Chapter 5
How Unstable Are Complex Financial Systems? Analyzing an Inter-bank Network of Credit Relations

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Abstract The recent worldwide economic crisis of 2007–09 has focused attention on the need to analyze systemic risk in complex financial networks. We investigate the problem of robustness of such systems in the context of the general theory of dynamical stability in complex networks and, in particular, how the topology of connections influence the risk of the failure of a single institution triggering a cascade of successive collapses propagating through the network. We use data on bilateral liabilities (or exposure) in the derivatives market between 202 financial intermediaries based in USA and Europe in the last quarter of 2009 to empirically investigate the network structure of the over-the-counter (OTC) derivatives market. We observe that the network exhibits both heterogeneity in node properties and the existence of communities. It also has a prominent core-periphery organization and can resist large-scale collapse when subjected to individual bank defaults (however, failure of any bank in the core may result in localized collapse of the innermost core with substantial loss of capital) but is vulnerable to system-wide breakdown as a result of an accompanying liquidity crisis.

5.1 Introduction

Isaac Newton, possibly the greatest physicist of all time, is believed to have once said that while he could calculate the motions of cosmic bodies, his theories are useless for understanding the madness of crowds [1]. This statement was supposedly made in the context of the mass frenzy that was seen among the general public during the height of the South Sea Bubble of 1720, one of the most famous episodes

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F. Abergel et al. (eds.), *Econophysics of Systemic Risk and Network Dynamics*, New Economic Windows,
DOI 10.1007/978-88-470-2553-0_5,

of financial speculation and the panic triggered by its subsequent collapse [2]. Indeed, until about the 1990s, the methods of physics were largely thought to be inapplicable for the study of society, of which economic activity is an integral part. However, in recent days, this perception has begun to change and it is instructive to learn that Paul Samuelson, one of the leading figures of contemporary mainstream economics, mentioned during an interview in 1998 that the physics of avalanches was a better guide for understanding the "irrational exuberance" of the overvalued markets of this period than what standard economics textbooks teach [3]. One might be tempted to think that physics has at last come of age to be applied fruitfully for understanding economic phenomena, even though we might be at the same stage in our search for a physical theory of Economics (a discipline that has often been referred to as *Econophysics* [4]) as natural philosophers were at the time of Newton in their quest for a theory of the physical universe.

The late 2000s financial crisis, possibly the worst economic disaster since the Great Depression of the 1930s, has brought to fore once again the poverty of mainstream economics when forced to explain the real world rather than idealized systems using elegant but unrealistic assumptions of perfect competition and complete knowledge. The inability of standard theories to understand the mechanisms that result in such system-wide failures of financial markets is deeply worrying, as these crises are damaging not just on their own account—often involving collapse of large financial institutions, extensive intervention in the financial sector by the government and sometimes involving bailouts costing the taxpayer enormous sums of money—but by affecting the stock market (and in the 2007–09 crisis, also the housing market) and constricting liquidity, they can depress the rest of the economy. The potential of financial crashes to drive the economy into severe recession is not a recent phenomenon but have been seen in earlier instances of market collapses [5]. The lack of availability of credit in the aftermath of such crises can result in failure of businesses causing large-scale unemployment. This in turn reduces the overall income and leads to a significant drop in consumer spending, which slows down the economy further. It is therefore of critical importance to come up with an alternative theoretical framework to understand the genesis of financial crisis, with the aim of averting disaster before it strikes by learning to recognize warning signs of an impending collapse. An even more desirable outcome will be to arrive at principles for designing robust financial structures that are much less likely to suffer system-wide failure than at present.

A theoretical approach to understand financial crisis has to consider what kind of conditions can make a large-scale collapse of financial institutions likely. This is in fact related to the general question of why and how do economic institutions fail, one of the most fascinating topics of modern economics [6]. As we know from our everyday experience that large events need not have been triggered by an extraordinary stimulus, an important related question in this context is whether the failure of a single economic entity can drive events so as to result in a cascading chain of successive collapses among several inter-connected institutions— eventually leading to a large-scale breakdown of the financial system [7]. Indeed, it was the specter of such a catastrophe that prompted governments across the world to spend enormous sums of money to try and save banks and financial institutions that were considered to be “too big to fail” in the sense of being so intricately and intimately connected to a large number of other institutions that they would bring down a significant fraction of them if allowed to fail. At an even larger scale, one can imagine that, if unchecked, such a series of failures can spread world-wide, aided by the existence of a densely connected global financial network that has been made possible by the communication revolution [8]. It is conceivable that the general collapse of the financial infrastructure coupled with the economic chaos that might ensue may well be enough to trigger the collapse of our civilization and usher in a new “dark age”, similar to the what has been repeatedly seen in history [9].

As collapse is usually manifested by a drastic reduction in the *complexity* of the system, be it in terms of the diversity of entities it is able to support or the types or nature of interactions between such entities which are allowed, it is natural to ask whether increasing complexity can itself make a system more prone to failure. This question has attracted a great deal of scientific interest for the past four or five decades, especially among ecologists [10]. At first it may appear counter-intuitive that a greater variety of elements and a strong degree of interactions among them can lead to instabilities. Indeed, the risk incurred through lending by financial institutions is sought to be reduced by securitization and selling debt instruments to other institutions, thereby connecting them together in a large web of mutual liabilities. The principle behind this practice appears to be that by sharing a large sum among many agents, the risk of default to each individual entity is reduced. In other words, as in selling insurance, if one increases and diversifies as much as possible the population which is insured, it reduces the risk that at any given time a significantly large fraction of the insured individuals will fail and that the insurer has to pay out large sums simultaneously. Arguments along these lines have been forwarded previously in other areas, for example in ecology, to contend that larger complexity actually makes a system more stable. However, in the early 1970s it was shown conclusively by Robert May [11] through linear stability analysis of large randomly connected networks that increasing the number of elements and/or the number of connections between them, as well as, increasing the strength of interactions, makes the system more unstable. In other words, a complex system is more likely to be knocked out of its equilibrium state by a small perturbation at any of its constituent elements, as compared to a simpler system.

Over the past four decades, the pioneering result of May has been debated intensely by scientists from various disciplines (dubbed as the *diversity-stability* debate [10]) and the exact conditions under which these results apply have been sought to be determined. A significant challenge to the general validity of these results had been that (a) the analysis was based on *linearization* of the system about a “fixed point” (or static) equilibrium, and (b) the system considered comprised *randomly connected* elements. However, subsequent studies of the global stability (e.g., measured in terms of the persistence of the constituent elements) of dynamical systems in various regimes, viz., exhibiting periodic and chaotic behavior apart from fixed point dynamics, has shown the original results to be valid even in this more general setting [12]. Similarly, the advent of new models of networks in the late 1990s, e.g., those exhibiting the “small-world” property [13] and those having “scale-free” distribution of degree[1] [14, 15], which arguably better represent the connection topology of complex systems seen in reality, has resulted in a series of studies of the stability when the complexity of such networks is increased. Again, it appears that increasing the size and connection density (as well as, strength of the connection weights) of these networks make them more, rather than less, unstable [16, 17]. Thus, it appears that despite the appealing intuition of the insurance hypothesis, increased connectance between a large number of dynamically evolving entities does increase the risk of overall system failure, a result whose implications for economic systems is obvious [18].

The recent crisis of 2007–09 has, therefore, brought forth calls by scientists (including from Robert May himself) to apply the lessons learnt in ecology through analyzing the stability of complex food webs to the problem of robustness in large, strongly connected networks of financial institutions [19, 20]. For example, structural properties of robust networks that can be identified as contributing to the dynamical stability of the system can be implemented in designing artificial entities such as the financial network to decrease their likelihood of failure when subjected to episodes of stress. It has been pointed out that “ecosystems are robust by virtue of their continued existence” [19], i.e., only those networks have survived (and are therefore seen today), whose structure enabled them to withstand the high degree of fluctuation in their environment and in the dynamics of their constituent species. On the other hand, financial networks have emerged very recently through the uncoordinated decisions of a large number of agents, often having divergent aims and interests. The connection topology of the network has not been developed based on robust design principles nor has the system been subjected to evolution through a series of successive failures and regrowth to attain a relatively stable configuration. In order to assess the fragility of the existing system (prior to redesigning it to make it more stable), we have to first reconstruct the network of interactions between financial institutions and study the dynamical stability implications of such a structure through simulations. Such an analysis can alert us to either “keystone” nodes in the network whose removal through failure can result in a significant number of other nodes failing in rapid succession.

Several such empirical studies of the inter-dependency networks of financial institutions have recently appeared in the literature. In particular, a very large network of over 7500 banks in USA connected through the Fedwire interbank payment network operated by the Federal Reserve System has been analyzed to reveal a sparsely connected system (only 0.3 % of the potential number of connections are actually observed) which nevertheless has relatively low average path length—a signature of the “small-world” phenomenon seen in many other networks—thus, indicating the existence of an extremely compact structure [21]. More importantly, the majority of the links correspond to weak flows, and focusing on the small set of high-value transactions reveals the existence of a core—a small set of 25 banks which are densely inter-connected—to which other banks (constituting the periphery) con-

[1] The degree of a node is the number of links it possesses.

nect. Such core-periphery organization, seen across many other complex systems, ranging from networks of neurons [22] to language [23], has also been reported for other networks of financial institutions such as the web of credit relations among Austrian banks [24]. A recent study on financial contagion propagation has claimed that applying stress in the core and in the periphery will have very different consequences for the system [25]. Another study has found a distinct bimodal capacity of nodes to propagate contagion, with those in the periphery having little or no effect while failure of nodes in the core can destroy the system [43]. The Fedwire network also exhibits scale-free degree distribution [21], another property it shares with many other inter-bank networks including those of Austria [24] and Japan [26]. Similar studies of the relation between properties of the connection topology of the credit network among financial institutions and its stability have also been carried out for other systems, including the European money market for overnight loans (required for maintaining liquidity) between Italian banks [27] and the Japanese credit network between banks and large firms [28, 29].

It is in this context that we report our analysis of a network of bilateral liabilities (or exposures) between 202 financial intermediaries (FI)[2] based in USA and Europe reconstructed from data for the last quarter of 2009 (the period during which the financial crisis of 2007–09 reached its denouement). Systemic risk in this system can be quantified as the probability that the failure of an individual entity results in a cascading series of defaults propagating through the network of mutual liabilities, with an institution failing when it is unable to honor its commitments to creditors as a result of its debtors failing and thereby defaulting on their commitments to it. Analysis of topological properties of the network reveal many of the same features seen in other financial networks, such as core-periphery organization and long-tailed distributions of degree and strength.[3] However, more important than such static properties is the dynamical response of the network to local perturbations (specifically, the failure of a particular FI). We have used a simple and intuitively appealing model of failure propagation in the network that takes into account the Tier-I core capital of each institution in addition to the information about bilateral liabilities, to study the impact of the collapse of each constituent FI on the rest of the network. This allows us to identify "super-spreader" nodes in the system whose collapse can trigger failure of a large fraction of elements in the network. A crucial parameter that affects this process is the critical fraction (q) of core capital of an FI that its net loss (as a result of failure of FIs connected to it via mutual liabilities) must exceed in order for it to collapse. Although the actual value of this critical fraction cannot be reliably determined from the empirical data, by studying the behavior of the system over a large range of q, it appears that a global or system-wide

[2] A financial intermediary is an institution, such as a bank, a credit union or a mortgage loan company, that transfers funds from investors (lenders) to those requiring capital (borrowers). For instance, a bank uses its deposits to provide loans or mortgages thereby mediating transactions between surplus and deficit agents [30].

[3] Strength of a node is the sum of weights of all links belonging to it.

collapse is unlikely except at very small (and possibly unrealistic) values of this parameter. Our observation that the propagation of disturbances along the network of explicit financial linkages (also referred to as *contagion* in the economics literature) is unlikely to cause system-wide collapse is good news and agrees with an earlier study [31]. The bad news is that the members of the highly clustered inner core who constitute the leading broker-dealers are potentially in need of tax-payer bailouts as failure of any member in the core may trigger failure of the entire core. We also see that an accompanying liquidity crisis can simultaneously decrease q as more FIs fail with time, thereby triggering even more failures and decreasing q even further. This coupling between the collapse of financial institutions and the reduction in availability of capital can thus drive a chain reaction of failures that can eventually cause the entire system to breakdown. Our results thus paint a nuanced picture of inter-bank networks, which can be viewed as "robust-yet-fragile" [32], and points out the importance of liquidity crisis that may accompany a cascading series of bank failures in triggering system-wide crisis in complex financial systems.

5.2 The Network of Financial Intermediaries

As already mentioned above, banks function as financial intermediaries between lenders who deposit money in the bank and borrowers who take out loans or mortgages. On the assumption that at any given time at most a small number of depositors will be withdrawing a substantial portion of their money, a bank holds only a small fraction of the total amount deposited in reserve to cover regular transactions with their customers and invests the rest in profit-earning enterprises [33]. While lending out their money provides banks income through interest payments, this also exposes them to credit risk of the borrowers defaulting on their promised payments. Another source of risk for banks can be a sudden devaluation in some of their external assets, such as, the drop in real estate prices following the end of a housing bubble. If such losses are a substantial fraction of its capital, a bank may find it difficult to honor its commitments to its lenders. Under such circumstances, if the bank faces a liquidity crisis and is unable to raise a loan to cover its liabilities, it can fail. Thus, banks need to have an optimized operational procedure in order to maximize their return (by lending out as large a fraction of their total deposits as possible) while at the same time minimizing the resulting risk.

A widely used method of risk reduction in modern finance is through the use of risk management instruments known as *derivatives* which are contracts between two parties specifying payoffs that will be made between them at some future date based on the value of an underlying asset such as foreign exchange rates, bonds/interest rates, commodities and equities [34]. As under normal circumstances a derivative and its corresponding underlying asset are expected to change their value in the same direction and by roughly the same amount, one can protect against loss by hedging, i.e., holding opposite positions in the underlying asset and derivative markets at the same time, so that losses in one market can be offset by gains in the other. In the specific context of credit risk, a credit default swap (CDS) is a type of derivative that can act as insurance for the lending agency against non-payment of debt. Thus, by purchasing CDS, a bank can transfer its credit risk (incurred by lending to a third party) to the seller of the swap, as in return for a series of payments (equivalent to an insurance premium) the seller agrees to compensate the buyer in the event of the default by paying off the debt. While for any individual institution such risk sharing by purchasing and selling derivatives may appear appealing, at the level of the overall system such practices bind together the different entities into a strongly interconnected entity where the failure of any one bank does not remain localized in its effects but spreads through the system. The systemic risk inherent in such a situation is worsened by a limited number of counterparties dominating the market in selling risk management instruments.

The subject of our study is the network of bilateral assets and liabilities of 202 financial intermediaries (listed in Table 5.1) aggregated over all categories of derivative products (including foreign exchange contracts, interest rate swaps, equities, CDS and commodities). In order to measure credit exposure of a FI, one first needs to identify the derivatives contracts which would result in loss of value to the institution if the counterparty defaults [35]. In the absence of bilateral netting[4] and any collateral from counterparties, the Gross Positive Fair Value (GPFV) is the aggregate fair value of all contracts where the FI is owed money by its counterparties. Thus, GPFV is the maximum credit exposure or losses which the FI can incur if its counter-parties default. Conversely, the sum total of values of all contracts where a FI owes money to its counterparties is referred to as Gross Negative Fair Value (GNFV), and it is the maximum loss incurred by the counterparties in the absence of netting agreement or bank collateral. Derivatives liabilities and assets are estimated by adjusting the gross payables and receivables (respectively) for collateral, bilaterally netting where agreements exist and summing over all counterparties.

The firm level data on derivative assets and liabilities used in our study were obtained from FDIC Call Reports for the fourth quarter of 2009 for US banks that operate solely as national associations, and from individual Annual Financial Statements for the global US banks and Europeans FIs. The firm level derivative liability (asset) is the positively (negatively) signed sum over all counterparties and products of the bilaterally netted market value of derivatives receivables and payables. An algorithm described in Ref. [37] is used to reconstruct a bilateral matrix for derivatives liability or asset between FIs from the firm-level data upto some margin of error. The starting point for the network reconstruction is this bilateral gross flow matrix between the FIs, $\mathbf{B}$, where B_{ij} represents the flow of financial obligation from the seller (row FI i) of the derivative to the buyer (column FI j). Thus, $N_i = \sum_j B_{ij}$ is the GNFV of bank i, representing the total derivatives obligations owed by it to other FIs, while $P_j = \sum_i B_{ij}$ is the GPFV of bank j, i.e., the total sum owed to it by all other FIs. The matrix will in general be asymmetric ($B_{ij} \neq B_{ji}$) and will have zeros along the diagonal ($B_{ii} = 0$) as banks do not lend to/borrow from themselves.

[4]Bilateral netting, whose primary purpose is to reduce exposure to credit risk, is an arrangement between two parties to exchange only the net difference in their obligations to each other [36].

Table 5.1 The list of 202 banks analyzed in this article arranged in decreasing order according to their Tier-I core capital

i	Financial intermediary	Core capital (billions USD)
1	Bank of America	111.92
2	Royal Bank of Scotland	98.28
3	Citibank	96.83
4	JP Morgan Chase	96.37
5	BNP Paribas	90.37
6	Barclays	77.56
7	Lloyds	74.27
8	UniCredit	56.07
9	Deutsche Bank	49.42
10	Morgan Stanley	46.67
11	Credit Agricole	44.53
12	Wells Fargo Bank	43.77
13	UBS	42.32
14	Wachovia Bank	39.79
15	Credit Suisse	39.49
16	HSBC	35.48
17	Societe Generale	34.69
18	Dexia	25.24
19	Standard Chartered	24.58
20	PNC Bank	24.49
21	Citibank (South Dakota)	19.71
22	Goldman Sachs	17.15
23	US Bank Natl Asso	16.25
24	Fifth Third Bank	13.57
25	Branch Banking & Trust Co	13.54
26	Suntrust Bank	11.97
27	State Street	11.38
28	Regions Bank	10.58
29	New York Mellon	10.15
30	TD Bank	9.27
31	Capital One	8.42
32	RBS Citizens	8.24
33	KeyBank Natl Asso	8.0
34	Union Bank	7.21
35	Comerica Bank	5.76
36	Manufacturers and Traders Trust Co	4.99
37	Bank of the West	4.80
38	Northern Trust Co	4.76
39	Compass Bank	4.58
40	Marshall & Ilsley Bank	3.95
41	Harris Natl Asso	3.52
42	First Tennessee Bank	3.36
43	Huntington Natl Bank	2.87
44	UBS Bank USA	2.52
45	Citizens Bank of Pennsylvania	2.43
46	RBC Bank (USA)	2.43
47	Zions First Natl Bank	1.81
48	Associated Bank	1.78
49	City Natl Bank	1.60
50	Frost Natl Bank	1.32
51	Amegy Bank	1.27
52	Webster Bank	1.27
53	BanCorpSouth Bank	1.14
54	Bank of Oklahoma	1.08
55	PrivateBank and Trust Co	1.06
56	Mizuho Corp Bank (USA)	1.05
57	Whitney Natl Bank	1.00
58	Susquehanna Bank	0.99
59	RaboBank	0.97
60	California Bank & Trust	0.96
61	Northwest Savings Bank	0.92
62	Arvest Bank	0.88
63	WesternBank Puerto Rico	0.84
64	Trustmark Natl Bank	0.84
65	Signature Bank	0.84
66	Firstmerit Bank	0.83
67	MB Financial Bank	0.82
68	Woodlands Commercial Bank	0.75
69	Bank of Hawaii	0.75
70	Investors Savings Bank	0.75
71	Israel Discount Bank of New York	0.72
72	United Community Bank	0.72
73	National Penn Bank	0.7
74	Doral Bank	0.69
75	Columbus Bank & Trust Co	0.67
76	Apple Bank for Savings	0.64

Table 5.1 (Continued)

i	Financial intermediary	Core capital (billions USD)
77	Banco Santander Puerto Rico	0.57
78	IberiaBank	0.55
79	Nevada State Bank	0.54
80	1st Source Bank	0.53
81	Natl Bank of Arizona	0.53
82	UMB Bank	0.53
83	Sterling Savings Bank	0.5
84	Texas Capital Bank	0.49
85	Southwest Bank	0.48
86	Safra Natl Bank of New York	0.48
87	Bank Leumi USA	0.45
88	Bank of North Georgia	0.43
89	Pinnacle Natl Bank	0.42
90	Natl Bank of South Carolina	0.4
91	Chemical Bank	0.4
92	Hancock Bank	0.38
93	Banco Bilbao Vizcaya Argentaria PR	0.36
94	Columbia State Bank	0.36
95	R-G Premier Bank of Puerto Rico	0.34
96	Rockland Trust Co	0.33
97	Sun Natl Bank	0.3
98	Hancock Bank of Louisiana	0.3
99	Sandy Spring Bank	0.29
100	Stellarone Bank	0.28
101	S & T Bank	0.27
102	Vectra Bank of Colorado	0.27
103	Centennial Bank	0.27
104	Wells Fargo HSBC Trade Bank	0.26
105	First American Bank	0.26
106	Mainsource Bank	0.23
107	Boston Pvt Bank & Trust Co	0.22
108	Bangor Savings Bank	0.21
109	First Security Bank	0.21
110	First Commercial Bank	0.2
111	Integra Bank Natl Asso	0.2
112	Berkshire Bank	0.2
113	Enterprise Bank & Trust	0.2
114	Frontier Bank	0.19
115	American Chartered Bank	0.19
116	Bank of Nevada	0.18
117	American Natl Bank	0.17
118	Stockman Bank of Montana	0.16
119	American Natl Bank of Texas	0.15
120	First United Bank & Trust Co	0.15
121	Bank of Kentucky	0.13
122	StockYards Bank & Trust Co	0.13
123	Wilson Bank & Trust	0.13
124	Bank of North Carolina	0.13
125	Bank Rhode Island	0.12
126	Community Bank of Texas	0.12
127	FSG Bank	0.12
128	Community Trust Bank	0.1
129	Commerce Bank of Washington	0.09
130	Paragon Commercial Bank	0.09
131	ICE Trust US LLC	0.08
132	Bryant Bank	0.08
133	Colorado Capital Bank	0.08
134	South Shore Savings Bank	0.08
135	D L Evans Bank	0.08
136	Commercial Bank	0.07
137	Capstar Bank	0.07
138	Northwestern Bank	0.07
139	Gulf Coast Bank & Trust Co	0.07
140	Business First Bank	0.06
141	Guaranty Bank	0.06
142	Guaranty Bond Bank	0.06
143	Avenue Bank	0.05
144	State Bank & Trust Co	0.05
145	Marine Bank	0.05
146	Northeast Bank	0.05
147	Horicon Bank	0.05
148	Citizens Natl Bank	0.05
149	Town North Bank	0.05
150	American State Bank	0.05
151	Community Natl Bank of Texas	0.05
152	First State Bank of East Detroit	0.05

Table 5.1 (Continued)

i	Financial intermediary	Core capital (billions USD)	i	Financial intermediary	Core capital (billions USD)
153	Clinton Savings Bank	0.05	179	Providence Bank	0.02
154	Jersey Shore State Bank	0.05	180	Carroll County State Bank	0.02
155	Passumpsic Savings Bank	0.04	181	State Bank of Faribault	0.02
156	Coastal States Bank	0.04	182	Summit Bank	0.02
157	Southern Bank	0.04	183	FirstBank	0.02
158	Lincoln Savings Bank	0.04	184	Touchmark Natl Bank	0.02
159	United Bank & Trust	0.04	185	State Bank & Trust Co	0.02
160	Oakworth Capital Bank	0.03	186	First Natl Bank	0.02
161	Central Bank	0.03	187	Commerce Bank of Oregon	0.01
162	Hometown Bank	0.03	188	Canyon Community Bank	0.01
163	Security Financial Bank	0.03	189	Nebraska Natl Bank	0.01
164	Progress Bank & Trust	0.03	190	First Natl Bank of Junction City	0.01
165	State Bank Financial	0.03	191	First Vision Bank of Tennessee	0.01
166	Cornerstone Bank	0.03	192	New Frontier Bank	0.01
167	Bank of South Carolina	0.03	193	Citizens State Bank	0.01
168	C US Bank	0.03	194	Keokuk County State Bank	0.01
169	Texas Bank	0.03	195	Boone Bank & Trust Co	0.01
170	Biddeford Savings Bank	0.03	196	Northwoods State Bank	0.01
171	Paragon Natl Bank	0.03	197	Cleveland State Bank	0.01
172	Cornerstone Community Bank	0.03	198	Farmers Bank	0.01
173	South Central Bank of Barren County	0.03	199	Farmers Savings Bank & Trust	0.01
174	Somerset Hills Bank	0.03	200	Business Bank	0.01
175	Platte Valley Bank	0.03	201	Mount Vernon Bank & Trust Co	0.01
176	Keysource Commercial Bank	0.02	202	West Town Savings Bank	0.01
177	First State Bank	0.02			
178	Premier Commercial Bank	0.02			

For simplicity we have then constructed an antisymmetric matrix $\mathbf{M}$ of netted positions between FIs, i.e., $M_{ij} = B_{ij} - B_{ji} = -M_{ji}$. For each FI i, a positive (negative) entry M_{ij} along the i-th row gives the net sum payable to (receivable from) the counterparty FI j. To analyze a chain of cascading failures following the collapse of bank i, only the positive entries of $\mathbf{M}$ are relevant—as the contagion flows from the failed FI to its net creditor FIs (i.e., those counterparties to which it owes more than what they have borrowed from it). Thus, the matrix $\mathbf{J}$ we use to construct the network of bilateral exposures among the FIs is obtained from $\mathbf{M}$, by replacing all negative matrix entries with zeros, i.e., $J_{ij} = M_{ij}$ if $M_{ij} \geq 0$ and $J_{ij} = 0$ otherwise. This represents a weighted, directed network of financial institutions, with a link being directed from a bank to its net creditors and the link weight being the net liability (in units of billions of US Dollars).

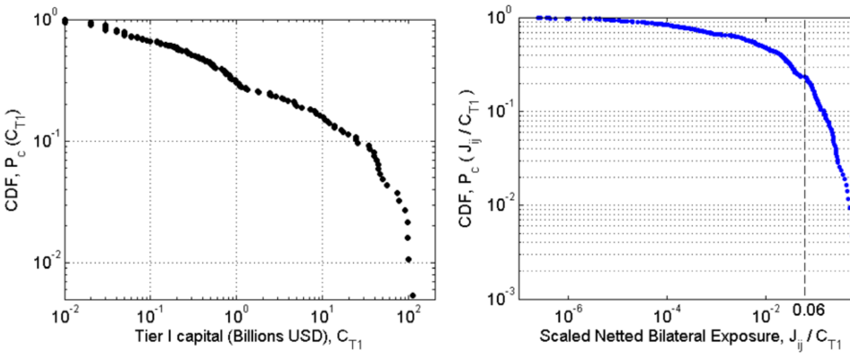

Fig. 5.1 The cumulative distribution function for (*left*) the core capital (C_{T1}) of the 202 FIs considered and (*right*) the netted bilateral exposure J_{ij} scaled by the Tier-I capital of the creditor bank. Assuming a value of 0.06 for the critical fraction q, all links to the right of the *broken vertical line* will spread contagion in the network

While most FIs in the network either send or receive at least one link, there are 15 nodes which have neither incoming nor outgoing links. In addition there is one other node for which the sum borrowed from another FI exactly equals the sum it has lent, so that on netting it does not have any net liability with respect to other FIs. The sixteen isolated FIs are City National Bank (node 49), Northwest Savings Bank (61), Apple Bank for Savings (76), Bangor Savings Bank (108), American National Bank of Texas (119), D L Evans Bank (135), Northeast Bank (146), Lincoln Savings Bank (158), Progress Bank & Trust (164), Providence Bank (179), Carroll County State Bank (180), Commerce Bank of Oregon (187), Canyon Community Bank (188), New Frontier Bank (192), Keokuk County State Bank (194) and Cleveland State Bank (197).[5] The largest connected component (LCC) of the network of netted bilateral obligations between FIs comprises $N_{LCC} = 186$ nodes, which have only 424 connections (out of the $N_{LCC}(N_{LCC} - 1)/2 = 17205$ total number of possible links) between them and is therefore very sparse.

In addition to the data on bilateral exposure, we also have information about the Tier-I capital, C_{T1} (in units of billions of USD) of each FI (Fig. 5.1(left)), which measures the financial strength of a bank and comprises the core capital consisting primarily of common stock and disclosed reserves (or retained earnings) [38]. Internationally set standards (the Basel agreements) specify the desired minimum ratio of the core capital of a bank to the total risk-weighted assets held by it in order to provide protection against defaults or sudden loss in value. In our model for failure propagation in the inter-bank network, we specify a critical fraction q of the Tier-I capital of an FI, which, if exceeded by the total net loss of the bank resulting from failures of one or more of its debtor counterparties, will cause its

[5]Except for the D L Evans Bank, for which the GNFV exactly equals the GNPV so that the total netted exposure is zero, all the other banks have no bilateral exposure at all with respect to any other bank in the network.

own failure. Figure 5.1(right) shows the distribution of the netted liabilities of the FIs scaled by the core capital of the creditor counterparty, i.e., J_{ij}/C_{T1}. If a node j defaults, then the resulting perturbation will bring down its neighboring node i only if $J_{ij}/C_{T1}(i) > q$ (a value of $q = 0.06$ is shown as a broken vertical line in the figure). Thus, the distribution of J_{ij}/C_{T1} determines the stability of nodes with respect to local perturbations (failure of a single FI).

5.3 Topological Properties

Calculating the standard topological properties from the directed network represented by J shows us that it shares many of the features of other inter-bank networks which have been reported in earlier studies. For example, it exhibits the characteristics of a "small-world" network [13] having both low average path length ($\langle l \rangle = 3.6$) and high clustering coefficient ($C = 0.24$). The undirected and non-weighted network shows disassortative mixing by degree (assortative coefficient $r = -0.28$), i.e., nodes with low degree preferentially connect to those having high degree. This may be related to the strong core-periphery structure seen in the network, where a small number of highly (and strongly) interconnected banks form the central nucleus to which most of the other banks of the network connect.

We use a generalization of the core decomposition technique applied to directed networks described in Ref. [22] to obtain the in-degree and out-degree k-core—a subnetwork containing only those nodes which have at least k incoming and outgoing links (respectively)—for the unweighted network corresponding to J. The cores corresponding to in-degree and out-degree need not be identical although they may have nodes in common, and this is indeed what is observed. We observe that 19 banks belong to both the innermost in-degree and out-degree cores (Nodes 1–7, 9–10, 12, 14–17, 20, 22, 26, 27, 29 and 33—see Table 5.1 for the identity of these FIs), while 4 banks belong only to the out-degree innermost core (Nodes 8, 11, 13 and 33) and only 1 bank (Node 19) belong only to the in-degree innermost core. Thus a set of 24 banks, all having relatively high core capital, form the highly interconnected central nucleus of the network to which the other banks connect.

While the in-degree and out-degree of an FI can give a sense of its "centrality" (i.e., importance) in the network, an even better measure is to use eigenvector centrality, which not only considers how many connections a node has, but also weighs this with the importance (or centrality score) of each neighbor. It is measured by simply considering the eigenvector corresponding to the largest eigenvalue of the adjacency matrix for the network, with the vector components corresponding to each node being their eigenvector centrality score. When a node has high eigenvector centrality, this could be either because it has many neighbors or it has relatively large number of important neighbors or both [39]. However, the standard method of determining eigenvector centrality does not work very well for directed networks, as is the case here. Using the Katz centrality, which works well for directed networks, also has limitations which can be overcome by using a variation, viz., the

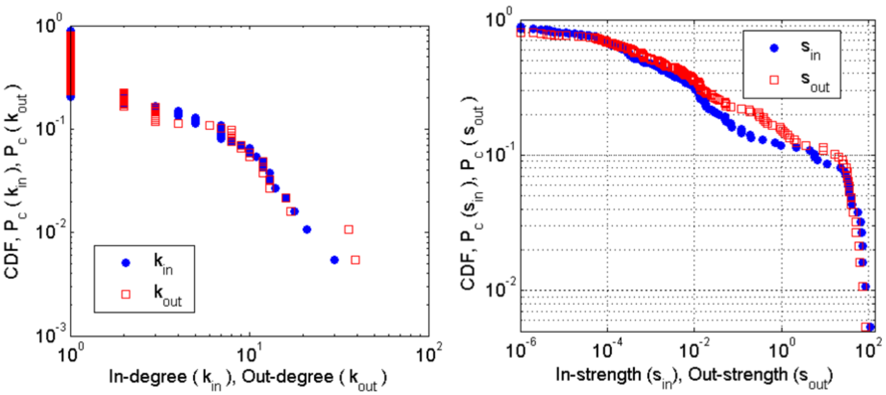

Fig. 5.2 The cumulative distribution function for (*left*) the in-degree k_{in} (*circles*) and out-degree k_{out} (*squares*) and (*right*) the in-strength s_{in} (*circles*) and out-strength s_{out} (*squares*) of the 202 FIs

Page Rank centrality measure used for example to assign importance to web pages. In particular, we use the Arnoldi iteration algorithm for Page Rank [40] and implemented in a package by David Gleich [41]. The free parameter α is set equal to 0.85 (as used for heuristic reasons by the Google search engine [39]). We find that the bank having highest Page Rank is JP Morgan Chase (Node 4), while the next nine banks in decreasing order of Page Rank are Societe Generale (Node 17), Bank of America (1), Morgan Stanley (10), Deutsche Bank (9), Royal Bank of Scotland (2), Lloyds (7), Goldman Sachs (22), HSBC (16) and BNP Paribas (5). Thus, banks with high centrality (as measured by Page Rank) not only have large core capital but are also the ones belonging to the innermost core for both in-degree and out-degree. There is thus a large degree of agreement among the topological measures used to identify the most crucial nodes of the network.

A recent paper that has looked at systemic risk from the perspective of ecosystem stability has stated that two topological features of inter-bank networks that are crucial are "First, diversity across the financial system… homogeneity bred fragility. …Second, modularity within the financial system… Modular configurations prevent contagion infecting the whole network in the event of nodal failure" [20]. We now proceed to verify whether the FI network indeed shows evidence of (a) heterogeneity in node properties, e.g., in terms of degree, strength, Tier-I capital, etc. and (b) the existence of modularity (i.e., multiple communities of nodes, with members of each community being more densely and/or strongly connected amongst themselves than with members of other communities).

We have already shown in Fig. 5.1 above the distributions for the Tier-I capital and the netted bilateral liabilities (scaled by the core capital), both of which span several orders of magnitude but exhibit sharply decaying tails. Figure 5.2(left) shows the distributions for both the in-degree and the out-degree of each node in the FI network, while the distributions for the in-strength and out-strength (corresponding to the aggregate of the net amounts lent and borrowed by a bank, respectively) are shown in Fig. 5.2(right). All the distributions have long tails; however, preliminary statistical tests do not appear to suggest a scale-free nature for them. JP Morgan Chase bank (Node 4) has the highest in-degree (30) and out-degree (39), as well as the largest out-strength (85.42), while Deutsche Bank (Node 9) has the highest in-strength (108.76). We note that there is a strong linear correlation between the in- and out-degrees of the nodes ($r = 0.88$ with p-value of 0) as well as between their in- and out-strengths ($r = 0.77$, p-value = 0). The degree and strength of nodes also show strong linear correlation of $r = 0.75$ and $r = 0.73$ respectively for the incoming and outgoing connections. Not surprisingly, the nodes having large Tier-I capital have high in-degree and out-degree (their linear correlation coefficients with C_{T1} being 0.78 and 0.83 respectively with zero p-values), as well as, high in-strength and out-strength (the linear correlation coefficients with C_{T1} being 0.80 and 0.85 respectively with zero p-values). In the LCC of 186 nodes, 21 have no in-degree, i.e., they are net borrowers in all of their bilateral interactions, while 36 have no out-degree, i.e., they are net lenders in all their bilateral interactions. 129 nodes (i.e., about 70 % of the LCC) has both incoming and outgoing connections so that they are net borrowers in some bilateral interactions while being net lenders in others.

In order to look for modularity in the FI network, we have used community detection techniques on both the unweighted and weighted LCC of the network. The spectral method for module determination [42] has yielded 13 communities in the unweighted network, the largest having 54 nodes (comprising all of the top 10 % of FIs according to their core capital except JP Morgan Chase) and the smallest containing 3 nodes. The smaller modules are seen to have a star-like topology with all other nodes having connections only to a central hub node of the module which links the community to the rest of the network. A generalized version of the spectral method has been used in the case of the weighted network, which results in the network being split into two modules: one containing 8 nodes and another having the remaining 178 nodes. The FIs in the smaller module (Royal Bank of Scotland, Lloyds, UniCredit, Deutsche Bank, Credit Suisse, Societe Generale, DEXIA and Standard Chartered) are all based in Europe, and this points to large credit flows between FIs whose base of operations are geographically close.

5.4 Dynamics of Failure Propagation

The topological properties of the FI network investigated above can alert us to the prominent role played by a small set of banks in the system, but do not by themselves explain how a series of failures can propagate through the network in a cascading process. In order to relate the static information contained in the weighted adjacency matrix J to a dynamic picture of how perturbing certain "keystone" nodes can trigger a significant fraction of the network to break down, we need to assume a specific mechanism for the propagation of the effects of the default of a particular FI to other FIs connected to it via credit relations. We have used a simple and intuitive model where the failure of a node results in the loss of the net sums lent to it by all its creditors (assuming the existence of bilateral netting agreements between all pairs of FIs). This can cause another node to fail if the total loss it faces as a result of all other failures in the network so far, exceeds a critical fraction of its core capital. In this process, as more and more nodes fail, the total loss faced by the remaining creditor nodes increases substantially thereby making it more likely for them to fail in subsequent time steps. Understandably, all nodes will not have similar impact upon the network; we are particularly interested in identifying "super-spreader" nodes, whose collapse will result in a system-wide breakdown in the network (or at least that of a large fraction of nodes belonging to it).

To describe the model of failure propagation, we first define the dynamical state of each node in terms of a binary variable s_i. At any time step t, if $s_i(t) = 1$ the node is solvent, whereas if $s_i(t) = 0$ it is understood to have failed (once a node has failed, it will remain so for all subsequent time steps). The netted bilateral exposures J_{ij} (i.e., how much is owed by bank i to bank j) describes the interactions between the nodes. In the event of a node i defaulting, all its creditors j lose the net sum $J_{ij} (> 0)$ lent to it. If the total loss of any node j as a result of such failures exceeds a critical fraction (q, a parameter in our simulations) of its Tier-I capital, $C_{T1}(j)$, it also fails. Thus, the dynamical evolution of each node i in the FI network is described by the discrete-time equation:

$$s_i^{t+1} = 1 - \Theta\left[\sum_j J_{ji}\left(1 - s_j^t\right) - q \cdot C_{T1}(i)\right], \tag{5.1}$$

where, Θ is the Heaviside step function (i.e., $\Theta(z) = 1$ if $z > 0$ and $= 0$, otherwise). The parameter q depends on the ease of credit availability in the system, with a liquidity crisis corresponding to a sharp decline in the value of q.

Initially, all nodes in our model are in the solvent state ($s = 1$). To simulate the propagation of failures, we then change the state of any one node to failed ($s = 0$) and observe whether this causes any of its neighbors to fail, and if so, whether the effect can propagate further along the network. We carry out the process repeatedly, choosing each node in the network in turn to be the initial failed node. While most nodes do not trigger any failure events among their neighbors, in a few cases the initial event can cause a series of failures to cascade along the network. We wait until the system reaches an equilibrium (i.e., the state of every node remains unchanged with time) and count the total fraction f_{failed} of nodes which have failed as a result of the initial single node failure. Figure 5.3(left) shows the largest of such cascade events for $q = 0.01$, when the initial default of Node 23 results in a total of 67 nodes to fail by the end of the cascade. We observe that there are several such nodes whose collapse affects the entire core of strongly connected FIs and tentatively identify them as "super-spreader" nodes (i.e., FIs whose failure results in a network-wide disturbance, in contrast to most other nodes which have no effect). Looking at the distribution of f_{failed} in Fig. 5.3(right) we note its strongly bimodal character. The large peak at extremely low values are due to the majority of nodes which have no effect on the rest of the network, while the smaller peak at higher values of f_{failed} correspond to the "super-spreader" nodes. We also note that decreasing q (corresponding to a constriction in the credit supply or capital buffers)

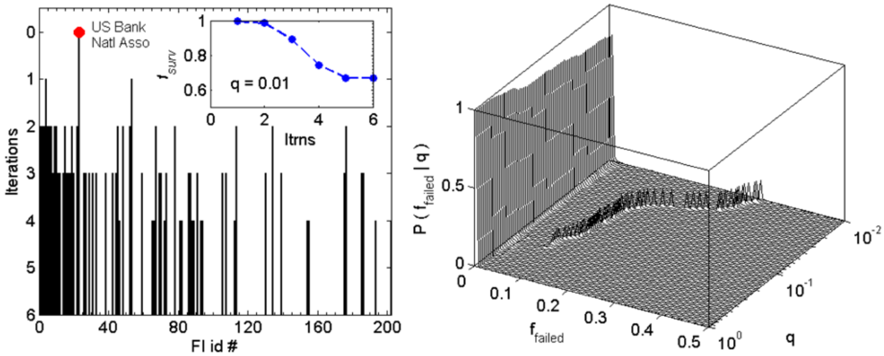

Fig. 5.3 (*Left*) The failure of a single node in the FI network (node 23) can initiate a series of cascading failures propagating through the network that results in a total of 67 nodes failing by the end of the cascade ($q = 0.01$). The inset shows the time-evolution of the cascade process with the fraction of surviving nodes, f_{surv}, declining from 1 at the initial time to a final value of 0.67. (*Right)* The distribution of failure cascade sizes (measured in terms of the total fraction of nodes in the system that fail over the duration of the cascade, f_{failed}) shown as a function of the parameter q. The distribution has a strongly bimodal character with a node failure either resulting in no effect on the rest of the network, or, bringing down a significant number of other nodes ("many-or-nothing" behavior). The size of cascades increase significantly with decreasing q (corresponding to tightening of credit availability)

increases the size of the cascades. However, the total number of nodes affected by an initial single node failure does not approach the size of the entire FI network, unless q has extremely low (and possibly unrealistic) values. Thus the propagation of disturbances along the network of bilateral liabilities is unlikely to be the sole cause of a system-wide collapse of financial institutions. This agrees with an earlier study [31] which found that perturbations transferred via explicit financial linkages are not enough for triggering large scale breakdown of financial systems. However, our results also identify the vulnerability of the innermost core of broker-dealers, which, though few in number, can through their failure result in over 75 % loss of Tier-I capital in the system. Finally, the cascade of failures can initiate an accompanying liquidity crisis, as the simultaneous default of multiple FIs may restrict credit availability with lender institutions reluctant to give out large loans and adopting a wait-and-watch policy. The resulting decrease in the parameter q will result in even more FIs failing, which in turn further decreases credit availability making the liquidity crisis more severe. Thus, a feedback process ensues with the failure propagation and liquidity crisis driving each other, eventually resulting in a global or system-wide collapse of the financial system. Thus, our results suggest that when evaluating the robustness of complex financial systems we need to focus not only on the network of explicit linkages between the institutions, but also on the overall environment in which they operate.

Acknowledgements We would like to thank S Raghavendra for earlier discussions on this topic and F Abergel, N Ganguly and S S Manna for useful comments on the work during presentations at meetings in Kolkata and Bangalore. We are grateful to E Schöll for stimulating discussions and hospitality at TU-Berlin where part of the work was done.

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