Is Inequality Inevitable in Society? Income Distribution as a Consequence of Resource Flow in Hierarchical Organizations

Sitabhra Sinha and Nisheeth Srivastava

The Institute of Mathematical Sciences, C.I.T. Campus, Taramani, Chennai – 600 113, India
sitabhra@imsc.res.in

> **society** (*noun*), from Latin *societas*, equiv. to *soci(us)* partner, comrade + -*etas*, var. of -*itas*- -ity
> **1.** an organized group of persons associated together for religious, benevolent, cultural, scientific, political, patriotic, or other purposes.
> **2.** a body of individuals living as members of a community.
> **4.** a highly structured system of human organization for large-scale community living that normally furnishes protection, continuity, security, and a national identity for its members.
>
> **hierarchy** (*noun*), from M.L. *hierarchia*, "ranked division of angels"
> **1.** any system of persons or things ranked one above another.
>
> *Dictionary.com Unabridged (v 1.1)*[1]

Almost all societies, once they attain a certain level of complexity, exhibit inequality in the income of its members. Hierarchical stratification of social classes may be a major contributor to such unequal distribution of income, with intra-class variation often being negligible compared to inter-class differences. In this paper, examples from different historical periods, such as 10th century Byzantium and the Mughal empire of India in the 15th century, and different kinds of organizations, such as a criminal gang in the USA and Manufacturing & IT Services companies in India, are shown to suggest a causal relation between the hierarchical structure of social organization and the observed income inequality in societies. Proceeding from the assumption that income inequality may be a consequence of resource flow in a hierarchically structured social network, we present a model to show that empirically observed long-tailed income distribution can be explained through a process of division of assets at various levels in a hierarchical organization.

[1] http://dictionary.reference.com/

1 Introduction

Human society, once it reaches a sufficient degree of complexity, is almost always marked by large inequalities in wealth and income among its members. This is as true of proto-states in the Bronze Age (as indicated by burial remains in the graves of the elite as opposed to common people) as it is in today's highly industrialized world. There have been several attempts at investigating the nature and causes of this pervasive inequality. Vilfredo Pareto (1848–1923) was possibly the first to attempt a quantitative evaluation of social inequality when he collected data on income distribution in several countries including England, Peru, several German states and a number of Italian cities [1]. Pareto claimed that in all cases the data fit a power law relation, with the number of households having income greater than x, $N(x) \sim x^{-\alpha}$. He further made the observation that values of α (now referred to as Pareto exponent) for all the countries observed were around 1.5, possibly the first report of *universality* in a power law relation from empirical data. Based on these observations, Pareto proposed an universal law for income distribution [2]. Later analysis with more accurate and extensive data showed that the power law fits only the long (upper) tail of the income distribution, with the bulk following either a log-normal or Gibbs distribution, and also that the Pareto exponent for different countries have a much wider range of variation than claimed by Pareto [3].

A striking feature of the income distribution predicted by Pareto's law is its extremely skewed nature, with the frequency distribution declining monotonically, beginning at a minimum income [1]. This was referred to by Pareto as the "social pyramid", and brings us to the question of the connection between the observed hierarchical structure of most societies and their income distribution. According to a standard introductory book in prehistoric archaeology, hierarchically organized institutions emerged together with larger, denser populations where decisions could no longer be effectively made by consensus [4]. In turn, hierarchical organizations served to consolidate as well as enhance existing inequalities, so that those having authority remained in power. Thus "civilized" societies are marked by increasing inequality, in wealth as well as in power. Therefore, it seems that the observed long-tailed income distributions can be partially explained to be a result of resource allocation among members belonging to a hierarchical structure, with the minority at the higher levels having a significantly higher income than the majority below. In comparison to inter-level inequality, differences in the income of individuals belonging to the same level, which is due to factors other than the stratified social structure, may be insignificant.

In this paper we present the results of our study involving empirical data analysis and theoretical modeling to show that hierarchical structures can indeed explain the power law form of income distribution in society. In the next section, we analyse the organizational structure and income data from different historical periods, such as that of the Byzantine army in 10th cen-

tury and the Mughal nobility in 15th century, and from different present-day organizations, such as an urban street gang of Chicago and Indian companies belonging to the Manufacturing and Information Technology sectors. This is followed by a section where we theoretically derive the power law distribution of income from a simple model of resource flow along a hierarchical structure (the tribute model).

2 Empirical Data

2.1 Military Wages in the Byzantine Empire, 10th Century

Between the 10th and 11th centuries, Byzantium was the richest state in the western world, with income levels coming close to the maximum that pre-industrial societies have achieved [5]. We focus on the wages of the Byzantine army, as military expenditure formed the bulk of the total annual budget of the state. This is also seen in its predecessor, the Roman empire, which spent about 80% of its annual budget on the army [6]. The total salaries paid each year to the *strategoi* (generals) of the Byzantine army by the emperor amounted to a total of 26,640 gold coins, which accounted for a significant portion of the state's regular expenditure [7]. Therefore, analyzing the military wages gives a good indication of the overall income distribution in Byzantine society.

Like most military organizations, the Byzantine army had a pyramidal structure, with each theme (a military unit) under the command of a strategos, who was assisted by a hierarchy of lower officers. Even a cursory look at their wages indicates that there was a high degree of inequality. Soldiers were paid between 12–18 nomismata, lower officers between 72–124 nomismata (1–2 pounds of gold), while the strategoi received 1440–2880 nomismata (20–40 pounds of gold), making the top-to-bottom income ratio > 200 [5]. Although the salaries of the strategoi may look extraordinarily high, it included funds to maintain the bodyguard, personal administrative staff and the official residence. It is therefore better to see the *roga* (salary) as an annual budget for running the administration, which included the salary of the official himself [7].

Figure 1 (left) shows the cumulative frequency distribution of wages for soldiers and officers of the *tagmata*, or central fleet, in 911 [8]. The distribution fits a power-law form with a Pareto exponent of $\alpha \sim 2.17$. The deviation of the income for the strategos, which appears too high compared to the general trend of wages for the other officers, is because it presumably includes the money earmarked for administrative expenses as mentioned above. At the lower end, the soldiers were partly paid in kind (grain, clothes, etc) and this could account for the slight deviation.

It is interesting to see that Byzantine army showed both hierarchical structure and the long-tailed income distribution that we associated with Pareto

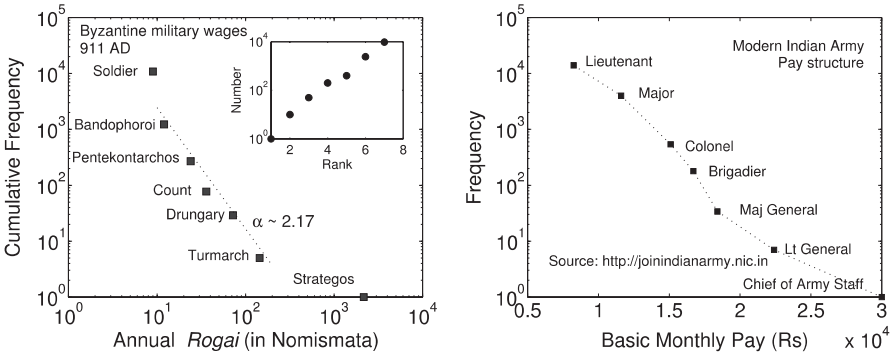

Fig. 1. (*left*) Cumulative frequency distribution of the annual military wages in the Byzantine army in 911 AD. The *dotted line* indicates a power-law fit to the data, with Pareto exponent $\alpha \sim 2.17$. The monetary unit is 1 nomisma, a coin of pure gold weighing 4.5 grams. The *inset* shows the approximate number of people at each rank in a theme (a Byzantine military unit), starting from the strategos. (*Right*) The frequency distribution of basic monthly pay (in Rupees) in the Indian army at present.

law. However, it is not necessary for the two to always coexist. Although modern armies retain the hierarchical organization of the Byzantine army, the salary structure is much more egalitarian. Figure 1 (right) shows the income distribution in the Indian army at present, which suggests an almost exponential decay, especially at the higher ranks. The data for the basic pay has been obtained from the Indian army web-site [9]. Similar kind of salary structure is seen in almost all governmental agencies, which in general show far less inequality than, e.g., companies belonging to the private sector. Presumably this is because such "non-competitive" organizations have other non-monetary benefits, such as, higher job-security.

2.2 Salaries of Mughal Imperial Officers in India, 15th Century

The Mughal empire of India was one of the largest centralized states of the pre-modern world [10]. Between 1556 and 1720 the empire was a large complex organization dominating most of the Indian subcontinent, with the emperor's orders being carried out in every province by a well-trained cadre of military-civilian bureaucrats. The centralized hierarchical administration was built largely during the reign of Emperor Akbar (1556–1605). The reigns of Akbar's successors, his son Jahangir (1605–1627) and grandson, Shah Jahan (1627–1658), saw steady growth in Mughal imperial power and consolidation of the centralized bureaucratic system. The higher administrative positions were filled by a select group of warrior-aristocrats, comprising royal princes and several hundred emirs (nobles). Each of them headed households and troop contingents ranging in size from several hundred to several thousand.

To meet the large expenses involved, the Mughal nobles were paid salaries that were probably the highest in the world at that time [11]. In 1647, during the reign of Emperor Shah Jahan, the 445 highest ranked administrators controlled over 60% of the total imperial revenue income. At the very top, 68 princes and nobles had a combined income of 36.6% of the total revenue, and their *jagirs* covered almost a third of the entire empire [12]. Thus, the income data for the Mughal administrators seems particularly suited for investigating the relation between hierarchical structure and long-tailed income distribution.

With the accession of Emperor Akbar in 1556, the Mughal empire went into a new phase of expansion involving long and costly campaigns. This necessitated streamlining of the army organization and finances, which led Akbar to start in 1577 the *Mansabdari* system (derived from the term *mansab* meaning rank) of hierarchical military nobility. All imperial officers in the civilian and military administration were given ranks, there being 33 grades of mansabdars ranging from commanders of 10 to commanders of 10,000. Each mansabdar was expected to maintain a prescribed number of soldiers, horses, elephants, equipment, etc. according to his *suwar* rank, while the *zat* rank determined his salary. The latter also denoted the standing of the officer in relation to the emperor, the higher the rank the more important the officer. During Akbar's reign, the highest rank an ordinary noble could hold was that of a commander of 5000, the grades between commanders of 7000 and 10,000 being reserved for the royal princes. During the reign of his successors, the grades were increased upto 20,000 or even more [13]. While some officers received their salary directly from the imperial treasury, most were paid through assignment of *jagirs*, the order to collect revenue from certain areas in lieu of salary. Unlike the feudal system in Europe, a mansabdar's property (including jagirs) was not hereditary. Their appointment, promotion or dismissal rested entirely with the emperor.

Detailed information about the salaries of the mansabdars from the middle of Akbar's reign is recorded in the Ain-i Akbari, a detailed statistical account of the empire compiled in 1596–7 by Abu 'l-Fazl (1551–1602), a high-ranking court official [14]. All 249 officers of and above the (zat) rank 500 are mentioned by name in Ain-i Akbari and this includes the three royal princes: Salim (later Emperor Jahangir), Murad and Danyal. In addition, those names of commanders between 500 and 200 who were alive at the time the list was made are also mentioned, while for ranks below 200, only the number of officers who were alive are given. In all, we get information about 1388 mansabdars. This group accounted for the major share of the imperial revenue income: towards the end of Akbar's reign, the mansabdars and their followers consumed 82% of the total annual budget of Rupees 99 million [10].

Figure 2 (left) shows the distribution of rank as well as the monthly salary of the mansabdars mentioned in Ain-i Akbari. The bulk of the officers follow a power law distribution with the Pareto exponent $\alpha \sim 0.69$, with only the royal princes deviating from the general trend. As seen in the inset, the rank

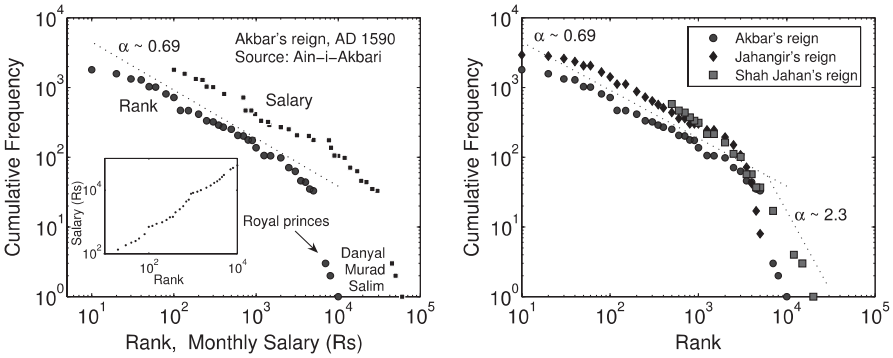

Fig. 2. (*left*) Cumulative frequency distribution of the Mansabdari rank (*filled circles*) and monthly salary in Rupees (*squares*) in the Mughal army around 1590 during the reign of Emperor Akbar. The *line* shows a power-law fit to the data giving a Pareto exponent of $\alpha \sim 0.69$. The *inset* shows the almost linear relation between rank and salary. (*Right*) A comparison of the cumulative frequency distribution of Mansabdari ranks during the successive reigns of Emperors Akbar, Jahangir and Shah Jahan. The data sources are Abu 'l-Fazl's Ain-i Akbari (Akbar), the Dutch traveler De Laët (Jahangir) and Lahori's Padishahnama (Shah Jahan). The latter does not include any information about officers below the rank of 500.

and the salary have an almost linear relationship, which enables us to form an idea of the income distribution in later reigns when sometimes only the rank distribution is available. Figure 2 (right) compares the distribution of mansabdar ranks during the successive reigns of Akbar, Jahangir and Shah Jahan, the bulk of the officers in all cases being described by a Pareto exponent ~ 0.69[2]. However, as in Akbar's reign, the distribution shows a deviation for the highest ranking officers in the later reigns also, with fewer occupants of these ranks than would be expected from the general trend. This is explained to some extent when we realize that these ranks in almost all cases were reserved for the royal princes. Trying to fit a different power law tail to this part of the data gives us a Pareto exponent of $\alpha \sim 2.3$.

2.3 Income of Members of a Chicago Criminal Gang, 1990s

From a consideration of the hierarchy and income distribution of societies from the long-distant past, we now travel to the present, where a unique study enables us to look at the wages of the members of a criminal organization [15]. The urban street gang which was the subject of this study, a branch of the

[2] It is expected that the nature of the income distribution would also have remain unchanged over this period. Differences would be almost entirely reflected as displacement along the salary axis: because of inflation, the salary of a mansabdar during Shah Jahan's reign was approximately 8 times that of a mansabdar of the same rank during Akbar's reign.

Black Gangster Disciple Nation, operated from Chicago's south side and its principal source of revenue (accounting for more than 75% of the total earnings) was through selling crack cocaine in the street. The street-level salesmen, referred to as foot-soldiers, formed the base of the hierarchical organization of the gang (shown in Fig. 3, left). Other sources of revenue included gang dues from rank-and-file members (full members did not pay dues) and extortion from businesses operating in the gang's territory.

The top level of the organization was made up by a elite group of individuals who were responsible for long-term strategy, maintaining relations with suppliers and affiliates in other regions, and overall gang management [15]. The next level comprised local leaders responsible for specific territories, each of whom were assisted by three "officers", who formed the next lower level. Below them, the foot-soldiers were organized in teams of six, with a team leader, carrier, lookout, etc. Foot-soldiers were paid a flat wage rather than a wage linked to the sales, with the team leader getting the highest and the lookout the lowest wage in the team. However, the actual wage data was not resolved to this level of detail, with only the total wages paid to the foot soldiers under a local leader having been recorded. Figure 3 (right) shows the distribution of monthly salary at the different levels of the gang. It is worth noting that the distribution is extremely skewed, with the local leaders earning 10–20 times more than the average foot soldier. The insets show the exponentially decreasing number of people at each level, and the exponentially increasing salary, up the hierarchy. The same trend has been observed in a study of the wage structure in a medium-sized US firm in a service industry over the years 1969–1988 [16]. We will see in the next section that such oppositely behaving

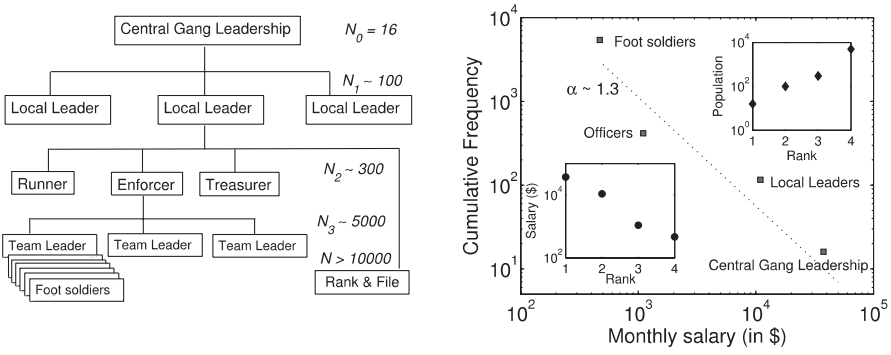

Fig. 3. (*left*) The organizational hierarchy of a Black Disciple Gangster Nation (BDGN) gang in Chicago around the 1990s. The number of gang members at each level is shown on the right. (*Right*) Cumulative frequency distribution of the monthly income for members of the BDGN gang. The *dotted line* is a power law fit with Pareto exponent ~ 1.3. The *upper inset* shows the exponentially increasing number of members at progressively lower levels in the hierarchy, while the *lower inset* shows exponentially decreasing income down the ranks.

exponential functions for the rank-population and rank-salary relations are related to the power law form of the income distribution.

2.4 Management Salary Structure in Indian Companies, 2006

Our final set of empirical data concerns the income of management employees in several Indian companies belonging to the Manufacturing and Information Technology (IT) sectors. Corporate salaries are often quite transparently a manifestation of the hierarchical organization structure of a company. We analysed the salary data from 21 Manufacturing Companies (e.g., Bajaj Auto, Cadbury, Himalaya Drug, etc) and 16 IT Companies (e.g., HCL, Oracle, Satyam, TCS, etc), the bulk of the data being obtained from an internet-based labor market survey site [17], and verified in selected cases with information from the company's annual report.

Figure 4 shows the organization structure of the management in a typical Indian company that was included in our study. The hierarchy starts at the level of executives, and goes up through managers and divisional heads all the way to the Chief Operating Officer or Vice President. The number of levels need not be same in all divisions, e.g., sales and marketing typically has more layers than other divisions, which is reflected in a relatively lower salary ratio between successive levels for people in sales. The salaries at different levels in selected key divisions in companies belonging to the Manufacturing and IT sectors are shown in Fig. 5. The data show exponentially increasing salary along the levels to the top of the hierarchy, a feature already observed in the previous sub-section. Our evidence seems to support that the salary structure

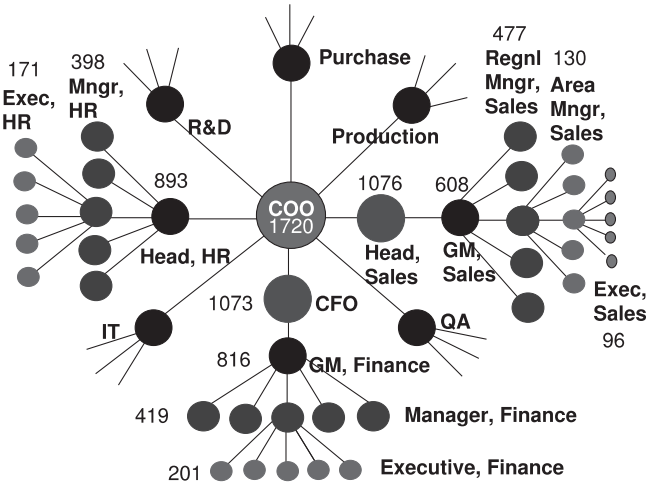

Fig. 4. Organization of the management in a typical Indian company belonging to the manufacturing sector. The numbers correspond to representative annual salaries (in thousands of Rupees) for the respective positions.

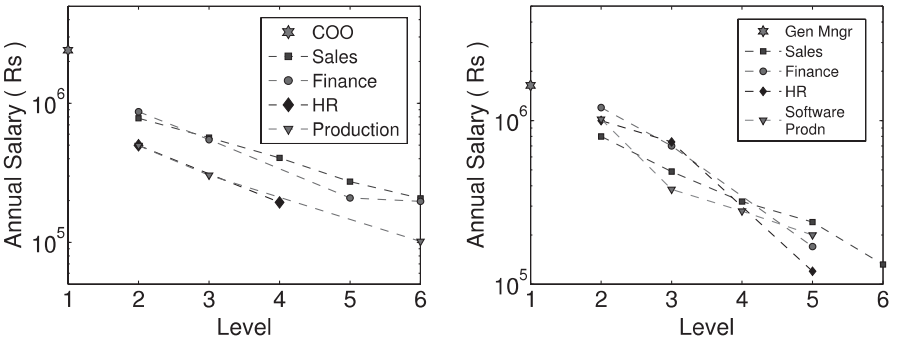

Fig. 5. The management salary structure of (*left*) an Indian company belonging to the manufacturing sector, different from the one whose organization structure is shown in Fig. 4, and (*right*) that of an Indian company belonging to the Information Technology sector, for the same few divisions in each. Note that salaries are shown on a logarithmic scale, so that exponential increase in salary up the hierarchy will be manifested as a linear trend in the figure.

in IT companies is *relatively* more egalitarian than Manufacturing companies, but, on the whole, all companies exhibit similarly skewed salary distribution along their hierarchical structure.

3 Income as Flow along Hierarchical Structure: The Tribute Model

Having seen evidence for a causal relation between long-tailed income distribution and the occurrence of hierarchical structures, we will now consider income distribution as a problem in physics. In particular, we consider income as resource flow along a hierarchically branched structure. We assume the existence of this hierarchy, and observe how power law distribution of resources at the various levels can arise as a result of flow along the structure.

We consider a strict hierarchy of N levels: at each layer, a node is connected to M subordinates belonging to the level below it. The salary of each node is proportional to the total information arriving at it for processing, i.e., the total number of nodes at the level below that it is connected to. Moreover, the income for a node is the difference between the total inflow from the nodes below it and the outflow to the node above it. We shall call this *the tribute model*, as the net flow up the hierarchy can be seen as tribute paid by agents belonging in the lower levels of the hierarchy to those above. An obvious realization of this model is in a criminal organization, where the people at the base put a fraction of their earnings for the disposal of their immediate boss, who in turn sends a fraction to his boss, and so on. We now show that, under certain circumstances, the resulting income distribution will have a power-law form.

Let the total number of nodes at layer q be n_q. Without loss of generality, we can assume that the income of each node at the base is 1. Each node sends a fraction f of its income to its immediate superior, so that income of node at level q is $I_q = fMI_{q+1}$, with net income at the base being $I_N = 1 - f$, while that at the top is $I_1 = (fM)^{N-1}$ (Fig. 6). Thus, the income of a level-q node in terms of the parameters N, M and f is

$$I_q = (1 - f)(fM)^{N-q}. \tag{1}$$

For the income distribution, we obtain n_q in terms of I_q as

$$n_q = n(I) \sim I^{-\log(1-f)/[\log M + \log f]}, \tag{2}$$

which has a power law form. For example, if $f = 1/2$ and $M = 3$, the distribution will be a power law having exponent $\simeq 1.7$. While it may appear that the parameters can be freely chosen to obtain any exponent whatsoever, there are certain restrictions in reality, such as, for a node at the upper layer to benefit from the arrangement, it is necessary that $fM > 1$.

Note that, the salary ratio between two consecutive levels is given by $SR = I_q/I_{q+1} = fM$, so that the top-to-bottom income ratio is

$$T_R = \frac{(fM)^N}{1 - f} = \frac{(SR)^N}{1 - (SR/M)}. \tag{3}$$

For example, if $N = 5$, $M = 5$ and $f = 1/2$, we obtain $T_R = 200$. If, as is empirically observed, T_R remains fairly constant for hierarchical organizations

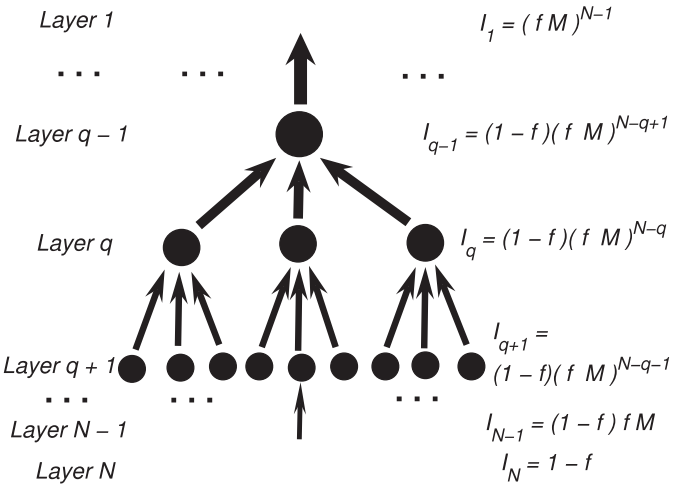

Fig. 6. A schematic diagram of the tribute model representing flow of resources along a hierarchical structure, with the income of nodes at each level q shown in the *right*.

with different number of levels, then it follows that as the number of levels, N, increases, the salary ratio between successive levels ($= fM$) decreases. This has indeed been observed by us in the data for management pay structure in Indian companies.

So far we had been concerned with inter-level differences in income. Within a level also, incomes for individuals will differ. However, as suggested by Gibrat [18], this is likely to be decided by large number of independent stochastic factors, such that the intra-level income distribution will most likely be log-normal, the outcome of a multiplicative stochastic process. When seen in conjunction with the multiple hierarchical levels which ensures that the gap between mean income at each level have exponentially increasing separation along with exponentially decreasing population at each level up the hierarchy, this will imply that within a given level the income distribution is log-normal, but a Pareto like power law behavior will describe the overall inequality, as inter-level differences will tend to dominate intra-level deviations within an organization.

4 Conclusions

In this paper, we have examined the co-occurrence of hierarchical social structures and long-tailed income distributions in many empirical examples, which suggest a causal relation between the two. By considering income distribution as a resource flow problem along a hierarchical structure, we see that it follows a power law form with Pareto exponents similar to those seen in reality. Although factors other than social stratification do play a role in deciding income of individuals, it may be that the broader features of long-tailed income distributions, including the Pareto law behavior at the upper tail, is explained in terms of organizational hierarchy. A future challenge lies in explaining why such hierarchical structures emerge spontaneously in society [19].

Acknowledgement. We thank R.K. Pan for helpful suggestions. SS would like to thank M. Sanyal of the Jadunath Sarkar Bhavan Library, Centre for Studies in Social Sciences, Kolkata for providing invaluable assistance in obtaining the income data for Mughal mansabdars.

References

1. Persky J (1992) Pareto's Law, J. Economic Perspectives 6: 181–192
2. Pareto V (1897) Cours d'Economique Politique, vol 2. Macmillan, London
3. Chatterjee A, Sinha S, Chakrabarti B K (2007) Economic inequality: Is it natural? E-print physics/0703201
4. Price T D, Feinman G M (1997) Images of the Past, 2nd edition. Mayfield Publishing, Mountain View, Calif.

5. Milanovic B (2006) An estimate of average income and inequality in Byzantium around year 1000, Review of Income & Wealth 52: 449–470
6. Duncan-Jones R (1994) Money and Government in the Roman Empire. Cambridge Univ Press, Cambridge
7. Oikonomides N (2002) The role of the Byzantine state in the economy. In: Laiou A E (ed) The Economic History of Byzantium. Harvard Univ Press, Cambridge, Mass.
8. Morrisson C, Cheynet J-C (2002) Prices and wages in the Byzantine world. In: Laiou A E (ed) The Economic History of Byzantium. Harvard Univ Press, Cambridge, Mass.
9. http://joinindianarmy.nic.in/nda13.htm
10. Richards J F (1993) The Mughal Empire. Cambridge Univ Press, Cambridge
11. Chandra S (1982) Standard of living: Mughal India. In: Raychaudhuri T, Habib I (eds) The Cambridge Economic History of India, Vol 1. Cambridge Univ Press, Cambridge
12. Habib I (1974) The social distribution of landed property in pre-British India: A historical survey. In: Sharma R S, Jha V (eds) Indian Society: Historical Probings. Peoples Publishing House, New Delhi
13. Mansabdar, entry in Banglapedia: National Encyclopedia of Bangladesh (http://banglapedia.net/)
14. Blochmann H (1927) Translation of Ain-i Akbari by Abu 'l-fazl Allami, Vol 1, 2nd edition (ed. D C Phillott). Asiatic Society, Calcutta
15. Venkatesh S A, Levitt S D (2000) The financial activities of an urban street gang, Qtrly. J. Econ. 115:755–789
16. Baker G, Gibbs M, Holmstrom B (1994) The internal economics of the firm: Evidence from personnel data, Qtrly. J. Econ. 109: 881–919
17. http://www.paycheck.in/
18. Gibrat R (1931) Les Inégalites Économiques. Librairie du Recueil Sirey, Paris
19. Watts D J, Dodds P S, Newman M E J (2002) Identity and search in social networks, Science 296: 1302–1305