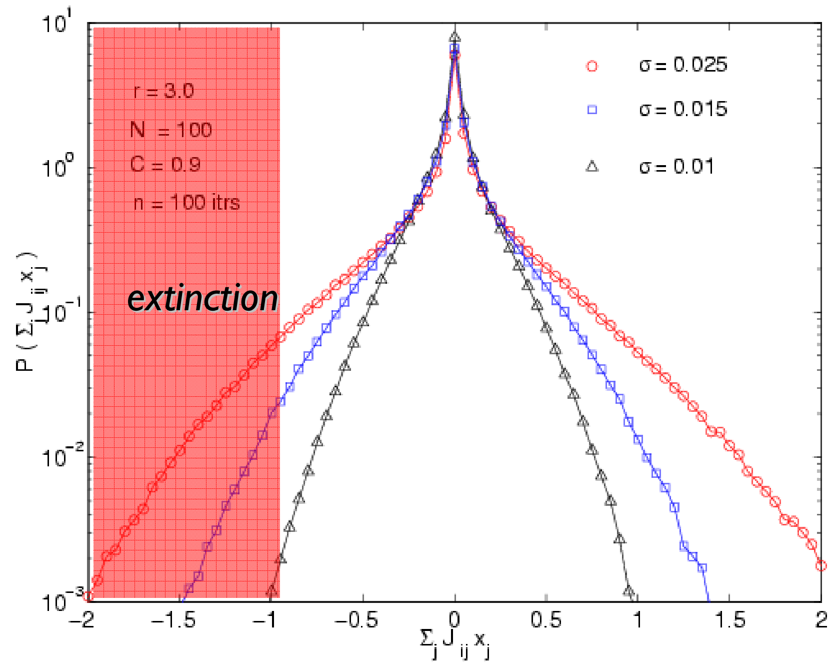
$$X_{n+1} = X_n \exp[r(1 - X_n)]$$



Fixed-point, periodic and chaotic dynamics

Network dynamics:

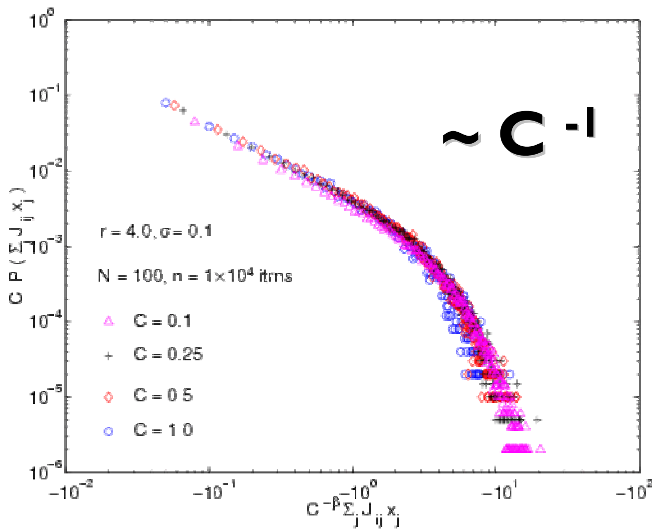
$$X_i(n+1) = F(X_i(n)[1+\Sigma J_{ij} X_j(n)])$$

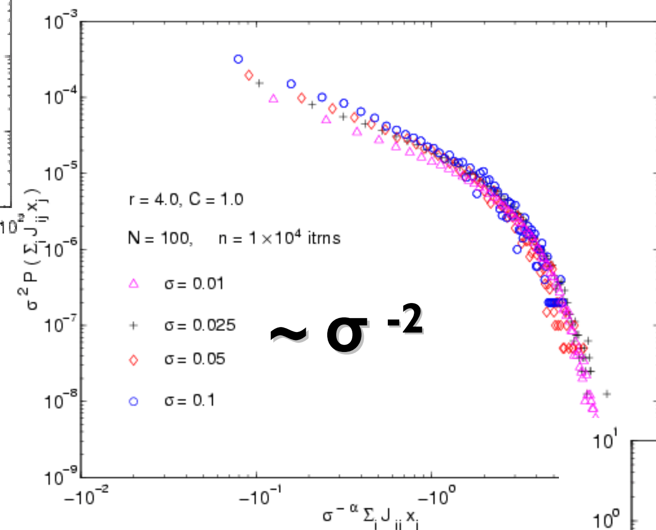


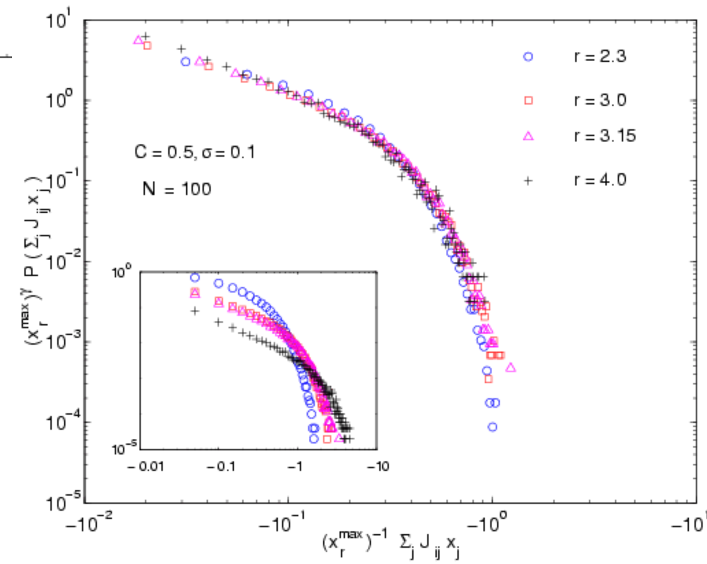
A node is extinct if $\Sigma J_{ij} X_j < -1$

Question: How many nodes survive asymptotically ?

Global stability of network ~ Probability of persistence of active nodes







Scaling of $\Sigma J_{ij} X_j$ distribution confirms the May-Wigner results

Probability of stability depends not on details of map dynamics …but on the extent of the attractor as ~ $[x_r^{max}]^{-3}$

Dynamical Systems $\rightarrow$ Finance

Can these results be used to understand the possibility of systemic risk in financial systems ?

Example: cascading defaults propagating along a network of inter-bank relations

For this we reconstruct and analyze a network of bilateral exposures between banks (US & European)

Data

Federal Deposit Insurance Corporation (FDIC) Call Report 2008 4th quarter

204 x 204 matrix : 202 European and US banks & financial institutions + aggregation of all Insurance companies + exposure to all banks outside Europe and USA considered together

Rows are gross negative fair value, i.e., market valued obligation from row bank to column bank

	JPMORGAN	BoA	Morgan Stanley	Deutsche Bank	Credit Suisse	CITIBANK
JPMORGAN	0	222.913336	138.374	129.276	109.635	105.287
BoA	221.42	0	124.155	116.339	104.958	100.795
Morgan Stanley	126.661	122.075497	0	70.7962	60.0402	57.6591
Deutsche Bank	118.784	114.4837229	71.0663	0	56.3063	54.0734
Credit Suisse	105.095	101.290621	62.8766	58.7422	0	47.842
CITIBANK	95.8675	92.39670082	57.3556	53.5843	45.4433	0

Units: Billions of dollars

Constructing the network of bilateral exposures

$$B = \begin{vmatrix} 0 & 222.91 & 138.37 & 129.28 & 109.64 & 105.29 & \ldots \\ 221.42 & 0 & 124.15 & 116.34 & 104.96 & 100.80 & \ldots \\ 126.66 & 122.08 & 0 & 70.80 & 60.04 & 57.66 & \ldots \\ 118.78 & 114.48 & 71.07 & 0 & 56.31 & 54.07 & \ldots \\ 105.10 & 101.29 & 62.88 & 58.74 & 0 & 47.84 & \ldots \\ 95.87 & 92.40 & 57.36 & 53.58 & 45.44 & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{vmatrix}$$

$C = B - B^T$: antisymmetric matrix of net amounts borrowed/lent

$C_{ij} > 0$ is net borrowing by node i from node j

$C_{ji} = -C_{ij}$ is corresponding amount lent by j to i

Considering only matrix of +ve values, i.e., $J_{ij} = C_{ij}$ if $C_{ij} > 0$, $J_{ij} = 0$ otherwise we obtain the weighted adjacency matrix for the directed network

$$J = \begin{vmatrix} 0 & 1.49 & 11.71 & 10.49 & 4.54 & 9.42 & \ldots \\ 0 & 0 & 2.08 & 1.86 & 3.67 & 8.40 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0.30 & \ldots \\ 0 & 0 & 0.27 & 0 & 0 & 0.49 & \ldots \\ 0 & 0 & 2.84 & 2.44 & 0 & 2.40 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{vmatrix}$$

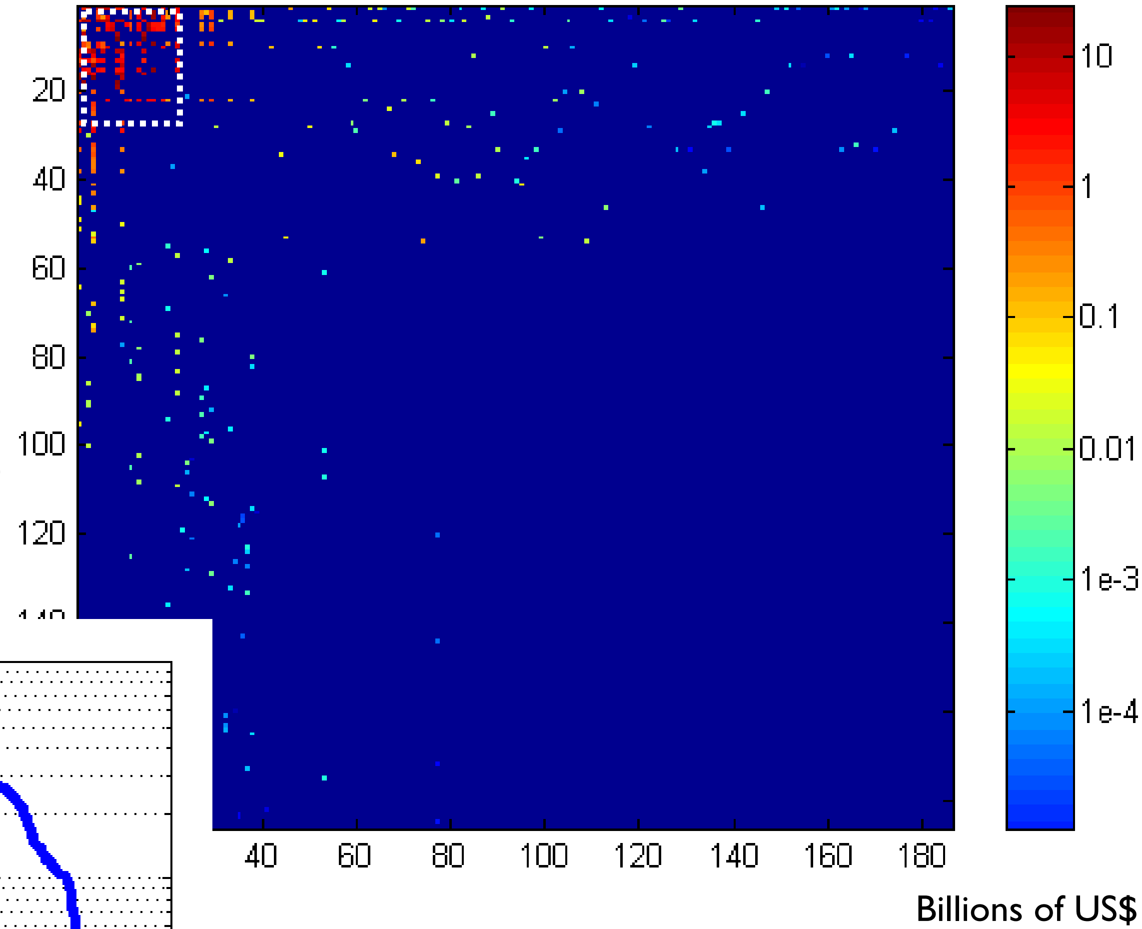
links point from the borrower to the lender (the direction of contagion)

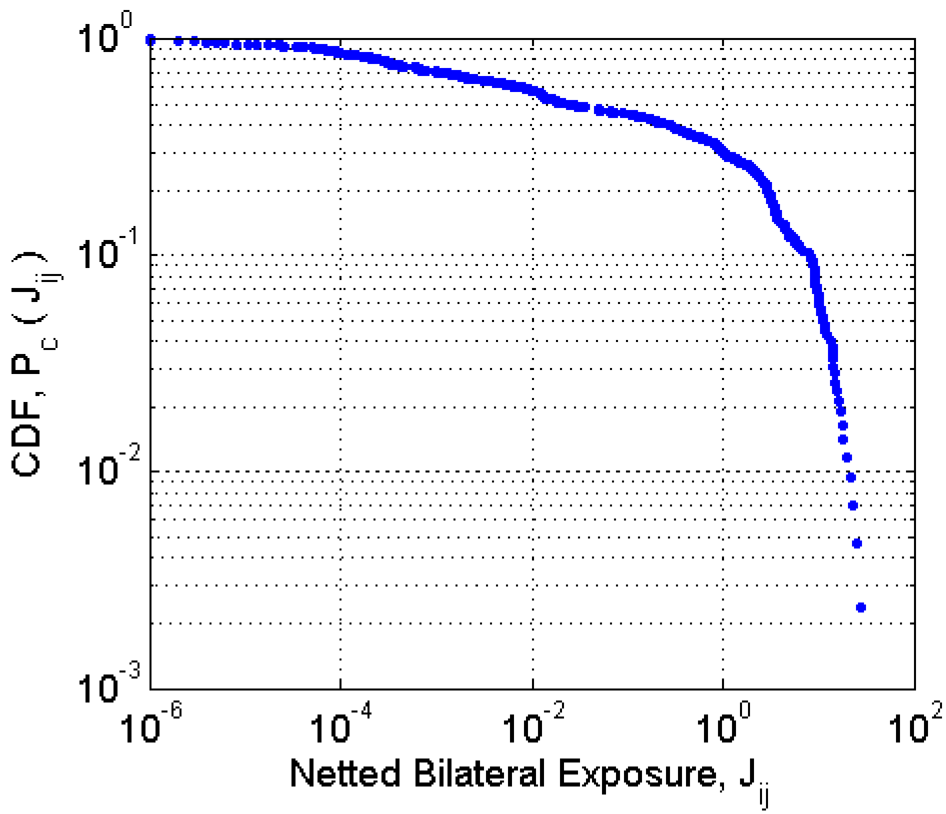
Weighted & directed network of bilateral exposures

16 nodes isolated

Largest connected component of 186 nodes considered

Financial institutions ordered acc to Tier I capital (decreasing order)





Apart from a group of strongly interacting nodes, the matrix is sparsely occupied: most nodes have few links to/from other nodes (majority of them with the strongly co-interacting group)

Distribution of netted bilateral exposures

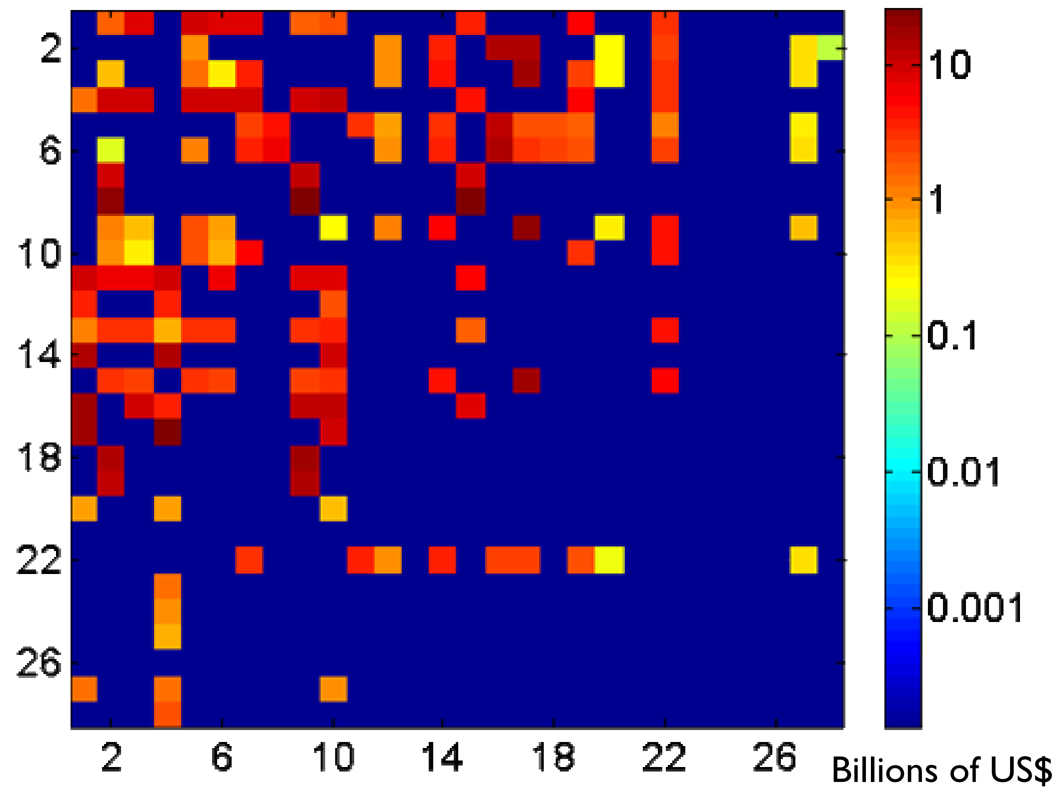
Weighted & directed network of bilateral exposures

16 nodes isolated

Largest connected component of 186 nodes considered

Financial institutions ordered acc to Tier I capital (decreasing order)

Interaction among Top 15% (28 banks)



Suggests **core-periphery organization**

• Few banks having high Tier I capital (*core*) have many & strong connections with each other
• Other banks (*periphery*) connect to one or few of these banks

Structural characterization of the network

PERSPECTIVE

20 JANUARY 2011 | VOL 469 | NATURE | 351

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Systemic risk in banking ecosystems

Andrew G. Haldane[1] & Robert M. May[2]

"From a public policy perspective, two topological features are the key. First, diversity across the financial system… homogeneity bred fragility. … Second, modularity within the financial system… Modular configurations prevent contagion infecting the whole network in the event of nodal failure."

Does the inter-bank network show evidence of

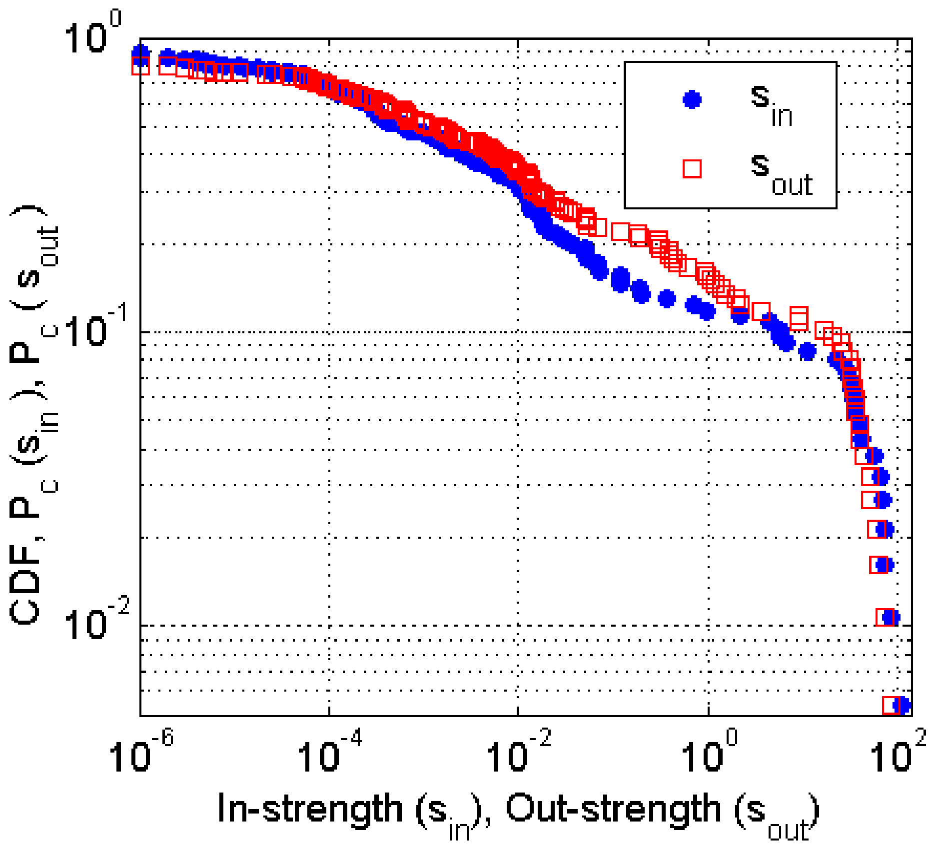
- Heterogeneity, e.g., in terms of strength, degree, tier-I capital, exposures, etc. ?
- Modularity ?

Strength distribution

In-strength $s_{in}\ (j) = \sum_i J_{ij}$: Total net amount lent by j to all nodes

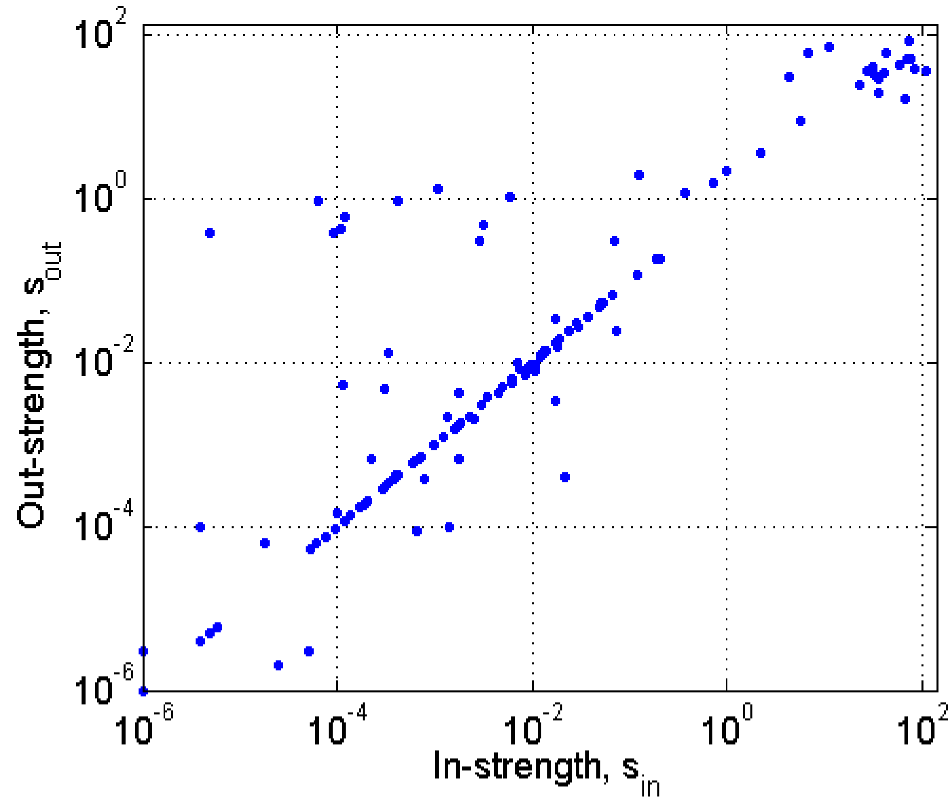
Out-strength $s_{out}\ (i) = \sum_j J_{ij}$: Total net amount borrowed by i from all nodes

21 nodes have no in-strength: <u>only</u> net borrowing
36 nodes have no out-strength: <u>only</u> net lending



Nodes with high in-strength also have high out-strength

Corr coeff r = 0.87 (p = 0)



Degree distribution

Unweighted adjacency matrix A:
A_{ij} = 1 if J_{ij} > 0; A_{ij}=0 if J_{ij}=0

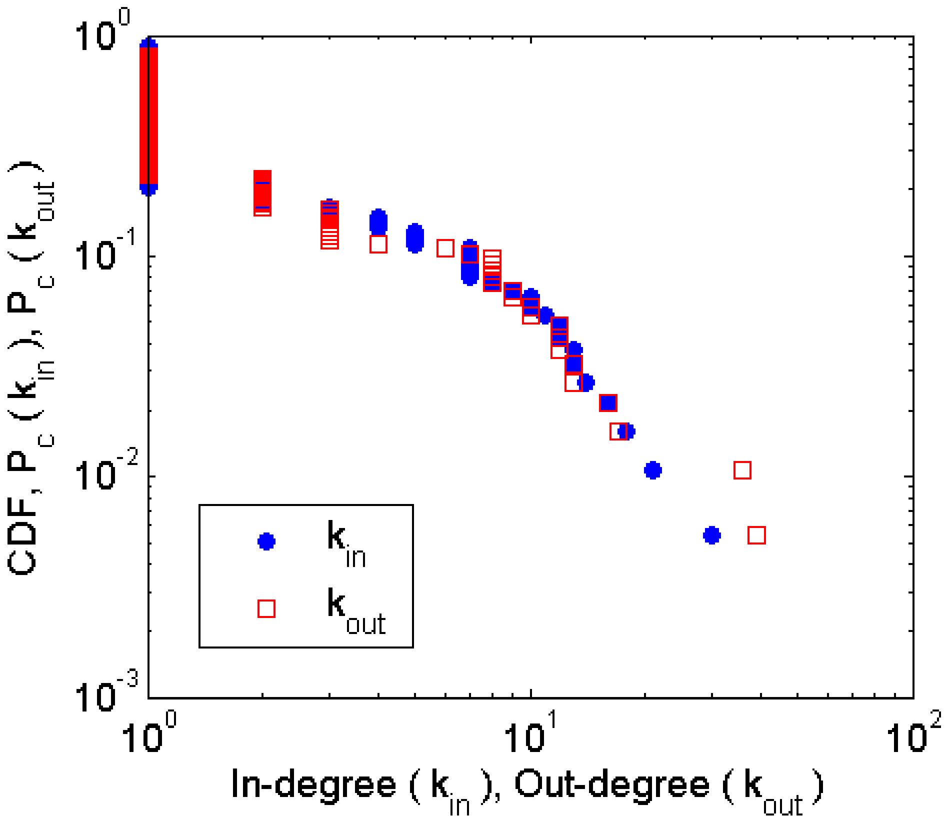
In-degree k_{in} (j)=$\sum_i A_{ij}$: Total number of nodes lent to by j

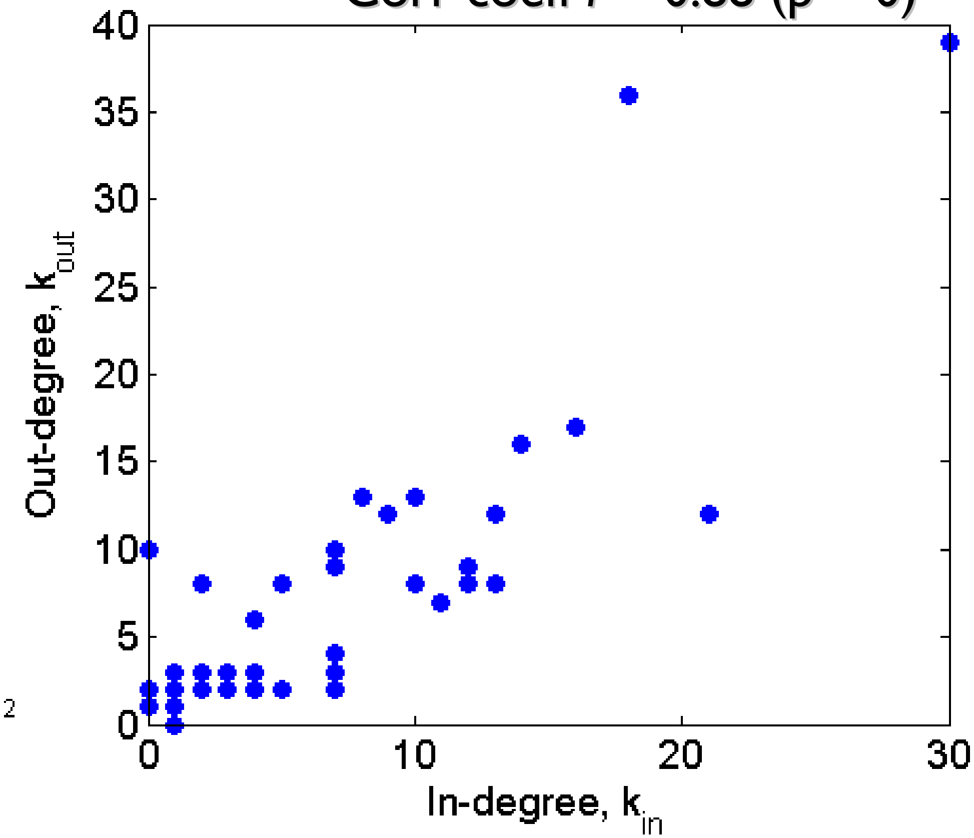
Out-degree k_{out} (i)=$\sum_j A_{ij}$: Total number of nodes i has borrowed from

129 nodes have both in-degree and out-degree
21 nodes have no in-degree & 36 nodes have no out-degree

Nodes with high in-degree also have high out-degree

Corr coeff r = 0.88 (p = 0)

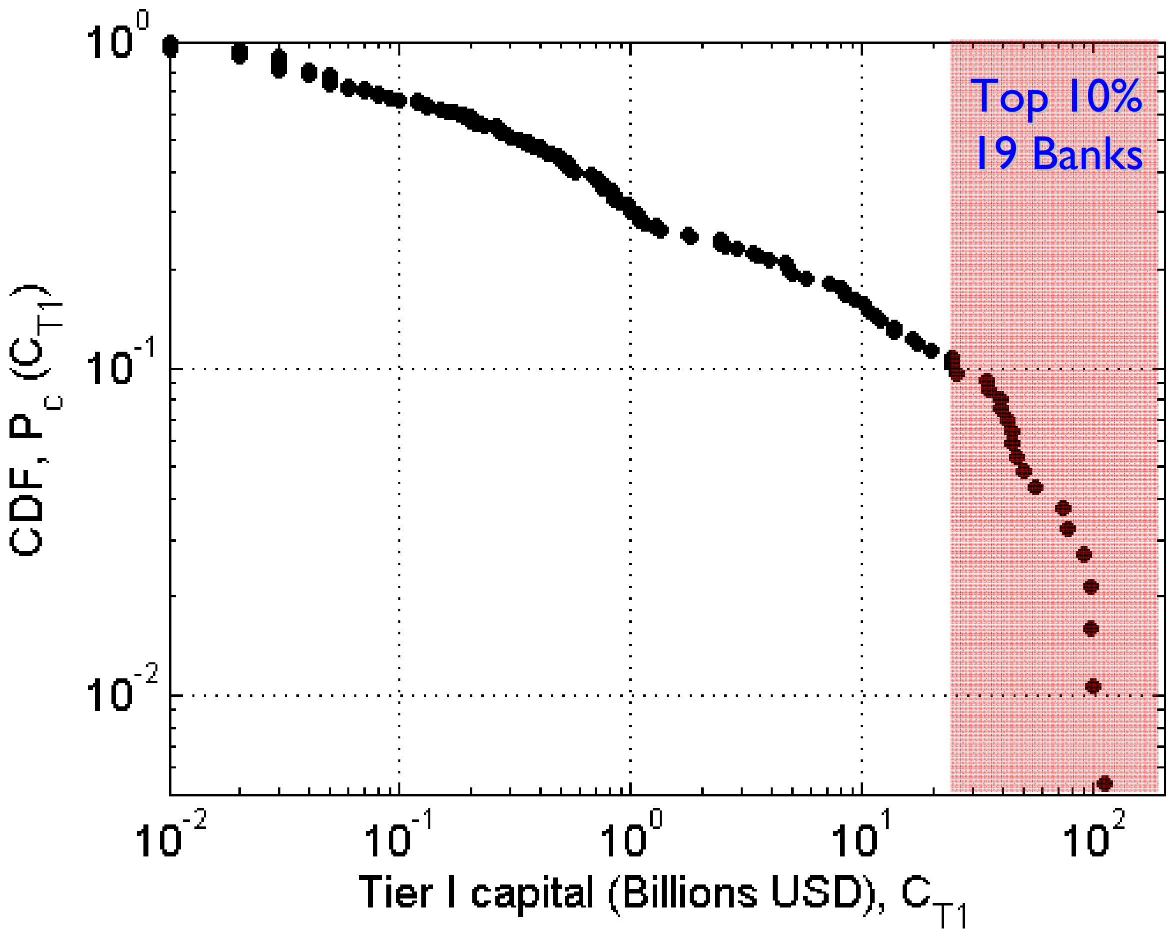


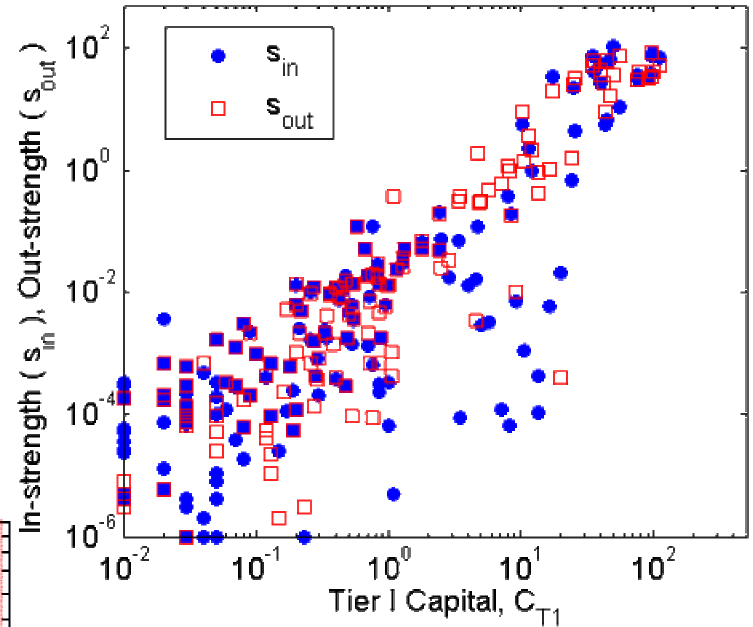


Tier I Capital

Measure of financial strength of a bank or financial institution used by regulators

Core capital consisting primarily of common stock & disclosed reserves



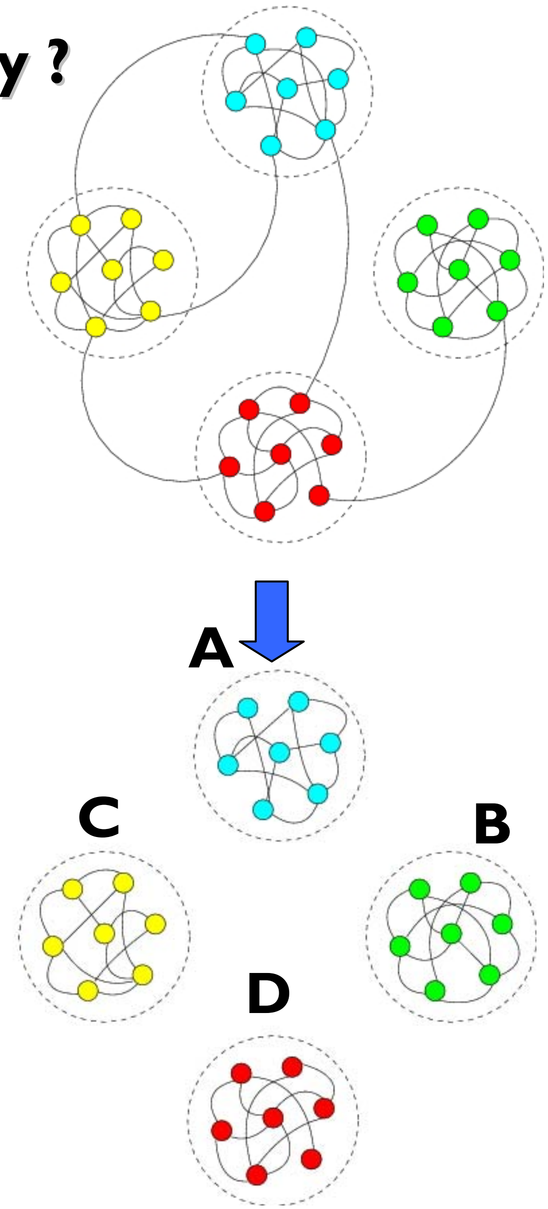


…
Morgan Stanley
Deutsche Bank
UniCredit
Lloyds
Barclays
BNP PARIBAS
J P Morgan
CITIBANK
RBS
BoA

In-strength, out-strength are strongly correlated to Tier I capital

Measuring modularity

How to quantify the degree of modularity ?



One suggested measure:

$$Q \equiv \frac{1}{2L}\sum_{i,j}\left[A_{ij} - \frac{k_i k_j}{2L}\right]\delta_{c_i c_j}$$

(Newman, EPJB, 2004)

A: Adjacency matrix
L : Total number of links
k_i : degree of *i*-th node
c_i : label of module to which *i*-th node belongs

For directed & weighted networks:

$$Q^W \equiv \frac{1}{L^W}\sum_{i,j}\left[W_{ij} - \frac{s_i^{\text{in}} s_j^{\text{out}}}{L^W}\right]\delta_{c_i c_j}$$

$(L^W = \sum_{i,j} W_{ij})$

W: Weight matrix
s_i : strength of *i*-th node

Modules determined through a generalization of the spectral method (Leicht & Newman, 2008)

Measuring modularity: explicit algorithm

We first define a modularity matrix B,

$$B_{ij} = W_{ij} - \frac{s_i^{\text{in}} s_j^{\text{out}}}{L^W}$$

To split the network into modules,

• the eigenvectors corresponding to the largest positive eigenvalue of the symmetric matrix $(B + B^T)$ is calculated

• the communities are assigned based on the sign of the elements of the eigenvector.

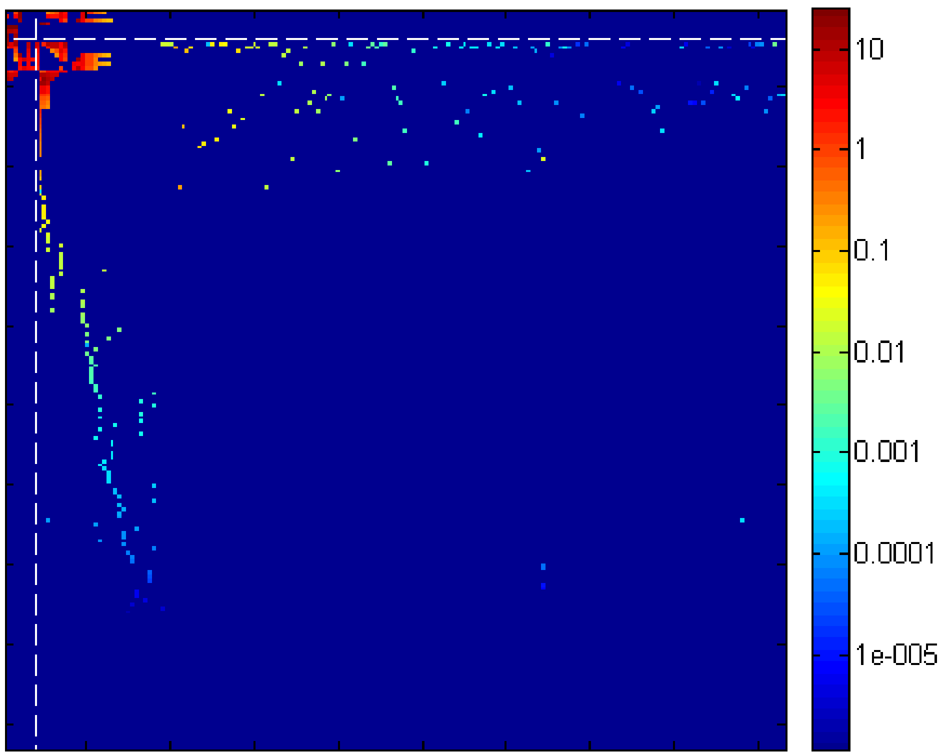
• This divides the network into two parts, which is refined further by exchanging the module membership of each node in turn if it results in an increase in the modularity.

• The process is then repeated by splitting each of the two divisions into further subdivisions.

• This recursive bisection of the network is carried out until no further increase of Q is possible.

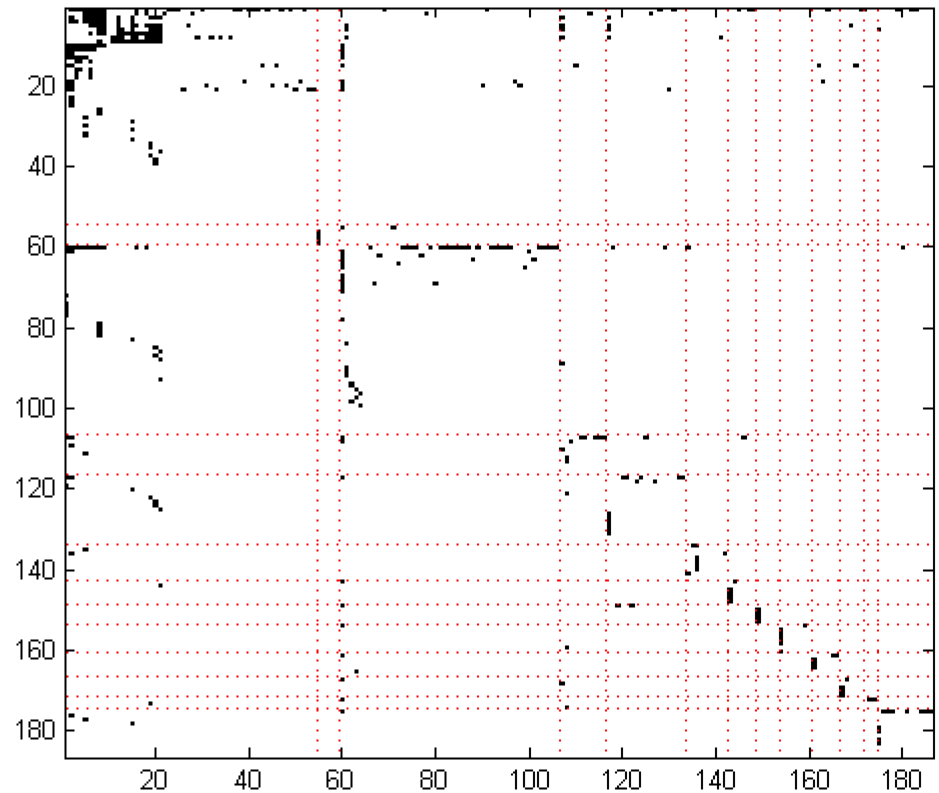
Measuring modularity: weighted & unweighted matrices

$Q^W = 0.14$



2 modules: A has 8 nodes, B has 178 nodes

$Q = 0.38$

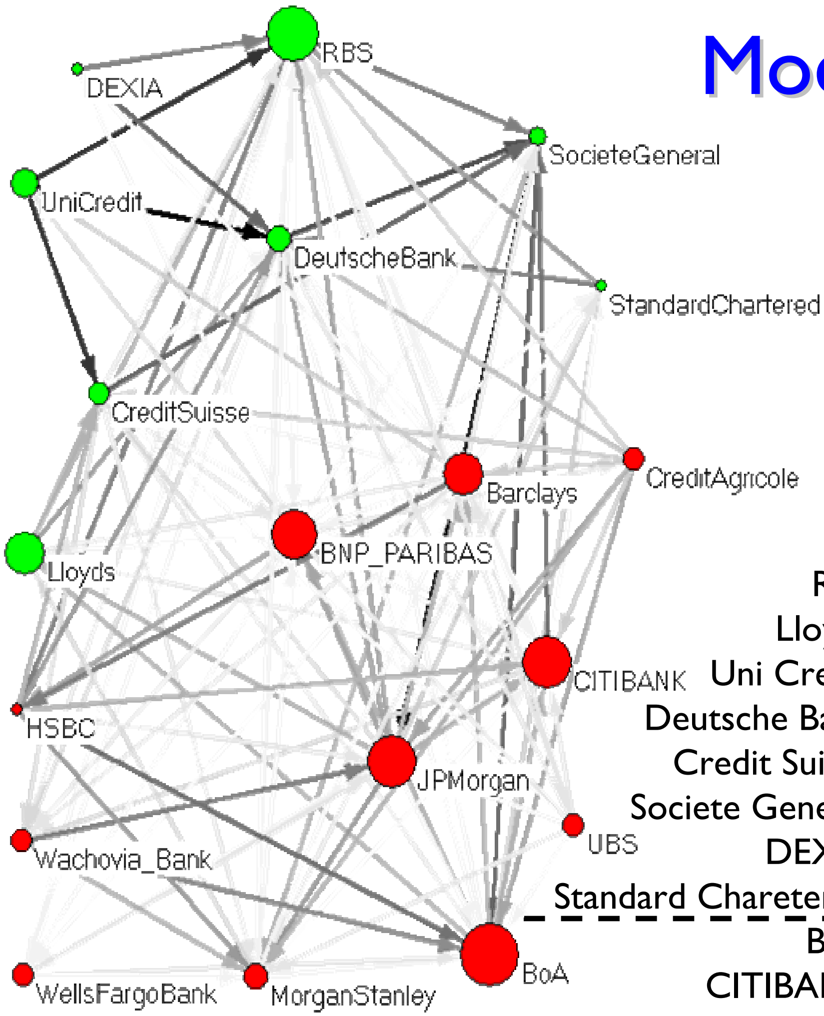


13 modules

Largest : 54 nodes - contains top 10% of nodes in terms of Tier I capital except one (JPMorgan)

Smallest : 3 nodes

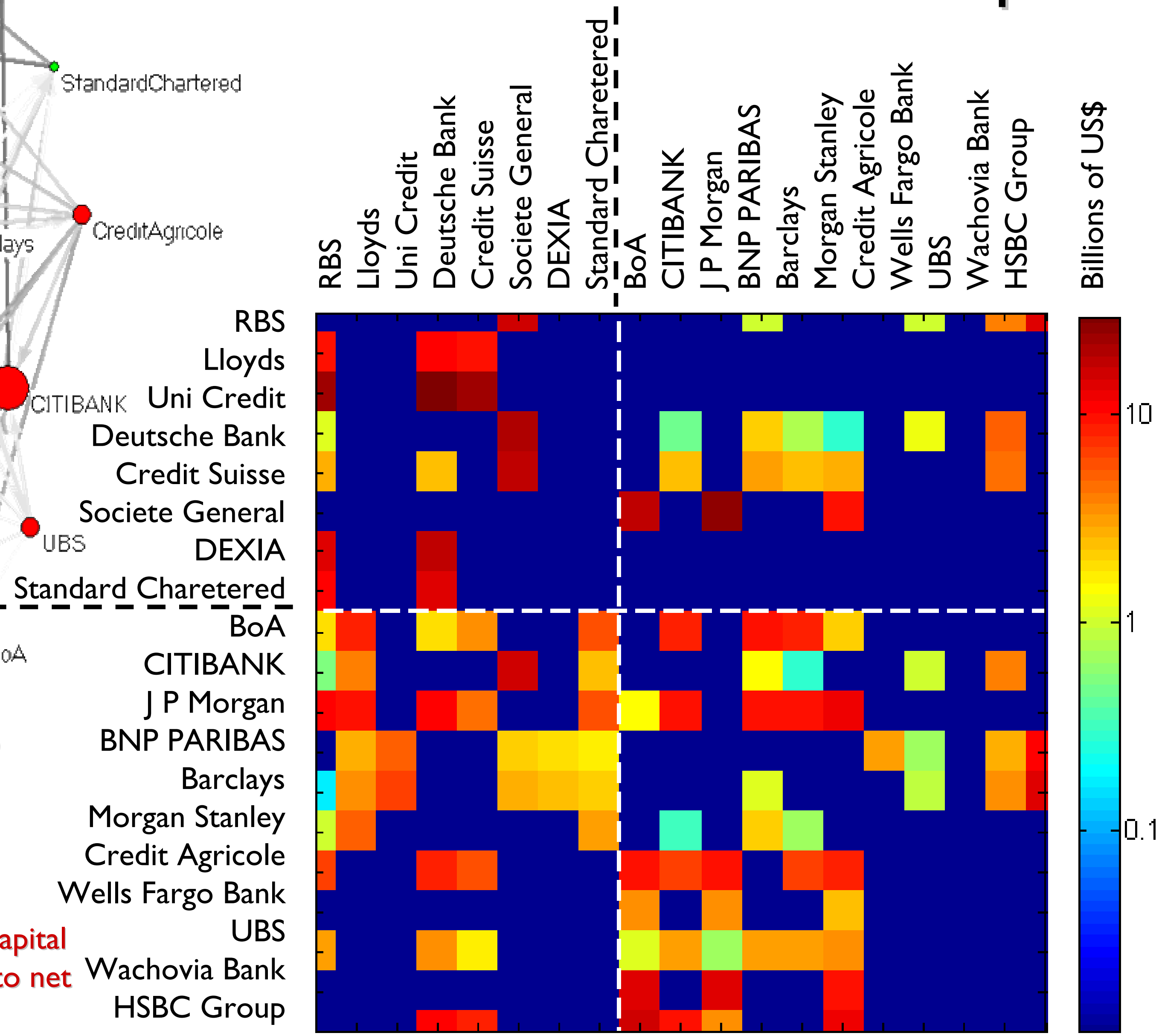
Modularity: Top 10 % of Banks in terms of Tier-I Capital



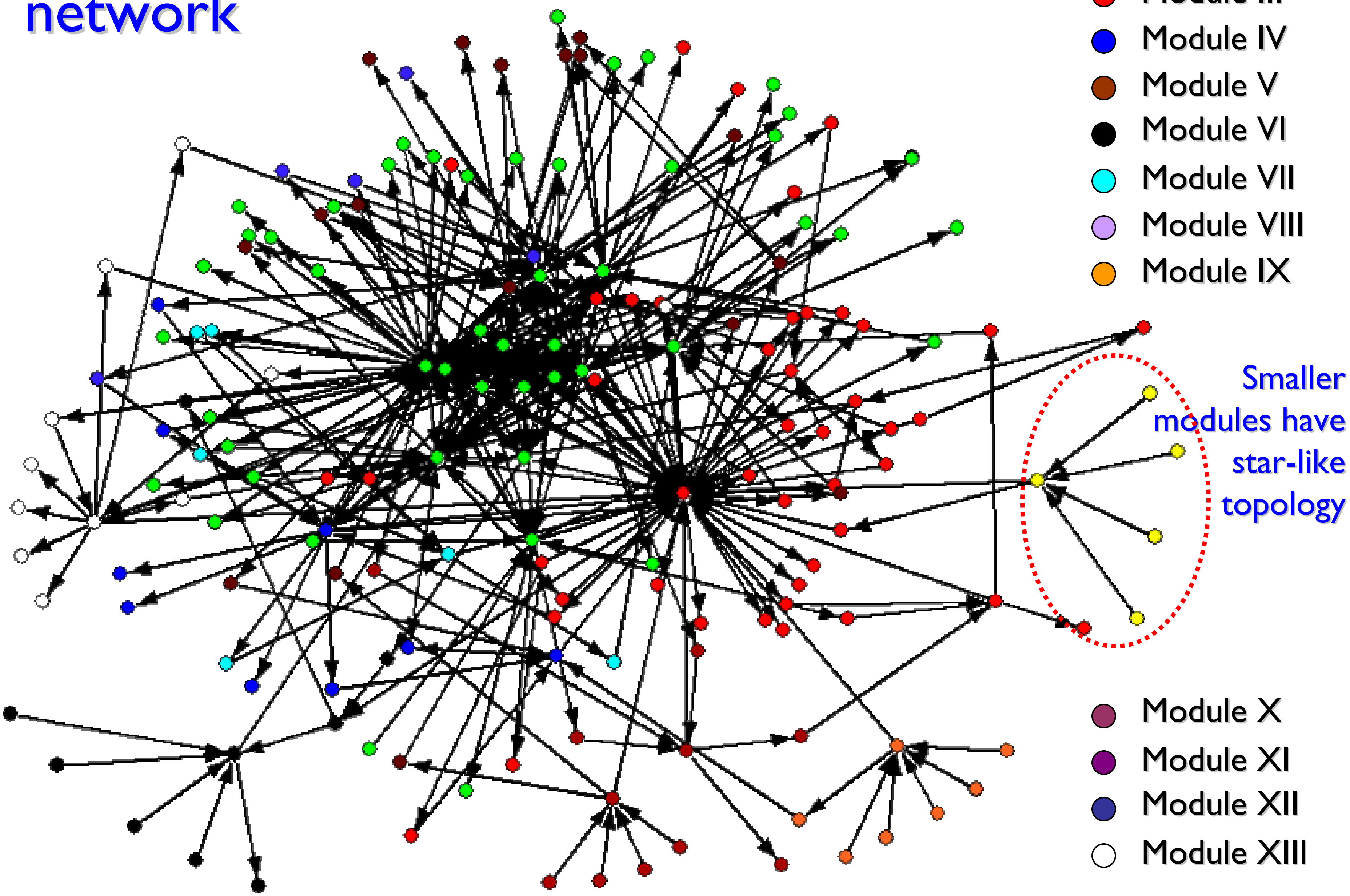
Module A (8 nodes)

Part of Module B (11 of 178 nodes)

Node size proportional to Tier-I capital
Link grayscale color proportional to net bilateral exposure



Modular organization of the unweighted network



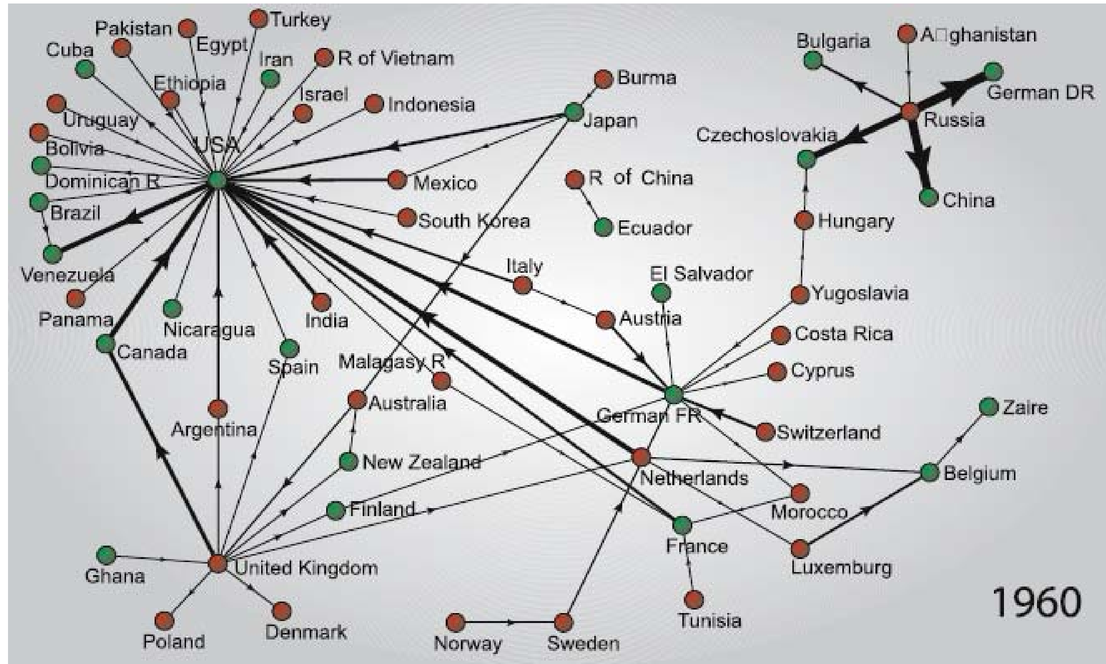
How to identify critical nodes using structural measures ?

- Backbone extraction
- Eigenvector Centrality
- k-Core analysis

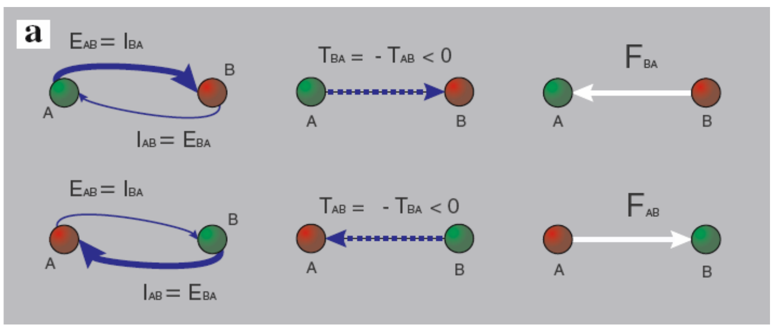
Patterns of dominant flow in a network: Backbone extraction

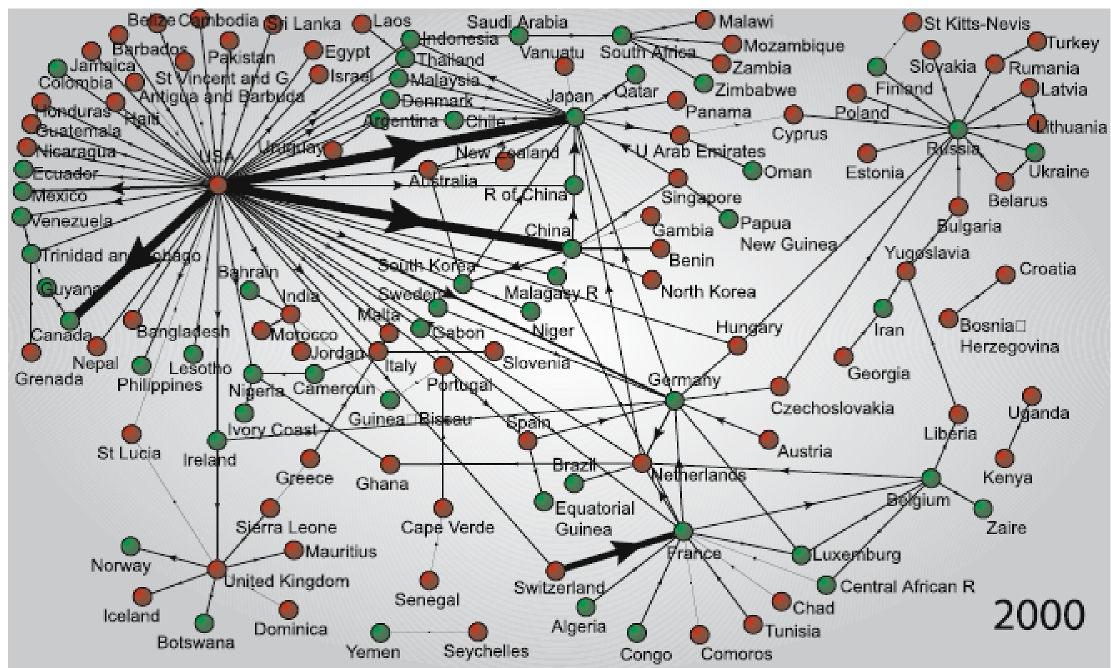
Example: International Trade Network

Serrano et al, JEIC 2007



Serrano et al, JEIC 2007

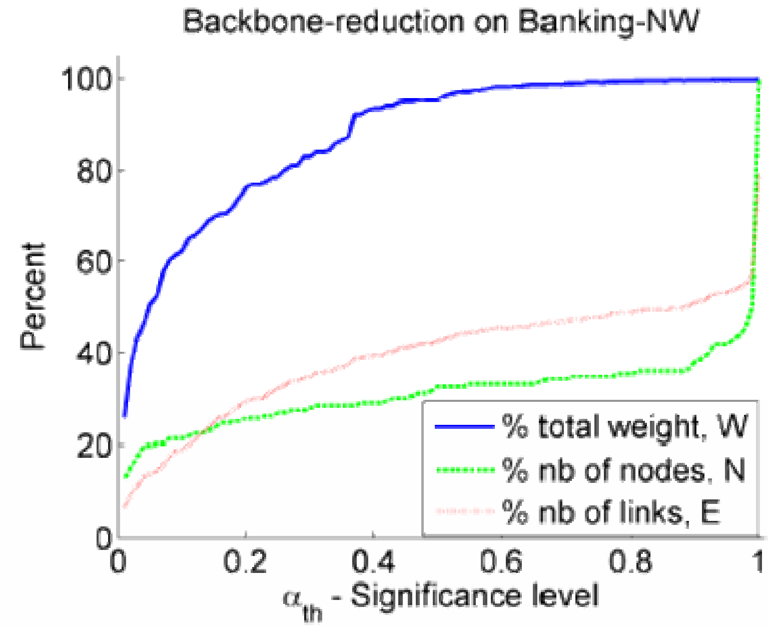


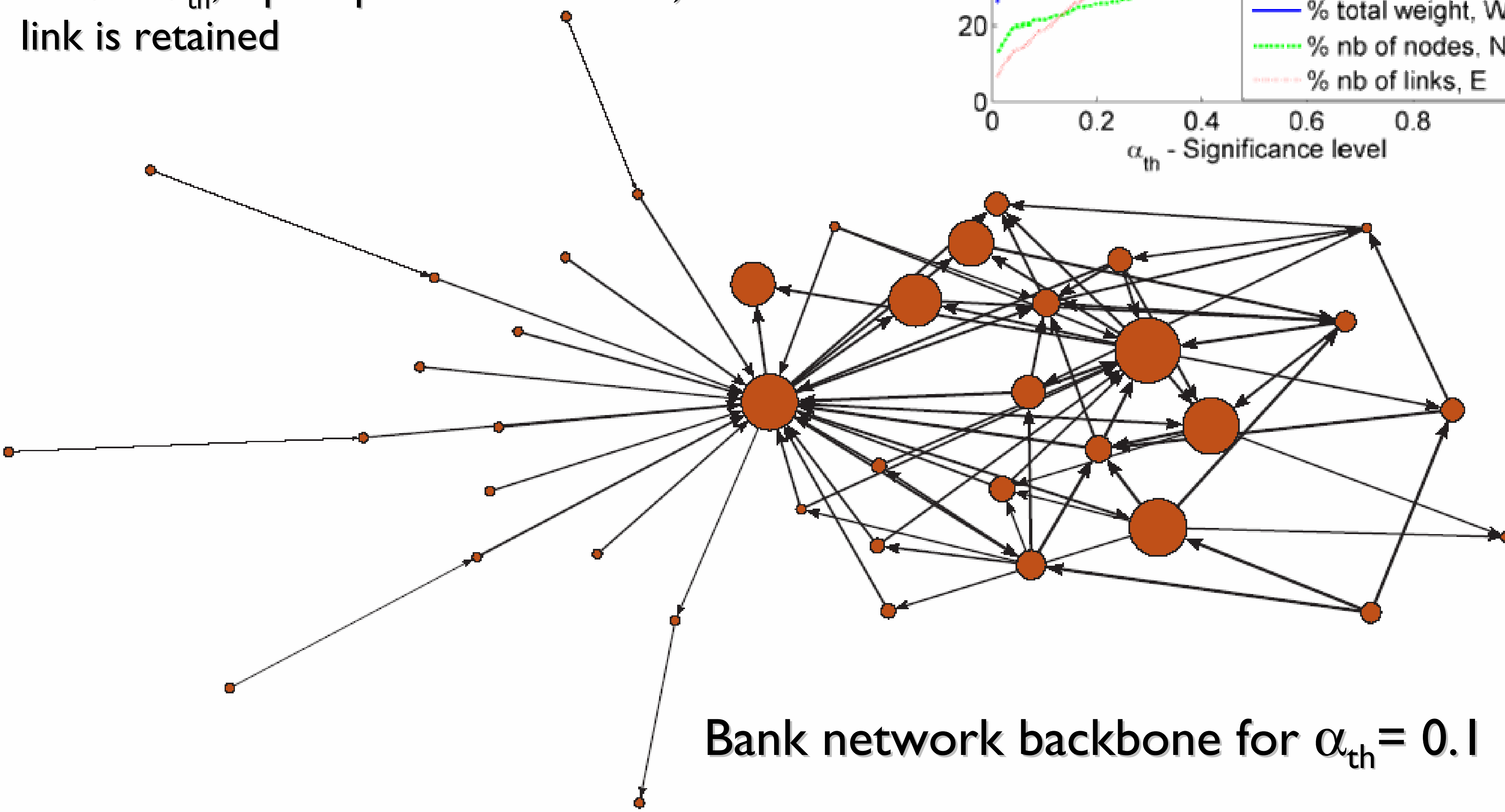


Idea: To reduce the number of links by retaining only the "most important" ones obtained by comparison with a null model

Identifying important nodes by Backbone reduction

- The probability α of each link to occur in a random network is computed
- If $\alpha < \alpha_{th}$, a pre-specified threshold, the link is retained





Bank network backbone for α_{th}= 0.1

Eigenvector Centrality

A variant is used in the Page Ranking algorithm used by Google

Centrality: a measure of the relative importance of a node within a network

Eigenvector centrality

Based on the idea that the centrality x_i of a node should be proportional to the sum of the centralities of the neighbors

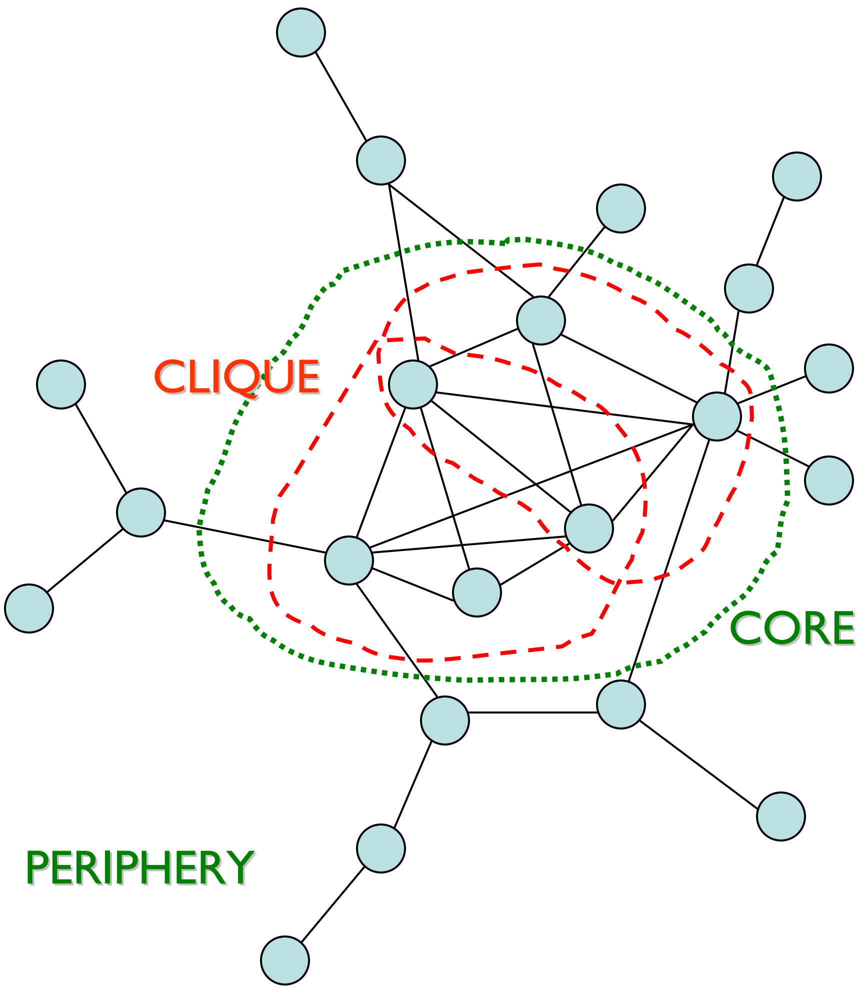
$$x_i = \frac{1}{\lambda} \sum_{j=1}^{n} A_{i,j} x_j$$

λ is a constant

The vector **x,** containing centrality values of all nodes is obtained by solving the eigenvalue equation $\lambda \mathbf{x} = A\mathbf{x}$
and selecting the eigenvector corresponding to the largest eigenvalue

Positive values for the centralities are guaranteed by Perron-Frobenius thm:
The eigenvector of the largest eigenvalue of a non-negative matrix A has only positive components.

Core-periphery organization



Core characterized by a central group of nodes that are densely/strongly connected to each other as well as to other nodes (in the periphery) which have very few links

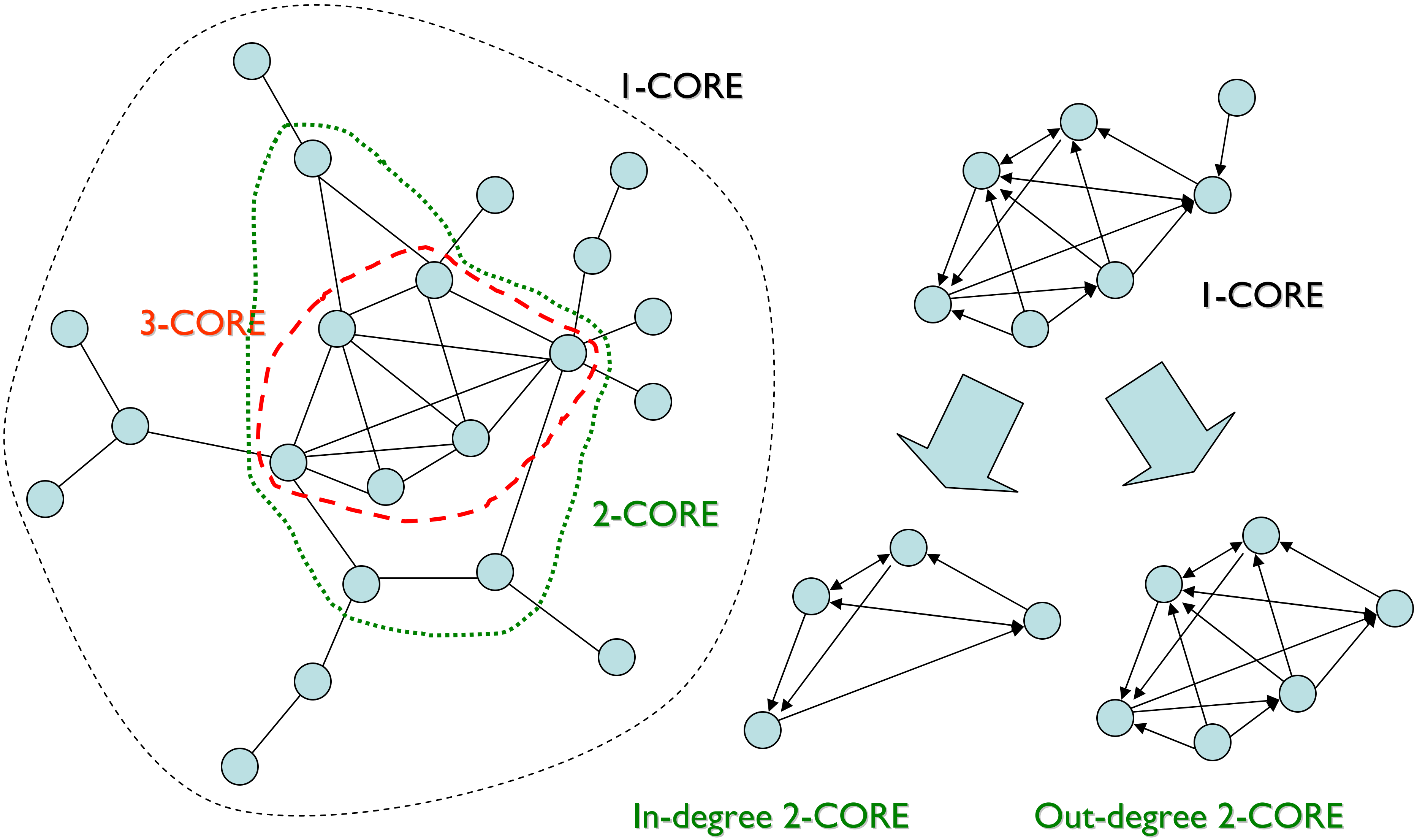
k-core is the largest subnetwork that contains only nodes with degree $\geq$ k.

The core number of a node is the largest k-value for which the node is still part of k-core.

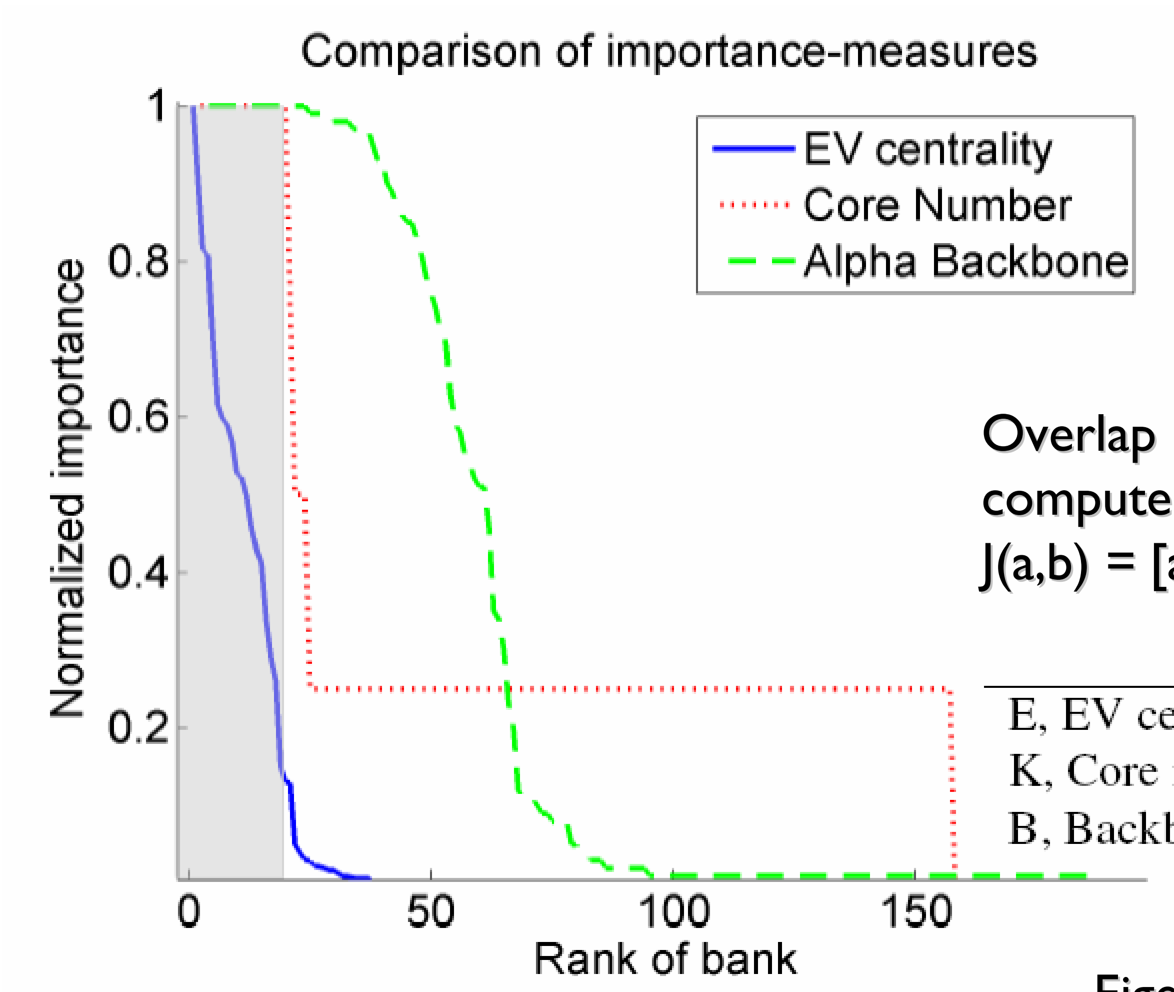
k-core is obtained computationally by recursively removing all nodes with degree < k.

k-Core: Undirected & Directed

can be generalized to weighted networks (s-core)



Comparing results



Shaded region marks top 10 % for each measure

The most important nodes (e.g., the top 10%) according to the three different measures are <u>not</u> identical

Overlap between the three measures computed by Jaccard index

$J(a,b) = [a \cap b] / [a \cup b]$

	E	K	B
E, EV centrality	1	0.63	0.39
K, Core number	0.63	1	0.38
B, Backbone-reduction	0.39	0.38	1

Eigenvector centrality is the best indicator in terms of structural measures for local and global stability

Dynamical model for failure propagation

Assumption: Let state of each node (bank/financial institution) S_i at any time be described by a binary variable

- S_i (t) = 1: "Healthy" or Solvent state
- S_i (t) = 0: "Sick" or Defaulted state [$\Rightarrow S_i(\tau) = 0$ for all $\tau > t$]

The netted bilateral exposures J_{ij} (how much i owes j) describes interactions between nodes

In the event of a node defaulting, all its creditors lose the net sum lent J_{ij}

If the loss of any neighboring node > critical fraction q of Tier I capital, the neighboring nodes also defaults

Dynamical evolution of the states occurs as:

$S_i(t+1) = 1 - F\{\sum J_{ij}[1 - S_j(t)] + q \,.\, C_{T1}(i)\}$

where $F(z)=1$ if $z > 0$;$F(z) = 0$ otherwise

Condition of Stability

Dynamical evolution of state of a bank:

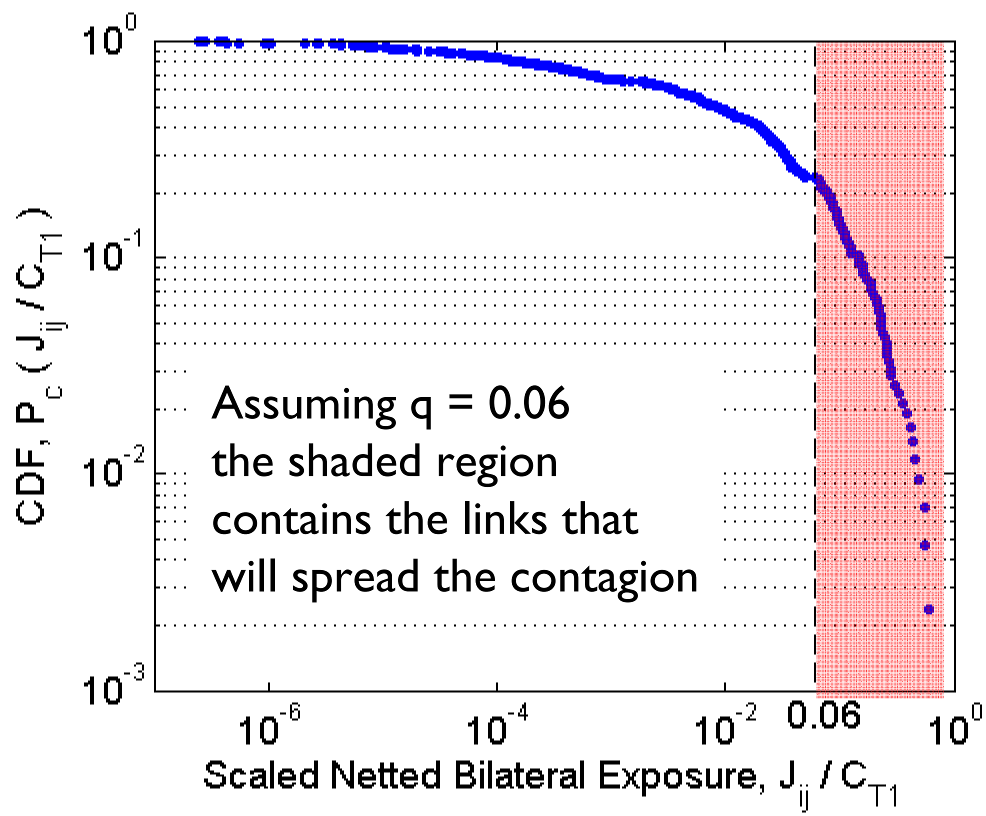
$S_i\,(t+1) = 1 - F\,\{ \sum_j J_{ij}\,[1 - S_j\,(t)] + q \,.\, C_{T1} \}$

where F(z)=1 if z > 0 ;F(z) = 0 otherwise

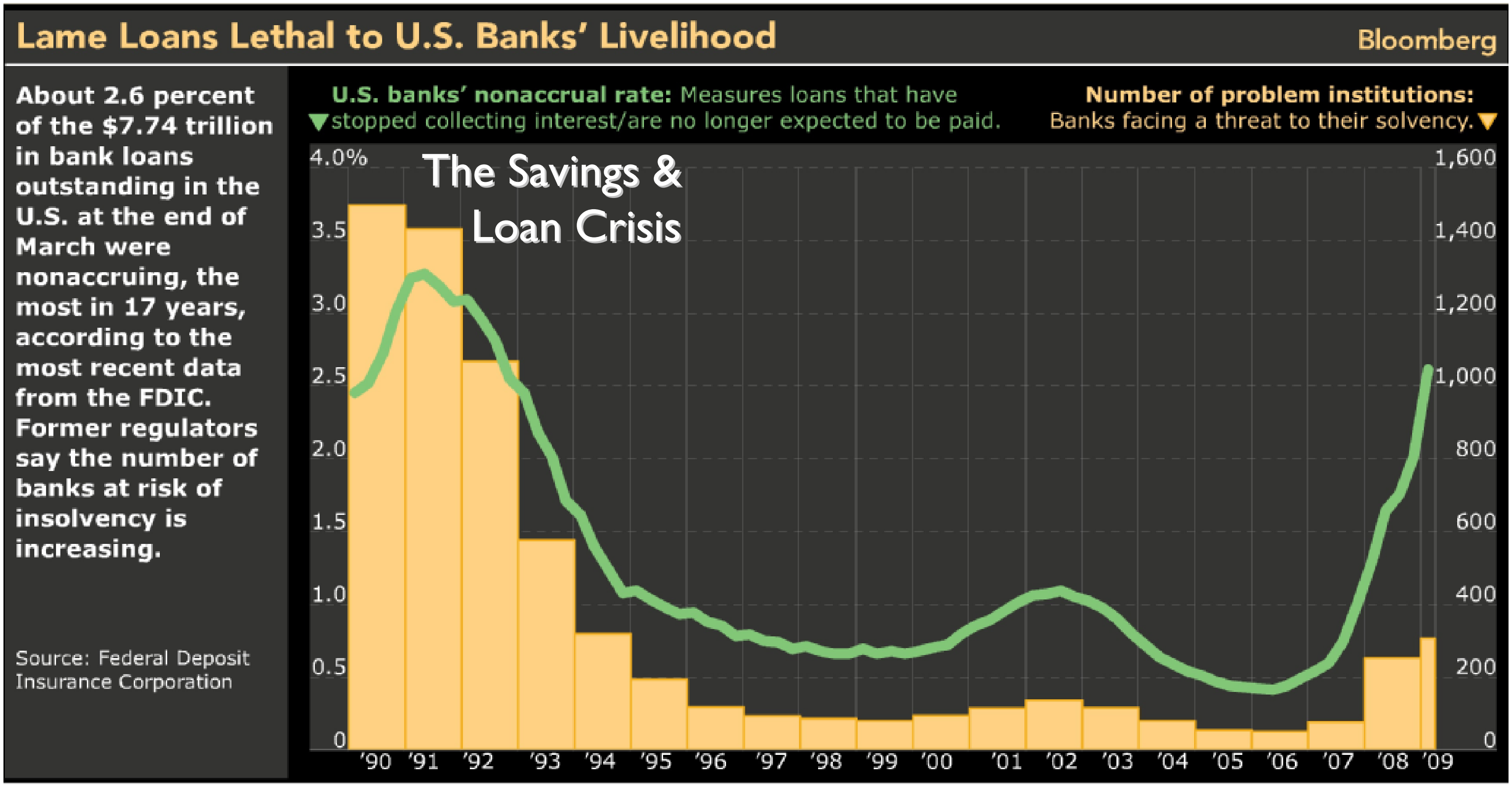
If only a single node *j* defaults, the perturbation will propagate to its neighbors *i* only if

$$J_{ji} / C_{T1}(i) > q$$

Thus, the distribution of netted bilateral exposures scaled by Tier I capital determines the stability of nodes w.r.t. small local perturbations



What determines the critical fraction *q* ?



"Toxic Loans Topping 5% May Push 150 Banks to Point of No Return"

Nonperforming loans: commercial and consumer debt that has stopped collecting interest or will no longer be paid in full.

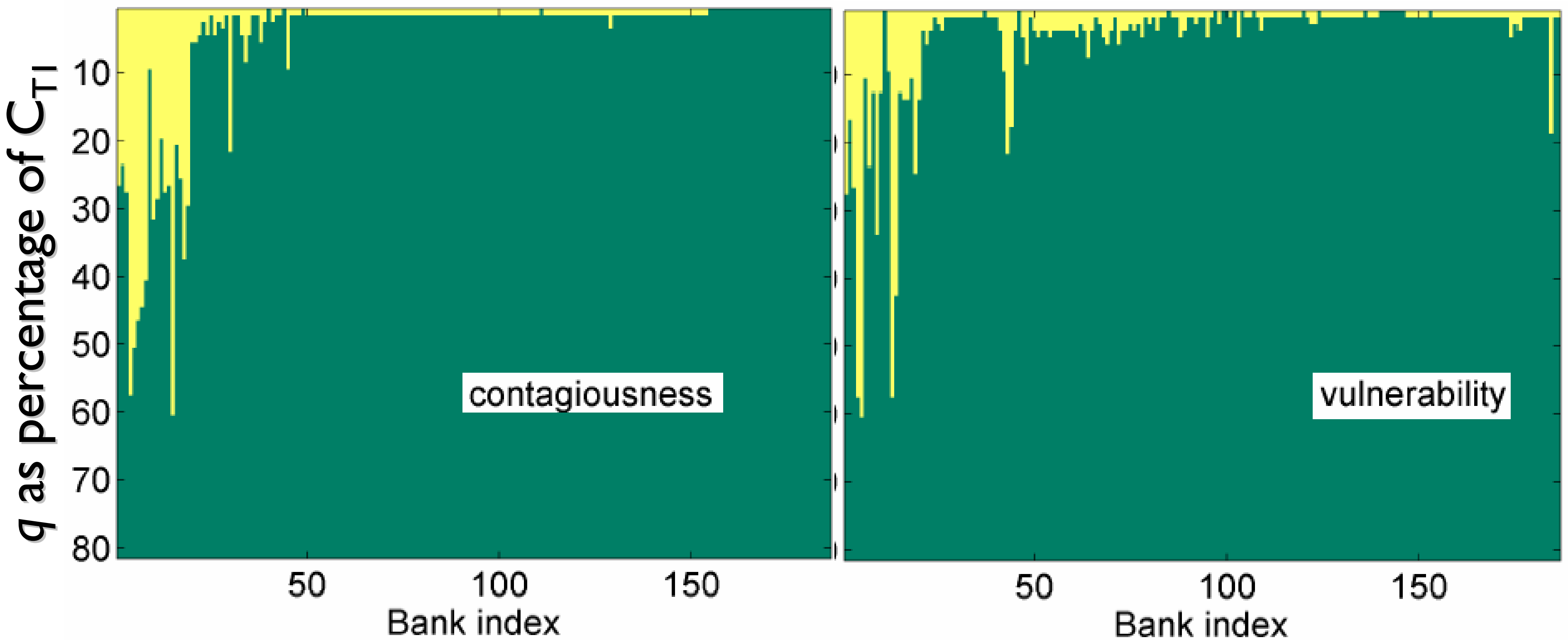
According to regulators, nonperforming loans > 5% of [a bank's] holdings… can wipe out a bank's equity and threaten its survival.

Local Stability

Calculated without explicit time-evolution

Contagiousness: A node is contagious at a given value of *q* if there is at least one neighbor that will fail if the node fails.
Vulnerability: A node is vulnerable at a given value of *q* if there is at least one neighbor whose default can lead to the default of the node.

For *q* = 0, almost all nodes are contagious and vulnerable



As *q* increases, nodes change from contagious/vulnerable (yellow) to healthy (green)

Global Stability

Calculated by explicit time-evolution

When a node fails, it can (depending on *q*) initiate a **sequential cascade of failure events** in the network

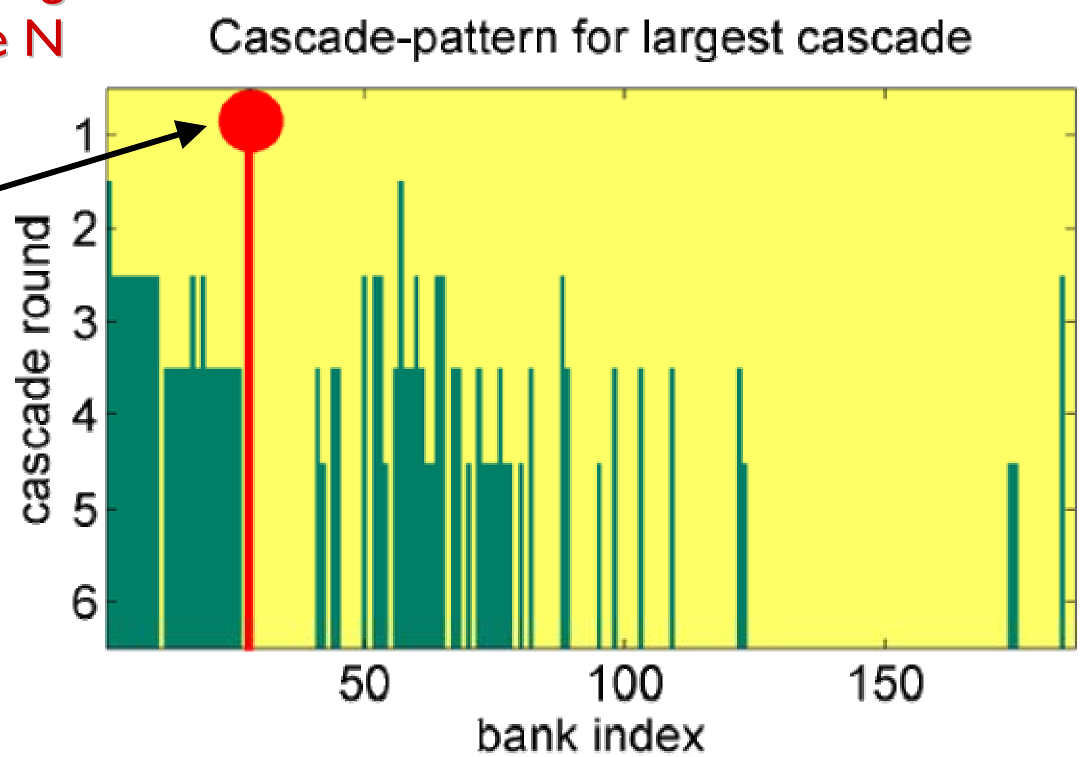
Global stability of the network is inversely related to the mean size of the failure avalanche triggered by a single node

Measured by g = [<Avalanche size>]$^{-1}$ with average taken over perturbation of each node of the network

If no propagation of failure occurs, g = 1

If every node in a network of size N fails, then g = 1/N

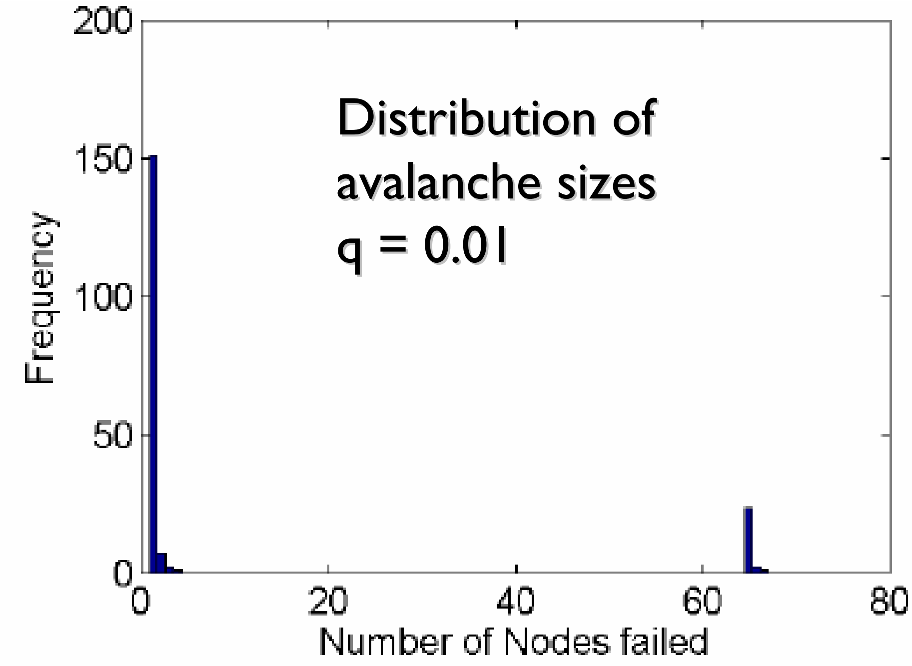
Initial perturbation: single default event



Propagation of failures for q = 0.01 in which a total of 67 banks default after the initial default of a single bank

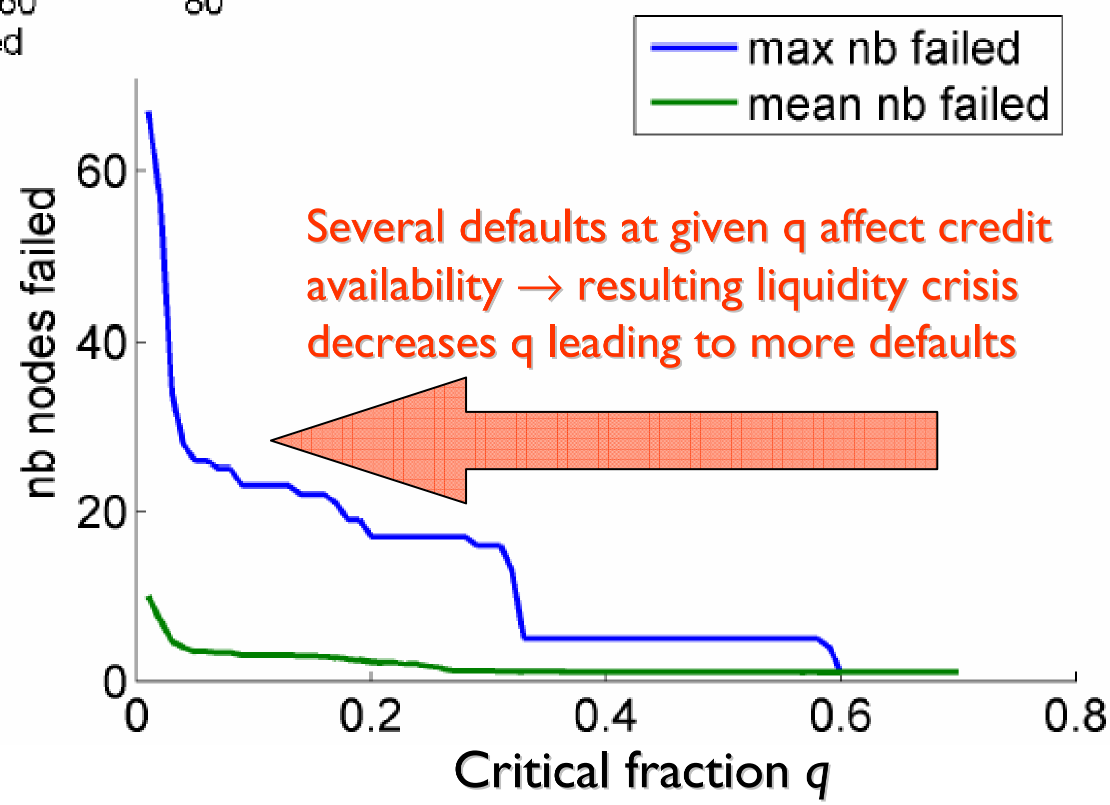
The disturbance affects the entire core of strongly connected banks

Global Failure & Liquidity Crisis



Bimodal distribution:
Cascades follow "many or nothing" behavior – either the failures don't spread at all from the initially perturbed node or many nodes dafault

Decreasing q increases the number of nodes affected in a failure cascade

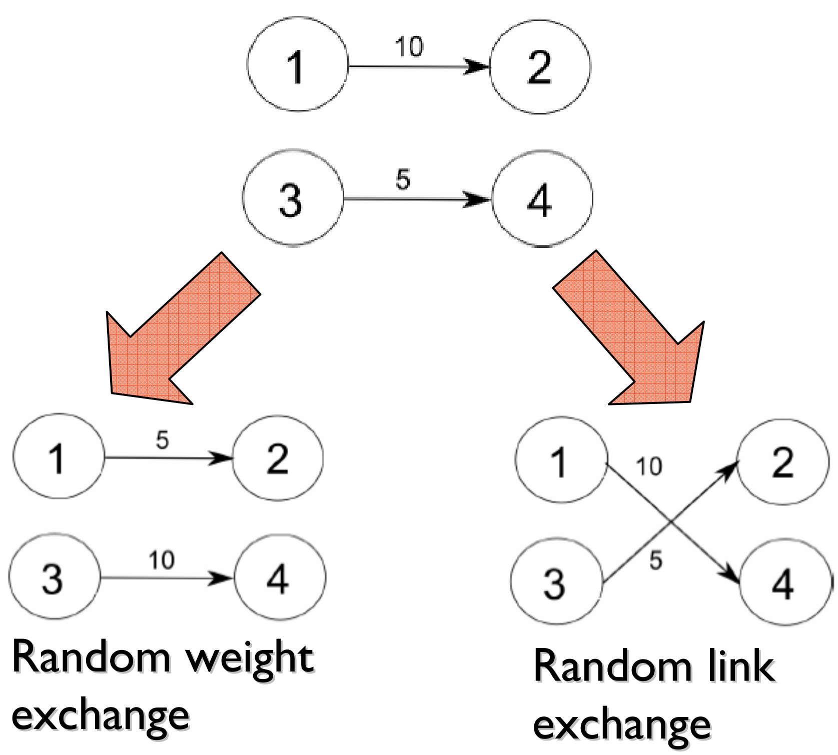


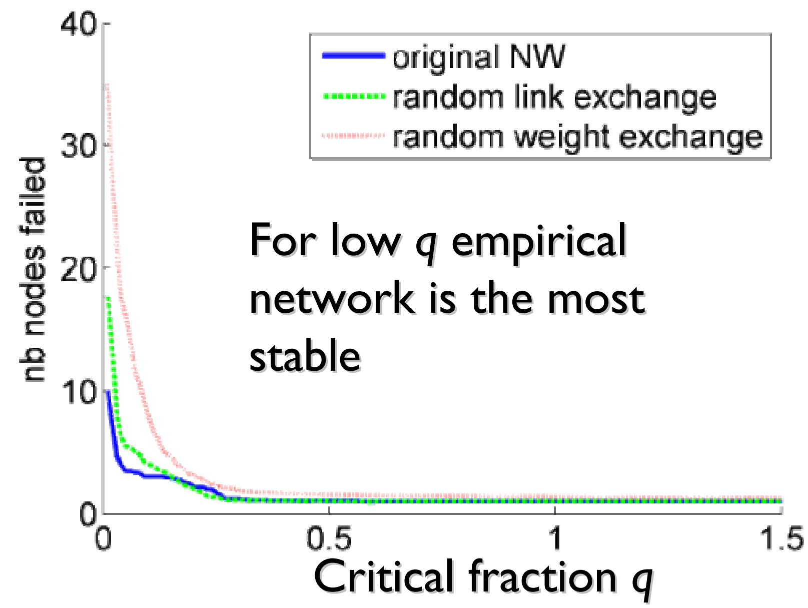
Q. But how can large-scale or global failure occur (affecting almost all nodes) ?

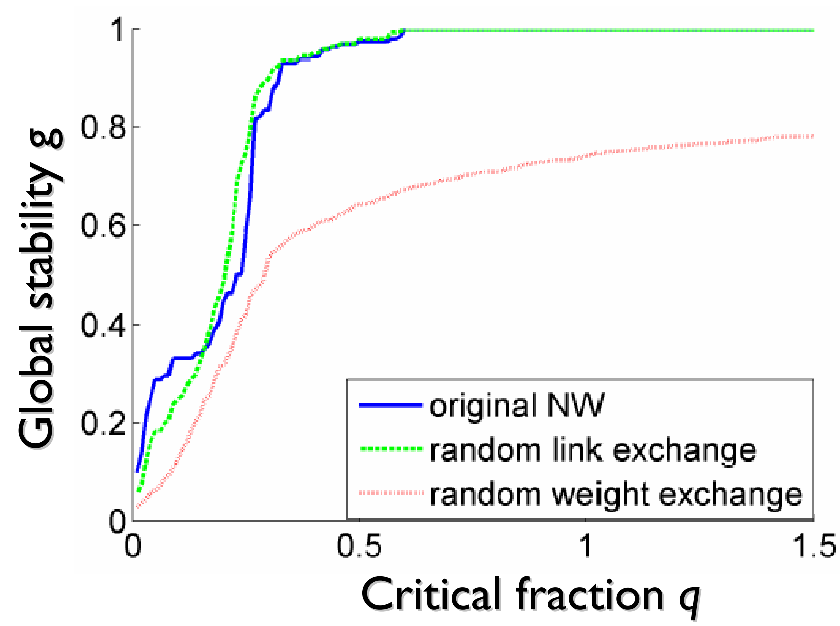
Ans: A feedback process involving liquidity crisis triggered by failures – which in turn causes more failures

Network Topology & Global Stability

To understand how the topological features of the inter-bank network affects the global stability, we consider ensemble of randomized networks







Predicting Dynamical Stability from Topology

Can the local stability (contagiousness and vulnerability) of banks and their impact on global stability (magnitude of cascade they can cause) be predicted solely from topological information about the network ?

Compute the overlap between the most important 10% banks identified by three structural measures and three dynamical measures using Jaccard index

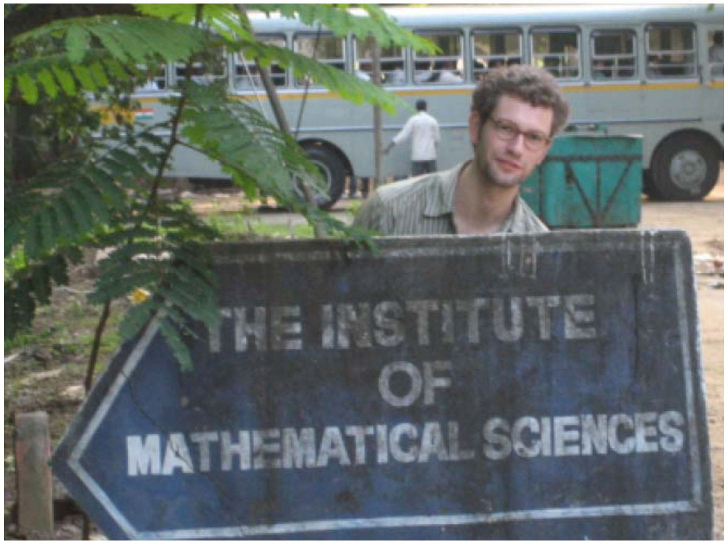
	E	K	B	C	V	G
E, EV centrality	1	0.63	0.39	0.9	0.65	0.73
K, Core number	0.63	1	0.38	0.56	0.62	0.70
B, Backbone-reduction	0.39	0.38	1	0.39	0.34	0.43
C, Contagious	0.9	0.56	0.38	1	0.58	0.67
V, Vulnerable	0.65	0.63	0.34	0.58	1	0.55
G, Global stability	0.73	0.70	0.43	0.67	0.55	1

Eigenvector centrality is best indicator for local and global stability: 90% overlap with contagious banks, 65 % with vulnerable and 73 % with global “superspreaders”

Conclusions

- Understanding Systemic Risk by using dynamical systems defined on complex networks
- Characterization of Inter-bank network from bilateral exposure data of US and European banks
- Investigating heterogeneity & modularity of the network
- The dynamics of cascading failures: local & global stability
- From network topology to dynamics: using structural measures to identify critical nodes
- Global failure: possible role of liquidity crisis ?

Thanks



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First Announcement

Workshop on Social Networks

IMSc Chennai and IIT-Madras

February 20-24, 2012

An event of the IMI Special Year on Networks

Local organizers: Sitabhra Sinha (IMSc)
Ravindran Balaraman (IIT-M)