Econophysics
An Emerging Discipline

Sitabhra Sinha, Bikas K Chakrabarti

Contemporary mainstream economics has become concerned less with describing reality than with an idealised version of the world. However, reality refuses to bend to the desire for theoretical elegance that an economist demands from his model. Modelling itself on mathematics, mainstream economics is primarily deductive and based on axiomatic foundations. Econophysics seeks to be inductive, to be an empirically founded science based on observations, with the tools of mathematics and logic used to identify and establish relations among these observations. Econophysics does not strive to reinterpret empirical data to conform to a theorist's expectations, but describes the mechanisms by which economic systems actually evolve over time.

Sitabhra Sinha (*sitabhra@imsc.res.in*) is with the Institute of Mathematical Sciences, Chennai and the National Institute of Advanced Studies, Bangalore; Bikas K Chakrabarti (*bikask.chakrabarti@saha.ac.in*) is with the Saha Institute of Nuclear Physics, Kolkata and the Economic Research Unit, Indian Statistical Institute, Kolkata.

> [Economics should be] concerned with the derivation of operationally meaningful theorems ... [Such a theorem is] simply a hypothesis about empirical data which could conceivably be refuted, if only under ideal conditions.
>
> – Paul A Samuelson (1947)

> I suspect that the attempt to construct economics as an axiomatically based hard science is doomed to fail.
>
> – Robert Solow (1985)

It had long been thought that the cyclical sequence of inflations and recessions that have buffeted most national economies throughout the 19th and 20th centuries are an inevitable accompaniment to modern capitalism. However, starting in the 1970s, economists allied with the influential Chicago school of economics started to promote the belief that the panacea to all economic ills of the world lay in completely and unconditionally subscribing to their particular brand of free-market policies. Their hubris reached its apogee at the beginning of the previous decade, as summed up by the statement of Nobel Laureate Robert Lucas (2003) at the annual meeting of the American Economic Association that "the central problem of depression prevention has been solved, for all practical purposes". This complacency about the robustness of the free-market economic system to all possible disturbances led not only most professional economists, but also, more importantly, government bureaucrats and ministers to ignore or downplay the seriousness of the present economic crisis in its initial stages – recall, for instance, the now infamous claim of British prime minister Gordon Brown (2007) that economic booms and busts were a thing of the past ("And we will never return to the old boom and bust") just a few months ahead of the global financial meltdown. As many of the recent books published in the wake of the financial systemic collapse point out, the mainstream economists and those whom they advised were blinded by their unquestioning acceptance of the assumptions of neoclassical economic theory (for example, Posner 2009). On hindsight, the following lines written by Canadian anthropologist Bruce Trigger (1998) a decade before the present crisis seem eerily prophetic.

> In the 1960s I never imagined that the 1990s would be a time when highly productive western economies would be accompanied by growing unemployment, lengthening breadlines, atrophying educational systems, lessening public care for the sick, and the aged, and the handicapped, and growing despondency and disorientation – all of which would be accepted in the name of a 19th century approach to economics that had been demonstrated to be dysfunctional already by the 1920s.

The late 2000s crisis (variously described as probably equal to or worse than the Great Depression of the 1930s in terms of severity) has by now led to a widespread discontent with mainstream economics. Several scientists, including physicists working on theories of economic phenomena (for example, Bouchaud 2008) and non-traditional economists who have collaborated with physicists (for example, Lux and Westerhoff 2009), have written articles in widely circulated journals arguing that a "revolution" is needed in the way economic phenomena are investigated. They have pointed out that academic economics, which could neither anticipate the current worldwide crisis nor gauge its seriousness once it started, is in need of a complete overhaul as this is a systemic failure of the discipline. The roots of this failure have been traced to the dogmatic adherence to deriving elegant theorems from "reasonable" axioms, with complete disregard to empirical data. While it is perhaps not surprising for physicists working on social and economic phenomena to be critical of mainstream economics and suggest the emerging discipline of *econophysics* as a possible alternative theoretical framework, even traditional economists have acknowledged that not everything is well with their discipline (Sen 2009).

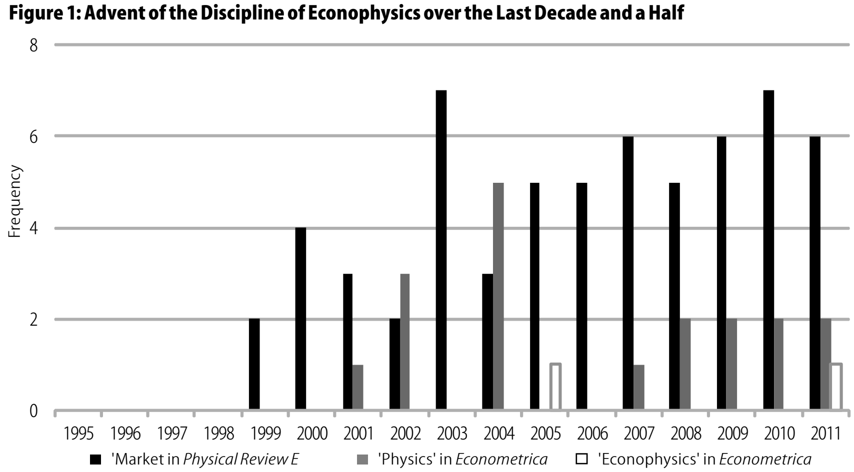
In response to the rising criticism of traditional economic theory, some mainstream economists have put up the defence that the sudden collapse of markets and banks is not something that can be predicted by economic theory as this contradicts their basic foundational principles of rational expectations and efficient markets. Thus, according to the conventional economic school of thought, bubbles cannot exist because any rise in price must reflect all information available about the underlying asset (Fama 1970). Although detailed analysis of data from markets clearly reveals that much of the observed price fluctuation cannot be explained in terms of changes in economic fundamentals, especially during periods of "irrational exuberance" (Shiller 2005), the unquestioning belief in the perfection of markets has prompted several economists in past decades to assert that the famous historical bubbles, such as Tulipomania in 17th century Holland or the South Sea Affair in 18th century England, were not episodes of price rise driven by irrational speculation as is generally believed, but rather were based on sound economic reasons (see, for example, Garber 1990)! This complete divorce of theory from observations points to the basic malaise of mainstream economics. What makes it all the more worrying is that despite the lack of any empirical verification, such economic theories have been used to guide the policies of national and international agencies affecting the well-being of billions of human beings.

In its desperate effort to become a rigorous science by adopting, among other things, the formal mathematical framework of game theory, mainstream economics has become concerned less with describing reality than with an idealised version of the world. However, reality refuses to bend to the desire for theoretical elegance that an economist demands from his/her model. Unlike the utility maximising agents so beloved of economists, in our day-to-day life we rarely go through very complicated optimisation processes in an effort to calculate the best course of action. Even if we had access to complete information about all the options available (which is seldom the case), the complexity of the computational problem would overwhelm our decision-making capabilities. Thus, most often we are satisfied with choices that seem "good enough" to us, rather than the best one under all possible circumstances. Moreover, our choices may also reflect non-economic factors such as moral values that are usually not taken into consideration in mainstream economics.

Econophysics: A New Approach to Understand Socio-economic Phenomena

Given that the hypotheses of efficient markets and rational agents cherished by mainstream economists stand on very shaky ground, the question obviously arises as to whether there are any alternative foundations that can replace the neoclassical framework. Behavioural economics, which tries to integrate the areas of psychology, sociology and economics, has recently been forwarded as one possible candidate (Sen 2009). Another challenger from outside the traditional boundaries of economics is a discipline that has been dubbed econophysics (Yakovenko and Rosser 2009; Sinha et al 2011). Although it is difficult to arrive at a universally accepted definition of the discipline, a provisional one given in Wikipedia is that it is "an interdisciplinary research field, applying theories and methods originally developed by physicists in order to solve problems in economics, usually those including uncertainty or stochastic processes and non-linear dynamics" (see http://en.wikipedia.org/wiki/Econophysics). This flourishing area of research that started in the early 1990s has already gone through an early phase of rapid growth and is now poised to become a major intellectual force in the world of academic economics. This is indicated by the gradual rise in appearance of the terms "physics" and "econophysics" in major journals in economics; as is also seen in the frequency with which the keyword "market" appeared in papers published in important physics journals (Figure 1). In fact,

Figure 1: Advent of the Discipline of Econophysics over the Last Decade and a Half



The number of papers appearing in *Physical Review E* (published by the American Physical Society) with the word "market" in the title published in each year since 1995 (when the term "econophysics" was coined) and those appearing in *Econometrica* (published by the Econometric Society) with the words "physics" and "econophysics" anywhere in the text published each year since 1999. Data obtained from respective journal websites.

even before the current economic crisis, the economics community had been grudgingly coming to recognise that econophysics can no longer be ignored as a passing fad, and the *New Palgrave Dictionary of Economics* published in 2008 has entries on "Econophysics" (which it defines as "...refers to physicists studying economics problems using conceptual approaches from physics" (Rosser 2008) as well as on "Economy as a Complex System". Unlike contemporary mainstream economics, which models itself on mathematics and is primarily deductive and based on axiomatic foundations, econophysics seeks to be inductive, that is, an empirically founded science based on observations, with the tools of mathematics and logic being used to identify and establish relations among these observations.

The Origins of Econophysics

Although physicists had earlier worked on economic problems occasionally, it is only since the 1990s that a systematic, concerted movement has begun which has seen more and more physicists using the tools of their trade to analyse phenomena occurring in a socio-economic context (Farmer et al 2005). This has been driven partly by the availability of large quantities of high-quality data and the means to analyse it using computationally intensive algorithms. In the late 1980s, condensed matter physicist Philip Anderson jointly organised with Kenneth Arrow a meeting between physicists and economists at the Santa Fe Institute that resulted in several early attempts by physicists to apply the then recently developed tools in non-equilibrium statistical mechanics and non-linear dynamics to the economic arena (some examples can be seen in the proceedings of this meeting, *The Economy as an Evolving Complex System*, 1988). It also stimulated the entry of other physicists into this interdisciplinary research area, which, along with slightly later developments in the statistical physics group of H Eugene Stanley at Boston University, finally gave rise to econophysics as a distinct field, the term being coined by Stanley in 1995 at Kolkata. Currently there are groups in physics departments around the world who are working on problems relating to economics, ranging from Japan to Brazil, and from Ireland to Israel.

While the problems they work on are diverse, ranging from questions about the nature of the distribution of price fluctuations in the stock market to models for explaining the observed economic inequality in society to issues connected with how certain products become extremely popular while almost equivalent competing products do not acquire significant market share, a common theme has been the observation and explanation of scaling relations (that is, the *power-law* relationship between variables x, y having the form $y \sim x^a$, that, when plotted on a doubly-logarithmic graph paper, appears as a straight-line with slope a, which is termed the *exponent*). Historically, scaling relations have fascinated physicists because of their connection to critical phenomena and phase transitions, for example, the phenomenon through which matter undergoes a change of state, say, from solid to liquid, or when a piece of magnetised metal loses its magnetic property when heated above a specific temperature. More generally, they indicate the absence of any characteristic scale for the variable being measured, and therefore the presence of universal behaviour, as the relationship is not dependent on the details of the nature or properties of the specific system in which it is being observed. Indeed, the quest for invariant patterns that occur in many different contexts may be said to be the novel perspective that this recent incursion of physicists have brought to the field of economics (for examples of unusual scaling relations observed in social and economic phenomena, see Sinha and Raghavendra 2004; Sinha and Pan 2007; Pan and Sinha 2010). This may well prove to be the most enduring legacy of econophysics.

Economics and Physics: The Past ...

Of course, the association between physics and economics is itself hardly new. As pointed out by Mirowski (1989), the pioneers of neoclassical economics had borrowed almost term by term the theoretical framework of classical physics in the 1870s to build the foundation of their discipline. One can see traces of this origin in the fixation that economic theory has with describing equilibrium situations, as is clear from the following statement of Pareto (1906) in his textbook on economics.

> The principal subject of our study is economic equilibrium. ... this equilibrium results from the opposition between men's tastes and the obstacles to satisfying them. Our study includes, then, three distinct parts: (1) the study of tastes; (2) the study of obstacles; (3) the study of the way in which these two elements combine to reach equilibrium.

Another outcome of this historical contingency of neoclassical economics being influenced by late 19th century physics is the obsession of economics with the concept of maximisation of individual utilities. This is easy to understand once we remember that classical physics of that time was principally based on energy minimisation principles, such as the Principle of Least Action (Feynman 1964). We now know that even systems whose energy function cannot be properly defined can nevertheless be rigorously analysed, for example, by using the techniques of non-linear dynamics. However, academic disciplines are often driven into certain paths constrained by the availability of investigative techniques, and economics has not been an exception.

There are also several instances where investigations into economic phenomena have led to developments that have been followed up in physics only much later. For example, Bachelier developed the mathematical theory of random walks in his 1900 thesis on the analysis of stock price movements and this was independently discovered five years later by Einstein to explain Brownian motion (Bernstein 2005). The pioneering work of Bachelier had been challenged by several noted mathematicians on the grounds that the Gaussian distribution for stock price returns as predicted by his theory is not the only possible stable distribution that is consistent with the assumptions of the model (a distribution is said to be *stable* when linear combinations of random variables independently chosen from it have the same functional form for their distribution).

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This foreshadowed the work on Mandelbrot in the 1960s on using Levy-stable distributions to explain commodity price movements (Mandelbrot and Hudson 2004). However, recent work by H E Stanley and others have shown that Bachelier was right after all – stock price returns over very short times do follow a distribution with a long tail, the so-called "inverse cubic law", but being unstable, it converges to a Gaussian distribution at longer timescales (for example, for returns calculated over a day or longer) (Mantegna and Stanley 1999). Another example of how economists have anticipated developments in physics is the discovery of power laws of income distribution by Pareto in the 1890s, long before such long-tailed distributions became interesting to physicists in the 1960s and 1970s in the context of critical phenomena.

With such a rich history of exchange of ideas between the two disciplines, it is probably not surprising that Samuelson (1947) tried to turn economics into a natural science in the 1940s, in particular, to base it on "operationally meaningful theorems" subject to empirical verification (see the opening quotation of this article). But in the 1950s, economics took a very different turn. Modelling itself more on mathematics, it put stress on axiomatic foundations, rather than on how well the resulting theorems matched reality. The focus shifted completely towards derivation of elegant propositions untroubled by empirical observations. The divorce between theory and reality became complete soon after the analysis of economic data became a separate subject called econometrics. The separation is now so complete that even attempts from within mainstream economics to turn the focus back to explaining real phenomena (as for example the work of Steven Levitt, which has received wide general acclaim through its popularisation in Levitt and Dubner 2005) has met with tremendous resistance from within the discipline.

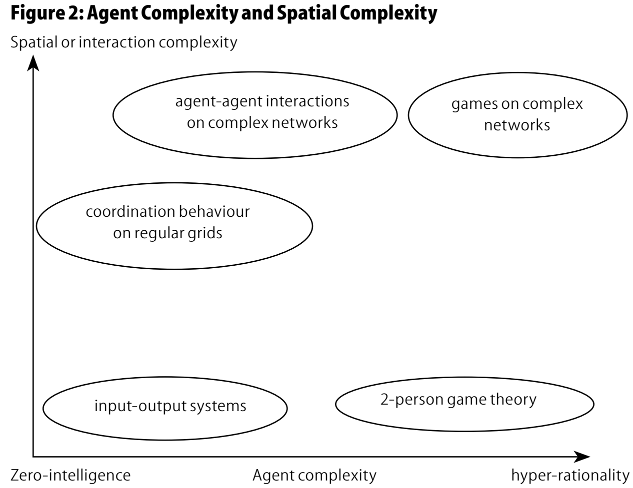
On hindsight, the seismic shift in the nature of economics in the 1950s was probably not an accident. Physics of the first half of the 20th century had moved so faraway from explaining the observable world that by this time it did not really have anything significant to contribute in terms of techniques to the field of economics. The quantum mechanics-dominated physics of those times would have seemed completely alien to anyone interested in explaining economic phenomena. All the developments in physics that have contributed to the birth of econophysics, such as non-linear dynamics or non-equilibrium statistical mechanics, would flower much later, in the 1970s and the 1980s.

Some economists have said that the turn towards game theory in the 1950s and 1960s allowed their field to describe human motivations and strategies in terms of mathematical models. This was truly something new, as the traditional physicist's view of economic agents was completely mechanical – almost like the particles described by classical physics whose motions are determined by external forces. However, this movement soon came to make a fetish of "individual rationality" by overestimating the role of the "free will" of agents in making economic choices, something that ultraconservative economists with a right-wing political agenda probably deliberately promoted. In fact, it can be argued that the game-theoretic turn of economics led to an equally mechanical description of human beings as selfish, paranoid agents whose only purpose in life is to devise strategies to maximise their utilities. An economist has said that (quoted in Sinha 2010b) this approach views all economic transactions, including the act of buying a newspaper from the street corner vendor, to be as complicated as a chess game between Arrow and Samuelson, the two most notable American economists of the post-second world war period. Surely, we do not solve complicated optimisation problems in our head when we shop at our local grocery store. The rise of bounded rationality and computable economics reflects the emerging understanding that human beings behave quite differently from the hyper-rational agents of classical game theory, in that they are bound by constraints in terms of space, time and the availability of computational resources.

Economics and Physics: ... and the Future

Maybe it is time again for economics to look at physics, as the developments in physics during the intervening period such as non-equilibrium statistical mechanics, theory of collective phenomena, non-linear dynamics and complex systems theory, along with the theories developed for describing biological phenomena, do provide an alternative set of tools to analyse, as well as a new language for describing, economic phenomena. The advent of the discipline of econophysics has shown how a balanced marriage of economics and physics can work successfully in discovering new insights. An example of how it can

Figure 2: Agent Complexity and Spatial Complexity



The wide spectrum of theories proposed for explaining the behaviour of economic agents, arranged according to agent complexity (abscissa) and interaction or spatial complexity (ordinate). Traditional physics-based approaches stress interaction complexity, while conventional game theory focuses on describing agent complexity.

go beyond the limitations of the two disciplines out of which it is created is provided by the recent spurt of work on using game theory in complex networks (see Szabo and Fath (2007) for a review). While economists had been concerned exclusively with the rationality of individual agents (see the horizontal or agent complexity axis in Figure 2), physicists have been more concerned with the spatial or interaction complexity of agents (see the vertical axis in Figure 2) having limited or zero intelligence. Such emphasis on only interaction-level complexity has been the motivating force of the field of complex networks that has developed over the last decade (Newman 2010). However, in the past few years, there has been a sequence of well-received papers on games on complex networks that explore both types of complexities – in terms of interactions between agents, as well as, decision-making by individual agents. There is hope that by emphasising the interplay between these two types of complexities, rather than focusing on any one of them (as had been done previously by economists using classical game theory or by physicists studying networks), we will get an understanding of how social networks develop, how hierarchies form and how interpersonal trust, which makes possible complex social structures and trade, can emerge.

The Indian Scene

Given that the term econophysics was coined in India, it is perhaps unsurprising that several Indian groups have been very active in this area. In 1994, at a conference organised in Kolkata several Indian economists (mainly from the Indian Statistical Institute; ISI) and physicists (including the authors) discussed possible formulations of certain economic problems and their solutions using techniques from physics. In one of the papers included in the proceedings of the meeting, possibly the first published joint paper written by an Indian physicist and an Indian economist, the possibility of ideal gas like models (discussed later) for a market was discussed (Chakrabarti and Marjit 1995). In recent times, physicists at Ahmedabad (Physical Research Laboratory; PRL), Chennai (Institute of Mathematical Sciences; IMSc), Delhi (University of Delhi), Kolkata (Indian Institute of Science Education and Research; IISER, ISI, Saha Institute of Nuclear Physics; SINP and Satyendra Nath Bose National Centre for Basic Sciences; SNBNCBS), Nagpur (University of Nagpur) and Pune (Indian Institute of Science Education and Research; IISER), to name a few, and economists collaborating with them (for example, from ISI Kolkata and Madras School of Economics, Chennai), have made pioneering contributions in the area, for example, modelling inequality distribution in society and the analysis of financial markets as complex networks of stocks and agents. The annual series of "Econophys-Kolkata" conferences organised by SINP (2005 onwards) and the meetings on "The Economy as a Complex System" (2005 and 2010) at IMSc Chennai have increased the visibility of this area to physicists as well as economists in India.

We shall now focus on a few of the problems that have fascinated physicists exploring economic phenomena.

Instability of Complex Economic Systems

Much of classical economic theory rests on the assumption that the economy is in a state of stable equilibrium, although it rarely appears to be so in reality. In fact, real economic systems appear to be far from equilibrium and share many of the dynamical features of other non-equilibrium complex systems, such as ecological food webs. Recently, econophysicists have focused on understanding a possible relation between the increasing complexity of the global economic network and its stability with respect to small variations in any of the large number of dynamical variables associated with its constituent elements (that includes firms, banks, government agencies, and the like). The intrinsic delays in communication of information through the network and the existence of phenomena that happen at multiple timescales suggest that economic systems are more likely to exhibit instabilities as their complexity is increased. Although the speed at which economic transactions are conducted has increased manifold through technological developments, arguments borrowed from the theory of complex networks show that the system has actually become more fragile, a conclusion that appears to have been borne out by the recent worldwide financial crisis during 2007-09. Analogous to the birth of non-linear dynamics from the work of Henri Poincare on the question of whether the solar system is stable, similar theoretical developments may arise from efforts by econophysicists to understand the mechanisms by which instabilities arise in the economy (Sinha 2010a).

Box 1: Dynamical Systems and Non-linear Behaviour

The time-evolution of economic variables, such as the price of a commodity, may, in principle, be expressed in terms of ordinary differential equations (ODEs). If we denote the price at any given time t as p(t), then its instantaneous rate of change can be described by the ODE: $dp/dt = f(p(t))$, where f is a function that presumably contains information about how the supply and/or demand for the product changes given its price at that instant. In general, f can be quite complicated and it may be impossible to solve this equation. Moreover, one may be interested in the prices of more than one commodity at a given time, so that the system has multiple variables that are described by a set of coupled ODEs: $dpi/dt = fi\ (p_1, p_2, \ldots, p_i, \ldots p_N)$ with $i = 1, 2, \ldots, N$. Any such description for the time-evolution of (in general) many interacting variables we refer to as a *dynamical system*. While an exact solution of a many – variable dynamical system with complicated functions can be obtained only under special circumstances, techniques from the field of non-linear dynamics nevertheless allow one to obtain important information about how the system will behave qualitatively.

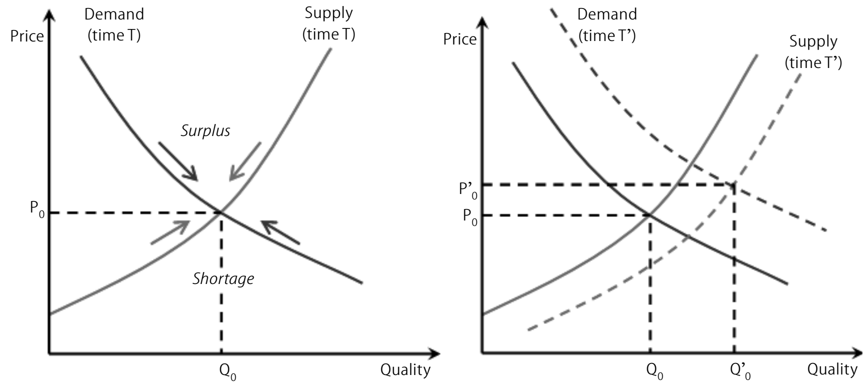
It is possible to define an *equilibrium state* for a dynamical system with price p* such that $f(p^*) = 0$, so that it does not change with time – for instance, when demand exactly equals supply. While for a given function f, an equilibrium can exist, we still need to know whether the system is likely to stay in that equilibrium even if somehow it is reached. This is related to the stability of the equilibrium p* which is measured by linearising the function f about p* and calculating the slope or derivative of the function at that point, that is, $f'(p^*)$. The equilibrium is stable if the slope is negative, with any change to the price decaying exponentially with a characteristic time $\tau = 1/|f'(p^*)|$ that is a measure of the rapidity of the price adjustment process in a market. On the other hand, if the slope is positive, the equilibrium is unstable – an initially small change to the equilibrium price grows exponentially with time so that the price does not come back to its equilibrium value. Unfortunately, linear analysis does not tell us about the eventual behaviour of the price variable as it is only valid close to the equilibrium; however, for a single variable ODE, only time-invariant equilibria are allowed (if one rules out unrealistic scenario of the variable diverging to infinity).

If we go over to the case of multiple variables, then other qualitatively different dynamical phenomena become possible, such as oscillations or even aperiodic chaotic activity. The state of the system is now expressed as a vector of the variables, for example, $p = \{p_1, p_2, \ldots, p_i, \ldots p_N\}$, the equilibria values for which can be denoted as p*. The stability of equilibria is now dictated by the Jacobian matrix J evaluated at the equilibrium p*, whose components, $J_{ij} = \partial f_i / \partial p_j$, are a generalisation of the slope of the function f that we considered for the single variable case. The largest eigenvalue or characteristic value of the matrix J governs the stability of the equilibrium, with a negative value indicating stability and a positive value indicating instability. Going beyond time-invariant equilibria (also referred to as fixed points), one can investigate the stability of periodic oscillations by using Floquet matrices. Even more complicated dynamical attractors (stable dynamical configurations to which the system can converge to starting from certain sets of initial conditions) are possible, for example, exhibiting chaos when the system moves aperiodically between different values while remaining confined within a specific volume of the space of all possible values of p.

Stability of Economic Equilibria

A widely cited example that shows the importance of non-linear dynamics (Box 1, p 48) in economics is the beer game devised by Jay Forrester at the Massachusetts Institute of Technology (MIT), which shows how fluctuations can arise in the system purely as a result of delay in the information flow between its components (Forrester 1961; also see Sterman 1989). In this game, various people take on the role of the retail seller, the wholesaler, the supplier and the factory, while an external observer plays the role of the customer, who places an order for a certain number of cases of beer with the retail seller at each turn of the game. The retailer in turn sends orders to the wholesaler, who places an order with the supplier, and so on in this way, all the way to the factory. As each order can be filled only once the information reaches the factory and the supply is relayed back to the retail seller, there is an inherent delay in the system between the customer placing an order and that order being filled. The game introduces penalty terms for overstocking (for example, having inventory larger than demand) and back-orders (for example, when the inventory is too small compared to the demand). Every person along the chain tries to minimise the penalty by trying to correctly predict the demand downstream. However, Forrester found that even if the customer makes a very small change in his/her pattern of demand (for example, after ordering two cases of beer for the first 10 weeks, the customer orders four cases of beer every week from the 11th week on until the end of the game), it sets off a series of perturbations up the chain which never settle down, the system exhibiting periodic or chaotic behaviour. Although the change in demand took place only once, the inherent instability of the system, once triggered by a small stimulus, ensures that equilibrium will never be reached. Based on this study, several scientists have suggested that the puzzle of trade cycles (where an economy goes through successive booms and busts, without any apparently significant external causes for either) may possibly be explained by appreciating that markets may possess similar delay-induced instabilities.

Figure 3: Price Mechanism Leading to Stable Equilibrium between Supply and Demand according to Traditional Economic Thinking



Left: The supply and demand curves indicate how increasing supply or decreasing demand can result in falling price or vice versa. If the available supply of a certain good in the market at any given time is less than its demand for it among consumers, its price will go up. The perceived shortage will stimulate an increase in production that will result in an enhanced supply. However, if supply increases beyond the point where it just balances the demand at that time, there will be unsold stock remaining which will eventually push the price down. This in turn will result in a decrease in production. Thus, a negative feedback control mechanism governed by price will move demand and supply along their respective curves to the mutual point of intersection, where the quantity available Q_0 at the equilibrium price P_0 is such that supply exactly equals demand.

Right: As the demand and supply of a product changes over time due to various different factors, the supply and demand curves may shift on the quantity-price space. As a result, the new equilibrium will be at a different price (P_0') and quantity (Q_0'). Until the curves shift again, this equilibrium will be stable, that is, any perturbation in demand or supply will quickly decay and the system will return to the equilibrium.

If the extrapolation from the beer game to real economics seems forced, consider this. Everyday the markets in major cities around the world, including those of Kolkata and Chennai, cater to the demands of millions of their inhabitants. But how do the merchants know how much goods to order so that they neither end up with a lot of unsold stock nor do they have to turn back shoppers for lack of availability of goods? How are the demands of the buyers communicated to the producers of goods without there being any direct dialogue between them? In this sense, markets are daily performing amazing feats of information processing, allowing complex coordination that in a completely planned system would have required gigantic investment in setting up communication between a very large number of agents (manufacturers and consumers). Adam Smith had, in terming it the "invisible hand" of the market, first pointed out one of the standard features of a complex system – the "emergence" of properties at the systems level that are absent in any of its components.

Economists often cite the correcting power of the market as the ideal negative feedback for allowing an equilibrium state to be stable. It is a very convincing argument that price acts as an efficient signalling system, whereby producers and consumers, without actually communicating with each other, can nevertheless satisfy each other's requirements. If the demand goes up, the price increases, thereby driving supply to increase. However, if supply keeps increasing, the demand falls. This drives the price down thereby signalling a cut-back in production. In principle, such corrections should quickly stabilise the equilibrium at which demand exactly equals supply. Any change in demand results in price corrections and the system quickly settles down to a new equilibrium where the supply is changed to meet the new level of demand (Figure 3). This is a classical example of self-organisation, where a complex system settles down to an equilibrium state without direct interaction between its individual components.

Unfortunately, this is only true if the system is correctly described by linear time-evolution equations. As the field of nonlinear dynamics has taught us, if there is delay in the system (as is true for most real-world situations), the assumptions underlying the situation described above break down, making the equilibrium situation unstable, so that oscillations appear. The classic analogy for the impact that delay can have in a dynamical system is that of taking a shower on a cold day, where the boiler is located sufficiently faraway that it takes a long time (say, a minute) to respond to changes in the turning of the hot and cold taps. The delay in the arrival of information regarding the response makes it very difficult to achieve the optimum temperature. A similar problem arises with timely information arrival but delayed response, as in the building of power plants to meet the changing needs for electrical power. As plants take a long time to build and have a finite lifetime, it is rarely possible to have exactly the number of plants needed to meet a changing demand for power. These two examples illustrates that a system cannot respond to changes that occur at a timescale shorter than that of the delays in the flow of information in it or its response. Thus, oscillations or what is worse, unpredictable chaotic behaviour, is the norm in most socio-economic complex systems that we see around us. Planning by forecasting possible future events is one way in which this is sought to be put within bounds, but that cannot eliminate the possibility of a rare large deviation that completely disrupts the system. As delays are often inherent to the system, the only solution to tackle such instabilities maybe to deliberately slowdown the dynamics of the system. In terms of the overall economy, it suggests that slowing the rate of economic growth can bring more stability, but this is a cost that many mainstream economists are not even willing to consider. While a freer market or rapid technological development can increase the rate of response, there are still delays in the system (as in gradual accumulation of capital stock) that are difficult to change. Thus, instead of solving the problem, these changes can actually end up making the system even more unstable.

Stability vs Complexity in Complex Systems

As already mentioned, traditionally, economics has been concerned primarily with equilibria. Figure 3 shows that the price mechanism was perceived by economists to introduce a negative feedback between perturbations in demand and supply, so that the system quickly settles to the equilibrium where supply exactly equals demand. Much of the pioneering work of Samuelson (1947), Arrow and Harwicz (1958); Arrow et al (1959) and others (for a review, see Negishi 1962) had been involved with demonstrating that such equilibria can be stable, subject to several restrictive conditions. However, the occurrence of complex networks (Box 2) of interactions in real life brings new dynamical issues to fore. Most notably, we are faced with the question: do complex economic networks give rise to instabilities? Given that most economic systems at present are composed of numerous strongly connected components, will periodic and chaotic behaviour be the norm for such systems rather than static equilibrium solutions?

This question has, of course, been asked earlier in different contexts. In ecology, it has given rise to the long-standing stability-diversity debate (see, for example, May 1973). In the network framework, the ecosystem can be thought of as a network of species, each of the nodes being associated with a variable that corresponds to the population of the species it represents. The stability of the ecosystem is then defined by the rate at which small perturbations to the populations of various species decay with time. If the disturbance instead grows and gradually propagates through the system affecting other nodes, the equilibrium is clearly unstable. Prior to the pioneering work of May in the 1970s, it was thought that increasing complexity of an ecosystem, either in terms of a rise in the total number of species or the density and strength of their connections, results in enhanced stability of the ecosystem. This belief was based on empirical observations that more diverse food webs (for example, in the wild) showed less violent fluctuations in population density than simpler communities (such as in fields under monoculture) and were less likely to suffer species extinctions. It has also been reported by Elton (1958) that tropical forests, which generally tend to be more diverse

Box 2: Complex Networks

Economic interactions in real life – be it in the nature of a trade, a credit-debit relation or formation of a strategic alliance – are not equally likely to occur between any and every possible pair of agents. Rather, such interactions occur along a network of relationships between agents that has a non-trivial structure, with only a few of all possible pair-wise interactions that are possible being actually realised.

Some agents can have many more interactions compared to others, a property that is measured by their degree (k), that is, the total number of other agents that the agent of interest has interactions with (its neighbours in the network). If the degree of an agent is much higher than the average degree for all agents in the network, it is called a hub. Hubs are commonly observed in networks with degree distribution having an extended tail, especially those referred to as scale-free networks that have a power-law form for the degree distribution $P(k) \sim k^{-\gamma}$. Other networks are distinguished by the existence of correlations between the degree of an agent and that of the other agents it interacts with. When agents having many interactions prefer to associate with other agents having many interactions, such a network is called *positively degree assortative* (that is, like connects with like); while in situations where agents with many interactions prefer to interact with other agents having few interactions, the network is referred to as *negatively degree assortative* (that is, like connects with unlike).

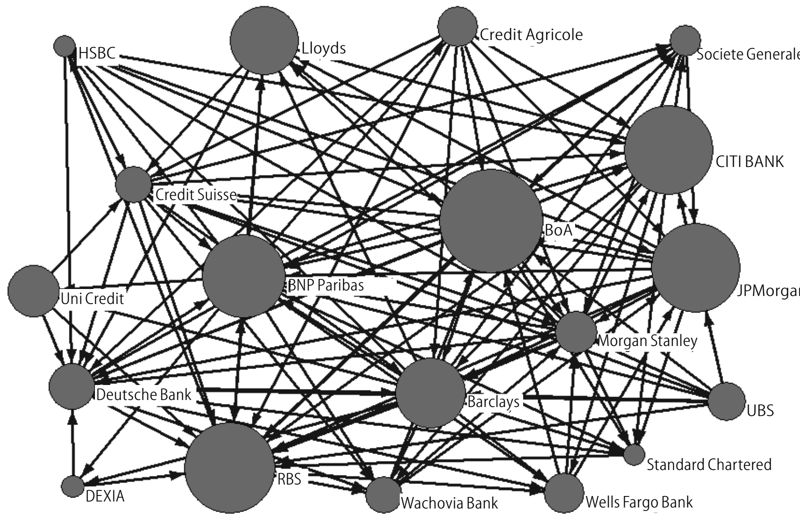
If the neighbours of an agent have many interactions between themselves, its neighbourhood is said to be cliquish (measured by the fraction of one's neighbours who are also mutual neighbours). The intensity of such cliquishness throughout the network is measured by the average *clustering*. The speed with which information can travel through the network is measured by the average *path length*, where the path length between any pair of agents is the shortest number of intermediate agents required to send a signal from one to the other. Many networks seen in real life have high clustering as well as short average path length and are often referred to as *small-world networks*, as any information can typically spread very fast in such systems, even though they have clearly defined local neighbourhoods. The properties so far described refer to either the network as a whole (global or macroscopic property) or an individual node or agent (local or microscopic property). Even if two networks share the same local as well as global properties, they can have remarkably distinct behaviour if they have different intermediate-level (mesoscopic) properties. One such property is the occurrence of *modularity* or community structure, where a module (or community) is defined as a subgroup of agents who have more interactions with each other than with agents outside the module. *Hierarchy* or the occurrence of distinct levels that constrain the types of interactions that agents can have with each other is another mesoscopic property seen in some social and economic networks.

If the distinction of different networks using the above-mentioned properties seems complicated, one should keep in mind that network structures may not be invariant in time. The topological arrangement of connections between agents can evolve, with the number of connections increasing or decreasing as new agents enter and old agents leave the system, as well as through rearrangements of links between existing agents. The past decade has seen an explosion of new models and results that go much beyond the classical results of graph theory (that had traditionally focused on random networks, where connections are formed with equal probability between any randomly chosen pair of nodes) or physics (which had been primarily interested in interactions arranged in periodic, ordered lattices that, while appropriate for many physical systems, are not suitable for describing socio-economic relations). Collectively, the newly proposed descriptions of networks are referred to as *complex networks* to distinguish them from both the random graphs and periodic lattices.

than subtropical ones, are more resistant to invasion by foreign species. It was therefore nothing short of a shock to the field when May (1972) showed that as complexity increases, linear stability arguments indicate that a randomly connected network would tend to become more and more unstable.

The surprising demonstration that a system which has many elements and/or dense connections between its elements is actually more likely to suffer potentially damaging large fluctuations initiated by small perturbations immediately led to a large body of work on this problem (see McCann 2000 for a review). The two major objections to May's results were (a) it uses linear stability analysis and that (b) it assumed random organisation of the interaction structure. However, more recent work which consider systems with different types of population dynamics in the nodes, including periodic limit-cycles and chaotic attractors (Sinha and Sinha 2005, 2006), as well as networks having realistic features such as clustered small-world property (Sinha 2005a) and scale-free degree distribution (Brede and Sinha 2005), have shown the results of increasing instability of complex networks to be extremely robust. While large complex networks can still arise as a result of gradual evolution, as has been shown by Wilmers et al (2002), it is almost inevitable that such systems will be frequently subject to large fluctuations and extinctions.

Figure 4: Network of Mutual Bilateral Exposures between Banks Having the Largest Core Capital in the OTC Financial Derivatives Market during the Last Quarter of 2009



The nodes represent financial intermediaries with the size of each node being proportional to the Tier-I capital of the bank, a measure of its financial strength. Arrows are directed from a bank to its net creditors. In the event of a default, the resulting disturbance will propagate through the network along the direction of the arrows (Sinha et al 2012).

Instability in Complex Economic Networks

The relevance of this body of work to understanding the dynamics of economic systems has been highlighted in the wake of the recent banking crisis when a series of defaults, following each other in a cascading process, led to the collapse of several major financial institutions. May and two other theoretical ecologists (2008) have written an article entitled "Ecology for Bankers" to point out the strong parallels between understanding collapse in economic and ecological networks. Recent empirical determination of networks occurring in the financial context, such as that of interbank payment flows between banks through the Fedwire real time settlement service run by the US Federal Reserve, has now made it possible to analyse the process by which cascades of failure events can occur in such systems. Soramaki et al (2007) have analysed such networks in detail and shown how their global properties change in response to disturbances such as the events of 11 September 2001. The dynamics of flows in these systems under different types of liquidity regimes have been explored by Beyeler et al (2007). Analogous to ecological systems, where population fluctuations of a single species can trigger diverging deviations from the equilibrium in the populations of other species, congestion in settling the payment of one bank can cause other pending settlements to accumulate rapidly, setting up the stage for a potential major failure event. It is intriguing that it is the very complexity of the network that has made it susceptible to such network propagated effects of local deviations making global or network-wide failure even more likely. As the world banking system becomes more and more connected (Figure 4), it may be very valuable to understand how the topology of interactions can affect the robustness of the network.

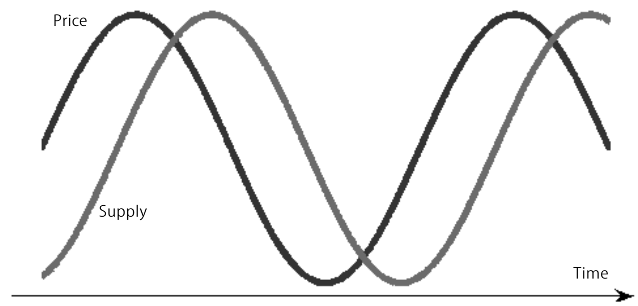
The economic relevance of the network stability arguments used in the ecological context can be illustrated from the following toy example (Sinha 2010a). Consider a model financial market comprising N agents where each agent can either buy or sell at a given time instant. This tendency can be quantitatively measured by the probability to buy, p, and its complement, the probability to sell, 1-p. For the market to be in equilibrium, the demand should equal supply, so that as many agents are likely to buy as to sell, that is, $p = 0.5$. Let us in addition consider that agents are influenced in their decision to buy or sell by the actions of other agents with whom they have interactions. In general, we can consider that out of all possible pairwise interactions between agents, only a fraction C is actually realised. In other words, the inter-agent connections are characterised by the matrix of link strengths $\boldsymbol{J}=\{J_{ij}\}$ (where $i,j=1, \ldots, N$ label the agents) with a fraction C of non-zero entries. If $J_{ij} > 0$, it implies that an action of agent j (buying or selling) is likely to influence agent i to act in the same manner, whereas $J_{ij} < 0$ suggests that the action of i will be contrary to that of j. Thus, the time-evolution of the probability for agent i to buy can be described by the following linearised equation close to the equilibrium $p_i = 0.5$ ($i=1,\ldots,N$):

$$dp_i/dt=\varepsilon_i\,(0.5 - p_i) + \Sigma_j\, J_{ij}\,(0.5 - p_j),$$

where ε_i is the rate of converge of an isolated node to its equilibrium state of equal probability for buying or selling. Without much loss of generality we can consider $\varepsilon_i = 1$ by appropriate choice of time units for the dynamics. If, in addition, we consider that for simplicity the interactions are assigned randomly from a Gaussian distribution with mean 0 and variance σ^2, then the largest eigenvalue of the corresponding Jacobian matrix $\boldsymbol{J}$ evaluated around the equilibrium is $\lambda_{max} = \sqrt{(NC\sigma^2\text{-}1)}$. For system parameters such that $NC\sigma^2 > 1$, an initially small perturbation will gradually grow with time and drive the system away from its equilibrium state. Thus, even though the equilibrium p=0.5 is stable for individual nodes in isolation, it may become unstable under certain conditions when interactions between the agents are introduced. Note that the argument can be easily generalised to the case where the distribution from which J_{ij} is chosen has a non-zero mean.

Another problem associated with the classical concept of economic equilibrium is the process by which the system approaches it. Walras, in his original formulation of how prices achieve their equilibrium value had envisioned the *tatonnement* process by which a market-maker takes in buy/sell bids from all agents in the market and gradually adjusts price until demand equals supply. Formally, it resembles an iterative convergence procedure for determining the fixed-point solution of a set of dynamical equations. However, as we know from the developments in non-linear dynamics over the past few decades, such operations on even simple non-linear systems (for example, the logistic equation; see May 1976) can result in periodic cycles or even chaos. It is therefore not surprising to consider a situation in which the price mechanism can actually result in supply and demand to be forever out of step with each other even though each is trying to respond to changes in the other. A simple situation in which such a scenario can occur is shown in Figure 5, where a delay in the response of the supply to the changes in price through variations in demand can cause persistent oscillations.

Figure 5: Persistent Price Oscillations Can Result from Delays in Market Response



Ideally the price mechanism should result in a transient increase (decrease) in demand to be immediately matched by a corresponding increase (decrease) in supply. However, in reality there is delay in the information about the rise or fall in demand reaching the producer; moreover, at the production end it may take time to respond to the increasing demand owing to inherent delays in the production system. Thus, the supply may always lag behind the price in a manner that produces oscillations – as price rises, supply initially remains low before finally increasing, by which time demand has fallen due to the high price which (in association with the increased supply) brings the price down. Supply continues to rise for some more time before starting to decrease. When it falls much lower than the demand, the price starts rising again, which starts the whole cycle anew. Thus, if the demand fluctuates at a timescale that is shorter than the delay involved in adjusting the production process to respond to variations in demand, the price may evolve in a periodic or even a chaotic manner.

Of course, the insight that delays in the propagation of information can result in oscillations is not new and can be traced back to the work of Kalecki (1935) on macroeconomic theory of business cycles. However, recent work on the role of network structure on the dynamics of its constituent nodes has produced a new perspective on this problem. If the principal reason for the instability is the intrinsic delay associated with responding to a time-evolving situation, one can argue that by increasing the speed of information propagation it should be possible to stabilise the equilibrium. However, we seem to have witnessed exactly the reverse with markets becoming more volatile as improvements in communication enable economic transactions to be conducted faster and faster.

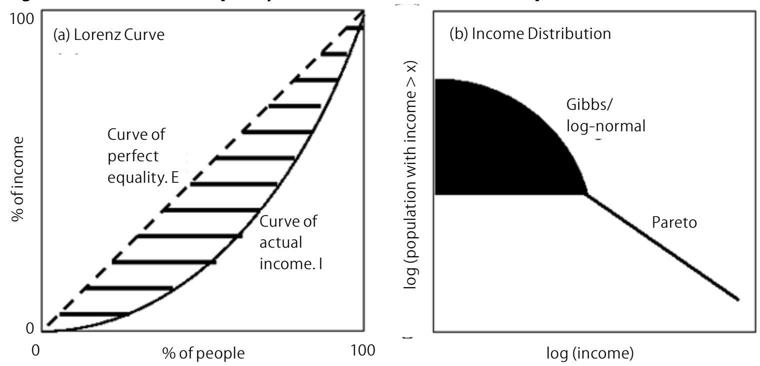
As Chancellor (1999) has pointed out in his history of financial manias and panics, "there is little historical evidence to suggest that improvements in communications create docile financial markets…". A possible answer to this apparent paradox lies in the fact that in any realistic economic situation, information about fluctuations in the demand may require to be relayed through several intermediaries before it reaches the supplier. In particular, the market may have a modular organisation, that is, segmented into several communities of agents, with interactions occurring significantly more frequently between agents belonging to the same community as opposed to those in different communities. This feature of modular networks can introduce several levels of delays in the system, giving rise to a multiple timescale problem – as has been demonstrated for a number of dynamical processes such as synchronisation of oscillators, coordination of binary decisions among agents and diffusion of contagion (see, for example, Pan and Sinha 2009; Sinha and Poria 2011).

In general, we observe that coordination or information propagation occurs very fast within a module (or community), but it takes extremely long to coordinate or propagate to different modules. For large complex systems, the different rates at which convergence to a local equilibrium (within a module) takes place relative to the time required to achieve global equilibrium (over the entire network) often allows the system to find the optimal equilibrium state (Pradhan et al 2011). Thus, increasing the speed of transactions, while ostensibly allowing faster communication at the global scale, can disrupt the dynamical separation between processes operating at different time-scales. This can prevent subsystems from converging to their respective equilibria before subjecting them to new perturbations, thereby always keeping the system out of the desired equilibrium state. As many socio-economically relevant networks exhibit the existence of many modules, often arranged into several hierarchical levels, this implies that convergence dynamics at several timescales may be competing with each other in sufficiently complex systems. This possibly results in persistent, large-scale fluctuations in the constituent variables that can occasionally drive the system to undesirable regimes.

Therefore, we see that far from conforming to the neoclassical ideal of a stable equilibrium, the dynamics of the economic system is likely to be always far from equilibrium (just as natural systems are always "out-of-equilibrium" (Prigogine and Stengers 1984)). In analogy with the question asked about ecological and other systems with many diverse interacting components, we can ask whether a sufficiently complex economy is bound to exhibit instabilities. After all, just like the neoclassical economists, natural scientists also at one time believed in the clockwork nature of the physical world, which in turn influenced

English philosopher Thomas Hobbes to seek laws for social organisation akin to Issac Newton's laws in classical mechanics. However, Poincare's work on the question of whether the solar system is stable showed the inherent problems with such a viewpoint and eventually paved the way for the later developments of chaos theory. Possibly we are at the brink of a similar theoretical breakthrough in econophysics, one that does not strive to reinterpret (or even ignore) empirical data to conform to a theorist's expectations but one which describes the mechanisms by which economic systems actually evolve over time. It may turn out that, far from failures of the market that need to be avoided, crashes and depressions may be the necessary ingredients of future developments, as has been suggested by Schumpeter (1975) in his theory of creative destruction.

Figure 6: Measures of Inequality: Gini Coefficient and Pareto Exponent



(a) The Gini coefficient, G, is proportional to the hatched area between the Lorenz curve (I), which indicates the percentage of people in society earning a specific per cent of the total income, and the curve corresponding to a perfect egalitarian society where everyone has the same income (E). G is defined to be the area between the two curves, divided by the total area below the perfect equality curve E, so that when G=0 everybody has the same income while when only one person receives the entire income, G=1.
(b) The cumulative income distribution (the population fraction having an income greater than a value x plotted against x) shown on a double logarithmic scale. For about 90-95% of the population, the distribution matches a Gibbs or Log-normal form (indicated by the shaded region), while the income for the top 5-10% of the population decays much more slowly, following a power-law as originally suggested by Pareto. The exponent of the Pareto tail is given by the slope of the line in the double-logarithmic scale, and was conjectured to be 1.5 for all societies by Pareto. If the entire distribution followed a power-law with exponent 1.5, then the corresponding Lorenz curve will have a Gini coefficient of 0.5, which is empirically observed for most developed European nations.

Explaining Inequality

The fundamental question concerning equality (or lack of it) among individuals in society is why neither wealth nor income is uniformly distributed? If we perform a thought experiment (in the best traditions of physics) where the total wealth of a society is brought together by the government and redistributed to every citizen evenly, would the dynamics of exchange subsequently result in the same inequality as before being restored rapidly? While such unequal distributions may to an extent be ascribed to the distribution of abilities among individuals, which is biologically determined, this cannot be a satisfying explanation. Distributions of biological attributes mostly have a Gaussian nature and, therefore, exhibit less variability than that seen for income and wealth. The distributions for the latter typically have extremely long tails described by a power law decay, that is, distributions that have the form $P(x) \sim x^{-\alpha}$ at the highest range of x where α is referred to as the scaling exponent. Indeed, econophysicists would like to find out whether inequality can arise even when individuals are indistinguishable in terms of their abilities (see Chatterjee et al 2007 for a review). It is of interest to note at this point that the functional form that characterises the bulk of the distribution of resources among individuals within a society appears to be similar to that which describes the distribution of energy consumption per capita by different countries around the world (Banerjee and Yakovenko 2010). As energy consumption provides a physical measure for economic prosperity and has been seen to correlate well with gross domestic product (GDP) per capita (Brown et al 2011), this suggests that there may be a universal form for the distribution of inequality, which applies to individuals as well as nations ("universal", in the sense used by physicists, indicate that the feature does not depend sensitively on system-specific details that vary from one instance to another).

Nature of Empirical Distribution of Income

Before turning to the physics-based models that have been developed to address the question of emergence of inequality distributions, let us consider the nature of the empirical distribution of inequality. Investigations over more than a century and the recent availability of electronic databases of income and wealth distribution (ranging from national sample survey of household assets to the income tax return data available from government agencies) have revealed some remarkable – and universal – features. Irrespective of many differences in culture, history, social structure, indicators of relative prosperity (such as GDP or infant mortality) and, to some extent, the economic policies followed in different countries, income distributions seem to follow an invariant pattern, as does wealth distribution. After an initial increase, the number density of people in a particular income bracket rapidly decays with their income. The bulk of the income distribution is well described by a Gibbs distribution or a lognormal distribution, but at the very high income range (corresponding to the top 5-10% of the population) it is fit better by a power law with a scaling exponent, between 1 and 3 (Figure 6). This seems to be a universal feature – from ancient Egyptian society through 19th century Europe to modern Japan. The same is true across the globe today: from the advanced capitalist economy of the US to the developing economy of India (Chatterjee et al 2007). Recently, the income distribution of Mughal *mansabdars*, the military administrative elite that controlled the empire of Akbar and his successors, has also been shown to follow a power-law form – a feature which has been sought to be explained through a model of resource flow in hierarchical organisations (Sinha and Srivastava 2007).

The power-law tail, indicating a much higher frequency of occurrence of very rich individuals (or households) than would be expected by extrapolating the properties of the bulk of the distribution, had been first observed by the Italian economist-sociologist Pareto in the 1890s. Pareto had analysed the cumulative income distribution of several societies at very different stages of economic development, and had conjectured that in all societies the distribution will follow a power-law decay with an exponent (later termed the Pareto exponent) of 1.5. Later, the distribution of wealth was also seen to exhibit a similar form. Subsequently, there have been several attempts, mostly by economists, starting around the 1950s to explain the genesis of the power-law tail. However, most of these models involved a large number of factors that made the essential reason behind the genesis of inequality difficult to understand. Following this period of activity, a relative lull followed in the 1970s and 1980s when the field lay dormant, although accurate and extensive data were accumulated that would eventually make possible precise empirical determination of the distribution properties. This availability of a large quantity of electronic data and their computational analysis has led to a recent resurgence of interest in the problem, specifically over the last one and half decades.

Although Pareto and Gini had respectively identified the power-law tail and the log-normal bulk of income distribution, demonstration of both features in the same distribution was possibly done for the first time by Montroll and Shlesinger (1982), in an analysis of fine-scale income data obtained from the US Internal Revenue Service (IRS) for the year 1935-36. They observed that while the top 2-3% of the population (in terms of income) followed a power law with Pareto exponent $\nu \sim 1.63$, the rest followed a lognormal distribution. Later work on Japanese personal income data based on detailed records obtained from the Japanese National Tax Administration indicated that the tail of the distribution followed a power law with a ν value that fluctuated from year to year around the mean value of 2 (Aoyama et al 2000).

Subsequent work by Souma (2000) showed that the power law region described the top 10% or less of the population (in terms of income), while the remaining income distribution was well described by the log-normal form. While the value of ν fluctuated significantly from year to year, it was observed that the parameter describing the log-normal bulk, the Gibrat index, remained relatively unchanged. The change of income from year to year, that is the growth rate as measured by the log ratio of the income tax paid in successive years, was observed by Fujiwara et al (2003) to be also a heavy-tailed distribution, although skewed, and centred about zero. Analysis of the US income distribution by Dragulescu and Yakovenko (2000) based on data from the IRS for the period 1997-98, while still indicating a power-law tail (with $\nu \sim 1.7$), has suggested that the lower 95% of the population has income whose distribution may be better described by an exponential form. A similar observation has been made for the income distribution in the UK for the period 1994-99. It is interesting to note that when one shifts attention from the income of individuals to the income of companies, one still observes the power-law tail. A study of the income distribution of Japanese firms by Okuyama et al (1999) concluded that it follows a power law with $\nu \sim 1$ (often referred to as *Zipf's law*). A similar observation has been reported by Axtell (2001) for the income distribution of US companies.

The Distribution of Wealth

Compared to the empirical work done on income distribution, relatively few studies have looked at the distribution of wealth, which consists of the net value of assets (financial holdings and/or tangible items) owned by an individual at a given point in time. Lack of an easily available data source for measuring wealth, analogous to income tax returns for measuring income, means that one has to resort to indirect methods. Levy and Solomon (1997) used a published list of wealthiest people to infer the Pareto exponent for wealth distribution in the US. An alternative technique was used based on adjusted data reported for the purpose of inheritance tax to obtain the Pareto exponent for the UK (Dragulescu and Yakovenko 2001). Another study by Abul-Magd (2002) used tangible asset (namely house area) as a measure of wealth to obtain the wealth distribution exponent in ancient Egyptian society during the reign of Akhenaten (14th century BC).

More recently, wealth distribution in India at present has also been observed to follow a power-law tail with the exponent varying around 0.9 (Sinha 2006). The general feature observed in the limited empirical study of wealth distribution is that wealthiest 5-10% of the population follows a power-law

Box 3: Kinetic Theory of Gases and Kinetic Exchange Models

According to the kinetic theory of gas, formulated more than 100 years ago, a gas of N atoms or molecules at temperature T, confined in a volume V and pressure P, satisfying the equation of state $PV=Nk_BT$ (where k_B is a proportionality constant referred to as Boltzmann constant) can be microscopically viewed as follows. At any given time, each atom or molecule of the gas is moving in a random direction with a speed that changes when it collides with another particle. In each such collision, the total momentum (given for each particle by the product of its mass and velocity and having the direction of the velocity) and total kinetic energy (given for each particle by half of the product of its mass and the square of its velocity) for the two colliding particles is conserved, that is, their values before and after the collision are identical. These collisions between pairs of particles, often referred to as scattering, keep occurring randomly.

According to this picture, the gas particles are constantly in motion, colliding randomly with each other. Because of the random nature of the motion of its constituent elements, the gas as a whole does not have any overall motion in any direction, and its internal kinetic energy is randomly distributed among the particles according to a given steady-state distribution. Even if one starts with each atom in the gas having the same initial kinetic energy, this initial equitable energy distribution rapidly gets destabilised following the random collisions of particles. Applying the entropy maximisation principle, one of the fundamental results of kinetic theory is that a single-humped Gamma distribution of energy among the particles is established, which is referred to as the Maxwell-Boltzmann distribution. In the steady-state (that is, when the distribution does not change with time), the average kinetic energy of any particle is decided by the temperature of the gas, while the pressure exerted by the gas on the walls of the container can be calculated from the rate of momentum transferred by the particles on a unit area of the wall. Using these, one can calculate the relation between P, V and T and confirm the above-mentioned equation of state that was originally obtained phenomenologically.

According to the kinetic exchange model of markets (discussed in this review), the traders are like gas atoms or molecules and the assets they hold are like the kinetic energy of the particles. Each trade between two traders is then identified as a collision (scattering) between particles, with each collision keeping the total asset before and after the trade unchanged (like energy for the gas) as none of the individual agents create or destroy these assets. In the market, such trades (collisions) between randomly chosen pair of traders keep occurring. As in the case of gas, even if all the traders are initially endowed with equal amount of assets, the random exchanges between traders will soon destabilise this initial equitable distribution. A single-humped Maxwell-Boltzmann like distribution of assets will soon get stabilised due to utility maximisation by the traders (demonstrated to be equivalent to entropy maximisation), for instance, when the traders each save a finite fraction of their assets at each trade. When the savings propensity of each trader differs, a Pareto tail of the asset distribution is observed (see, for example, Chakrabarti et al 2012).

Box 4: Thermodynamic System

A thermodynamic system is a macroscopic physical system (for example, a gas occupying a container of volume V), made up of a large number N (of the order of Avogadro number, for example, 10^{23}) of smaller constituents (for example, atoms or molecules) in contact with a heat bath at absolute temperature T (measured in degree centigrade + 273). By definition, the heat bath is of infinite capacity so that a small amount of heat added to or subtracted from it does not change the temperature of the bath. However, the thermodynamic system is of finite capacity, so that its temperature can change when heat is added to or subtracted from it. The thermodynamic state of such a system is often expressed by an equation of state. For example, the equation of state of an ideal gas is $PV = Nk_BT$, where k_B is a proportionality constant referred to as the Boltzmann constant. The equation describes how, for example, the pressure of the gas increases with temperature if the volume is kept fixed.

One can use such a thermodynamic system to convert heat energy (random kinetic energy of the gas atoms) to useful mechanical energy, for example, by using the work done by the expanding or contracting gas in pushing up or down a piston attached to the container of the gas to drive a motor. Such systems for transforming heat to work are called heat engines. For this, the thermodynamic system (for example, the gas in the container of volume V) has to be alternately brought in contact with a heat bath (called the *source*) at high temperature T_{source} (higher) and another heat bath (called the *sink*) at low temperature Tsink periodically so as to transfer heat energy from one to the other and in the process convert heat energy into mechanical energy. The maximum efficiency of any possible heat engine is the fraction of heat that can be converted to work by it and is measured by $\eta = 1-(T_{sink}/T_{source})$. Thus, complete conversion (that is, perfect efficiency corresponding to $\eta = 1$) of heat energy to useful mechanical energy is ruled out unless the heat sink is at a temperature of absolute zero (that is, −273 degree centigrade).

According to some models of econophysics (see, for example, "A Thermodynamic Formulation of Economics" by J Mimkes, in *Econophysics & Sociophysics*, B K Chakrabarti, A Chakraborti and A Chatterjee (ed.), Wiley-VCH, 2006, pp 1-33), the temperature of an economy can be identified with the average money in circulation. In such models, the production of goods in the economy is analogous to the functioning of a heat engine. This "economic engine" converts available raw commodities into the desired product. In line with the discussion above of source and sink heat baths, such an engine will work most efficiently between a country with cheaper labour (say, India or China) and a country with richer consumers (say, the US). According to such models, the greater the income difference among the "source" and "sink" economies, the higher the efficiency of such economic engines.

while an exponential or log-normal distribution describes the rest of the population. The Pareto exponent as measured from the wealth distribution is found to be always lower than the exponent for income distribution, which is consistent with the general observation that, in market economies, wealth is much more unequally distributed than income.

Theoretical Models for Explaining Inequality

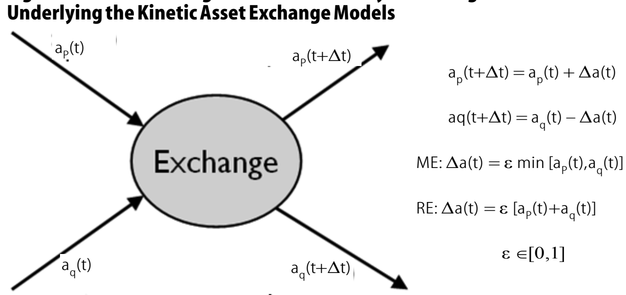
The striking regularities observed in income distribution for different countries have led to several new attempts at explaining them on theoretical grounds. Much of the current impetus is from physicists' modelling of economic behaviour in analogy with large systems of interacting particles, as treated, for example, in the kinetic theory of gases (see Box 3, p 54; also Sinha et al 2011). According to physicists working on this problem, the regular patterns observed in the income (and wealth) distribution may be indicative of a natural law for the statistical properties of a large complex system representing the entire set of economic interactions in a society, analogous to those previously derived for gases and liquids. It is interesting to note here that one of the earliest comprehensive textbooks on the kinetic theory of heat written by Indian physicists, Meghnad Saha and B N Srivastava (1931), had used the example of reconstructing a distribution curve for incomes of individuals in a country to illustrate the problem of determining the distribution of molecular velocities in kinetic theory. Although the analogy was not meant to be taken very seriously, one can probably consider this to be the first Indian contribution to econophysics; indeed, it anticipates by about seven decades the result that the bulk of the income distribution follows a Gibbs-like distribution.

By viewing the economy as a thermodynamic system (Box 4), one can identify income distribution with the distribution of energy among particles in a gas. In particular, a class of kinetic exchange models has provided a simple mechanism for understanding the unequal accumulation of assets (for a non-technical introduction, see Hayes 2002). Many of these models, while simple from the perspective of economics, have the benefit of coming to grips with the key factor in socio-economic interactions that results in different societies converging to similar forms of unequal distribution of resources.

Simple Physics-Inspired Models of Asset Distribution

To introduce the simplest class of physics models that reproduces the distribution of assets as seen in reality let us think of economic activity to be composed of a large number of pairwise exchanges between agents (Dragulescu and Yakovenko 2000). Note that instead of actual commodities, only their values in terms of a uniform asset (for example, money) are being considered here. In an asset exchange "game", there are N agents or players, each of whom has an initial capital of 1 unit. N is considered to be very large, and the total asset $M = N$ remains fixed over the duration of the game as is the number of players.

Figure 7: Schematic Diagram of the Two-Body Scattering Process Underlying the Kinetic Asset Exchange Models



The asset owned by agent a_p at time t changes due to an exchange (for example, through trade) with a randomly chosen agent a_q. The scattering process conserves the total amount of assets for the pair but can be of different types, for example, random exchange (RE) or minimal exchange (ME).

In the simplest version, called the *Random Exchange* (RE) model, the only allowed move at any time is that two of these players are randomly chosen who then divide their pooled resources randomly among themselves (Figure 7, RE). As no debt is allowed, none of the players can end up with a negative amount of assets. As one can easily guess, the initial distribution of assets (with every player having the same amount) gets destabilised with such moves and the state of perfect equality, where every player has the same amount, disappears quickly. The eventual steady-state distribution of assets among the players after many such moves is well known from the molecular physics of gases developed more than a century ago – it is the Gibbs distribution: $P(m) \sim \exp[-m/T]$, where the parameter $T = M/N$ corresponds to the average asset owned by an agent.

We now consider a modified move in this game (Chakraborti and Chakrabarti 2000). Each player "saves" a fraction λ of his/her total assets during every step of the game, from being pooled, and randomly divides the rest with the other (randomly chosen) player. If everybody saves the same fraction λ, what is the steady-state distribution of assets after a large number of such moves? It is Gamma-function like, whose parameters depend on λ: $P(m) \sim m^{\alpha} \exp[-m/T(\lambda)]$; $\alpha = 3\lambda/(1-\lambda)$. Although qualitative explanation and limiting results for $\lambda \to 0$ or $\lambda \to 1$ are easy to obtain, no exact treatment of this problem is available so far.

What happens to the steady-state asset distribution among these players if λ is not the same for all players, but is different for different players? Let the distribution $\rho(\lambda)$ of saving propensity λ among the agents be such that $\rho(\lambda)$ is non-vanishing when $\lambda \to 1$. The actual asset distribution in such a model will depend on the saving propensity distribution $\rho(\lambda)$, but the asymptotic form of distribution will become Pareto-like: $P(m) \sim m^{-(1+\nu)}$; with $\nu = 1$ for $m \to \infty$ (Chatterjee et al 2004; Chatterjee and Chakrabarti 2007; Chakrabarti and Chakrabarti 2010). This is valid for all such distributions (unless $\rho(\lambda) \propto (1-\lambda)^{\delta}$, when $P(m) \sim m^{-(2+\delta)}$). However, for variation of $\rho(\lambda)$ such that $\rho(\lambda) \to 0$ for $\lambda < \lambda_0$, one will get an initial Gamma function form for $P(m)$ for small and intermediate values of m, with parameters determined by λ_0 $(\neq 0)$, and this distribution will eventually become Pareto-like for $m \to \infty$ with $\nu = 1$. Analytical understanding is now available and a somewhat rigorous analytical treatment of this problem has been given recently (Mohanty 2006). It may be mentioned that there are a large number of random multiplicative asset exchange models to explain the Pareto (power-law) tail of wealth or income distribution. The advantage of the kind of model discussed above is that it can accommodate all the essential features of $P(m)$ for the entire range of m, not only the Pareto tail.

One can of course argue that the random division of pooled assets among players is not a realistic approximation of actual trading carried out in society. For example, in exchanges between an individual and a large company, it is unlikely that the individual will end up with a significant fraction of the latter's assets (Hayes 2002). Strict enforcement of this condition leads to a new type of game, the *Minimum Exchange* (ME) model, where the maximum amount that can change hands over a move is a fraction of the poorer player's assets (Figure 7, ME). Although the change in the rules from the RE game does not seem significant, the outcome is astonishingly different – in the steady state, one player ends up with all the assets (see, for example, Sinha 2003). In the language of physics, the system has converged to a *condensate* corresponding to an asset distribution having two sharp peaks, one at zero and the other at M. If we now relax the condition that the richer player does not completely dictate the terms of exchange, so that the amount exchanged need not be limited by the total asset owned by the poorer player, we arrive at a game which is asymmetric in the sense of generally favouring the player who is richer than the other, but not so much that the richer player dominates totally (Sinha 2005b). Just like the previously defined savings propensity for a player, one can now define "thrift" τ, which measures the ability of a player to exploit its advantage over a poorer player. For the two extreme cases of minimum ($\tau = 0$) and maximum ($\tau = 1$) thrift, one gets back the RE and ME models respectively. However, close to the maximum limit, at the transition between the two very different steady-state distributions given by the two models (that is, the Gibbs distribution and a condensate, respectively), we see a power-law distribution. As in the case of the model with saving propensity λ, we can now consider the case when instead of having the same τ, different players are endowed with different thrift abilities. For such heterogeneous thrift assignment in the population, where τ for each player is chosen from a random distribution, the steady-state distribution reproduces the entire range of observed distributions of income (as well as wealth) in the society – the tail follows a power-law, while the bulk is described by an exponential distribution. The tail exponent depends on the distribution of τ, with the value of $\nu = 1.5$ suggested originally by Pareto, obtained for the simplest case of uniform distribution of τ between [0, 1]. However, even extremely different distributions of τ (for example, U-shaped) always produce a power-law tailed distribution that is exponentially decaying in the bulk, underlining the robustness of the model in explaining inequality (Sinha 2005b).

All the gas-like models of trading markets discussed above are based on the assumptions of (a) conservation of total assets (both globally in the market; as well as locally in any trading) and (b) the interactions between agents being at random and independent of each other. One can naturally question the validity of these assumptions. It is of course true that in any asset exchange process, one agent receives some good or service from another and this does not appear to be completely random, as assumed in the models. However, if we concentrate only on the "cash" exchanged, every trade is an asset-conserving one. In more recent models, conservation of asset has been extended to that of total wealth (including money and commodities) and the introduction of the notion of price that fluctuates in time has effectively allowed a slight relaxation of this conservation, but the overall distribution has still remained the same. It is also important to note that the frequency of asset exchange in such models defines a timescale in which total asset in the market does not change. In real economies, total asset changes relatively slowly, so that at the timescale in which exchanges between agents take place, it is quite reasonable to assume the total resource to be conserved in these exchange models.

Assumption of Random Trading

How justified is the assumption of random trading among agents? Looked at from the point of view of an individual, this assumption may appear incorrect. When one maximises his/her utility by exchanging money for the p-th commodity, he/she may choose to go to the q-th agent and for the r-th commodity he/she will go to the s-th agent. But since $p \neq q \neq r \neq s$ in general, when viewed from a global level these trading events will look random (although for individuals this is a defined choice or utility maximisation). It may be noted in this context that in the stochastically formulated ideal gas models in physics (developed in the late 19th and early 20th centuries), physicists already knew for more than a century that each of the constituent particles (molecules) follows a precise equation of motion, namely that due to Newton. However, when one is considering a collection of an enormously large number of particles, using deterministic equations for each of the individual particles is no longer a practical approach and calculations based on the assumption of random interactions between the particles give a very accurate description of the state of the system. The assumption of stochasticity in asset-exchange models, even though each agent might follow a utility maximising strategy (just as molecules follow the deterministic Newton's equation of motion), is therefore not unusual in the context. Further, analysis of high-quality income data from the UK and the US shows Gamma distributions for the low- and middle-income ranges, which is strong evidence in favour of models discussed above.

The 'Universal' Nature of Market Movements

Given that the wealth and income of the highest bracket in the population (which exhibits the Paretian power-law tail) can be attributed mostly to their investment in financial instruments, it is probably expected that scientists would look for power laws in such market movements. Indeed, one of the most active sub-fields within econophysics is the empirical characterisation of statistical properties of financial markets. Starting from the work of Mantegna and Stanley (1999), several important results are now known about such markets which appear to be universal, in the sense that they are invariant with respect to the systems being considered, the time-period under consideration and the type of data being analysed. One of the best examples of such universal features of financial markets is the *inverse cubic law* (discussed below) for the distribution of fluctuations in price as well as in the index (Jansen and de Vries 1991; Gopikrishnan et al 1998; Plerou et al 1999a). Not only has it been observed to hold across several different time-scales and across different types of stocks (and market indices), but more surprisingly, it appears to be valid irrespective of the stage of development of the market (Pan and Sinha 2007a, 2008). Similar universal power-law functional forms have been claimed for the distributions of trading volume and the number of trades within a given interval of time, but they are still somewhat controversial (see, for example, Vijayraghavan and Sinha 2011). Financial markets have also proved a fertile ground for uncovering the structure of interactions between the different components of an economic system. In particular, the transactions between agents buying and selling different stocks in the market are reflected in the correlated movements of the prices of different stocks.

Analogous to the process of inferring the movement of air molecules by watching the Brownian motion of suspended particles, we can have a coarse-grained view of the interaction dynamics between individuals in the market by reconstructing the network of significantly correlated stocks (that is, correlated in terms of their price fluctuations). Comparison of such stock interaction networks for different markets has hinted that a financial market at a later stage of development possesses many more strongly bound clusters of co-moving stocks that are often from the same business sector (Pan and Sinha 2007b). Such markets tend to have identical statistical properties in terms of the distributions of price or index fluctuations, but differ significantly in the topological structure of the interactions between their components. Thus, network analysis can provide us with a window into the process of economic development.

The 'Inverse Cubic Law' for Price Fluctuations

Financial markets can be considered as complex systems that have many interacting elements and exhibit large fluctuations in their associated observable properties, such as stock price or market index. The state of the market is governed by interactions among its components, which can be either traders or stocks. In addition, market activity is also influenced significantly by the arrival of external information. The importance of interactions among stocks, relative to external information, in governing market behaviour has emerged only in recent times. The earliest theories of market activity, for example, Bachelier's random walk model, assumed that price changes are the result of several independent external shocks, and therefore, predicted the resulting distribution to be Gaussian. As an additive random walk may lead to negative stock prices, a better model would be a multiplicative random walk, where the price changes are measured by logarithmic returns. While the log-return distribution calculated from empirical data is indeed seen to be Gaussian at long time scales, at shorter times the data show much larger fluctuations than what we would expect from this distribution. Such deviations were also observed in commodity price returns, for example, in Mandelbrot's analysis of cotton price variation, which was found to follow a Levy-stable distribution. However, it contradicted the observation that the distribution converged to a Gaussian at longer time scales. Later, it was discovered that while the bulk of the return distribution for a market index (the S&P 500) appears to be fit well by a Levy-stable distribution, the asymptotic behaviour shows a much faster decay than expected. Hence, a truncated Levy distribution, which has exponentially decaying tails, was proposed as a model for the distribution of returns (Mantegna and Stanley 1995). Subsequently, it was shown that the tails of the cumulative return distribution for this index actually follow a power-law, with an exponent of -3 (Gopikrishnan et al 1998). This is the so-called *inverse cubic law* where the exponent lies well outside the stable-Levy regime (of exponent value between 0 and 2).

This is consistent with the fact that at longer timescales the distribution converges to a Gaussian. Similar behaviour has been reported for the DAX, Nikkei and Hang Seng indices (see, for example, Lux 1996). These observations are somewhat surprising, although not at odds with the "efficient market hypothesis" in economics, which assumes that the movements of financial prices are an immediate and unbiased reflection of incoming news and future earning prospects. To explain these observations, various multi-agent models of financial markets have been proposed, where the scaling laws seen in empirical data arise from interactions between agents (see, for example, Lux and Marchesi 1999). Other microscopic models, where the agents (that is, the traders comprising the market) are represented by mutually interacting elements that can be in any one of several discrete states (referred to as "spins" in the language of statistical physics) and the arrival of information by external fields which affect the orientation of the spins, have also been used to simulate the financial market. Among non-microscopic approaches, multi-fractal processes have been used extensively for modelling such scale-invariant properties (Bacry et al 2001a). The multi-fractal random walk model (Bacry et al 2001b) has generalised the usual random walk model of financial price changes and accounts for many of the observed empirical properties.

Universal Distributions

Recently, there has been a debate in the literature concerning the range of applicability of the inverse cubic law for price fluctuation distribution. As most previous reported observations were from developed markets, a question of obvious interest was whether the same distribution holds for developing or emerging financial markets. If the inverse cubic law is a true indicator of self-organisation in markets, then observing the price fluctuation distribution as the market evolves gradually over the years will inform us about the process by which this complex system converges to the non-equilibrium steady state characterising developed markets. Recent analysis of high-frequency trading data from the National Stock Exchange (NSE) of India shows that this emerging market exhibits the same inverse cubic law as all other developed markets, despite commencing operations only in 1994. In fact, by analysing the data from the inception of the NSE to the present (in 2005 it became the third largest financial market in the world in terms of transactions) it is possible to study the nature of the return distribution as a function of time. Thus, if markets do show discernible transition in return distribution during their time-evolution, the Indian market data is best placed to spot evidence for it, not least because of the rapid transformation of the Indian economy in the liberalised environment since the 1990s. However, the results show that the power-law nature of the return distribution can be seen even in the earliest days of the market, from which time it has remained essentially unchanged (Pan and Sinha 2007a). The convergence of the return distribution to a power-law functional form is thus extremely rapid, so that a market is effectively always at the non-equilibrium steady state characterised by the inverse cubic law regardless of its stage of development.

Inferring the Structure of Markets from Cross-Correlation between Stocks

So, if emerging markets do not differ from developed ones in terms of the properties of price fluctuations, are there still other observables that will allow us to distinguish between them? It now appears that the cross-correlation behaviour between the price fluctuations of the stocks in a market may have a very different nature depending on the state of development of the market. The observation of correlated movement in stock prices gives us a proxy variable for studying the interactions between stocks mediated through the action of agents who are buying/selling different stocks. As the dynamics of individual investors are being only indirectly inferred based on the dynamics of price for the different stocks, this is somewhat akin to a "Brownian motion" picture of the market, analogous to the process of inferring the dynamics of air molecules by observing the movement of suspended pollen grains with which the molecules are colliding.

The existence of collective modes in the movement of stock prices had been earlier inferred from the study of market dynamics, although such studies had almost exclusively focused on developed markets, in particular, the New York Stock Exchange (NYSE). A recent detailed analysis of the cross-correlation between stocks in the Indian market has demonstrated that an emerging market differs from more developed markets in that the former lacks clusters of co-moving stocks having distinct sector identities.

How do Price Movements of Different Stocks Affect Each Other?

To uncover the structure of interactions among the elements in a financial market, physicists primarily focus on the spectral properties of the correlation matrix of stock price movements. Pioneering studies by Laloux et al (1999) and Plerou et al (1999b) have investigated whether the properties of the empirical correlation matrix differ from those of a random matrix that would have been obtained had the price movements been uncorrelated. Such deviations from the predictions of random matrix theory (RMT) can provide clues about the underlying interactions between various stocks. It was observed that while the bulk of the eigenvalue (or characteristic value) distribution for the correlation matrix of the NYSE and Tokyo Stock Exchange follow the spectrum predicted by RMT, the few largest eigenvalues deviate significantly from this. The largest eigenvalue has been identified as representing the influence of the entire market, common for all stocks, whereas the remaining large eigenvalues are associated with the different business sectors, as indicated by the composition of their corresponding eigenvectors. The interaction structure of stocks in a market can be reconstructed by using filtering techniques implementing matrix decomposition or maximum likelihood clustering. Apart from its use for understanding the fundamental structure of financial markets, correlation matrix analysis has applications in the area of financial risk management, as mutually correlated price movements may indicate the presence of strong interactions between stocks. Such analyses have been performed using asset trees and asset graphs to obtain the taxonomy of an optimal portfolio of stocks (Mantegna 1999; Onnela et al 2002).

While it is generally believed that stock prices in emerging markets tend to be relatively more correlated than the developed ones, there have been very few studies of the former in terms of analysing the spectral properties of correlation matrices. Most studies of correlated price movements in emerging markets have looked at the *synchronicity*, which measures the incidence of similar (that is, up or down) price movements across stocks. Although related to correlation, the two measures are not same, as correlation also gives the relative magnitude of similarity. By analysing the cross-correlations among stocks in the Indian financial market over the period 1996-2006, it has been found that, in terms of the properties of its collective modes, the Indian market shows significant deviations from developed markets. As the fluctuation distribution of stocks in the Indian market follows the same "inverse cubic law" seen in developed markets like the NYSE, the deviations observed in the correlation properties should be almost entirely due to differences in the nature of interaction structure in the two markets. The higher degree of correlation in the Indian market compared to developed markets is found to be the result of a dominant market mode affecting all the stocks, which is further accentuated by the relatively very few number of clusters of mutually interacting stocks as compared to, say, the NYSE (Pan and Sinha 2007b). These results imply that one of the key features signifying the transition of a market from emerging to developed status is the appearance and consolidation of distinct sector or group identities.

The Dynamics of Interacting Economic Agents

One of the principal features distinguishing the individual agents operating in economic systems from the inanimate particles that were traditionally studied by physics is that the former are capable of making informed choices from among a set of possible actions. In the following subsections, we provide a glimpse of the range of different problems involving the dynamics of such interacting agents that have been treated using physics-based approaches.

Reproducing the Stylised Facts of Financial Markets

There have been several attempts at reproducing the universal features in the dynamics of markets mentioned above (often referred to as "stylised facts" in the economics literature), including the inverse cubic law for the distribution of price or index fluctuations (as measured by the logarithmic return) and volatility clustering. Many of these models stress endogenous interactions between the market players as the underlying cause for the generation of such patterns rather than exogenous factors such as news breaks and variations in macroeconomic indicators. While most such attempts have assumed explicit interaction between agents who are involved in buying and selling assets from each other and/or make a priori assumptions about individual trading strategies (for example, chartists vs fundamentalists), one could also view the exchanges between agents to be indirect, mediated by the market through the price mechanism. The coordination of agent behaviour through a global signal such as asset price allows the theory of mean-field coupling (that replaces all individual agent-agent interactions with the interaction between a single agent and an effective field representing all other agents) that is regularly used in physics to be used for explaining the statistical features of financial markets. A recently proposed model (Vikram and Sinha 2011) where the agents do not interact directly but respond to fluctuations in the asset price by deciding to buy, sell or hold on to an asset reproduces all the known stylised facts of financial market dynamics. In this model, the trading occurs in a two-step process, with each agent first deciding whether to trade or not at that given instant based on the deviation of the current price from an agent's notion of the "true" or fundamental price of the asset (estimated from the long-time moving average of the observed price). Next, the agents who have decided to trade make the choice of either to buy or sell. It turns out that the exact details of this process do not affect the results of the model and could be chosen either to be decided at random or based on information about the prevalent demand-supply ratio as measured by the log return of the price.

The Model

A simplified view of a financial market is that it consists of a large number of agents (say, N) trading in a single asset. Considering time to evolve in discrete units, the state of each trader i can be represented by the variable $S_i(t)$ $(i = 1, \ldots, N)$ at a given time instant t. It can take values +1, –1 or 0 depending on whether an agent buys or sells (a unit quantity of asset) or decides not to trade (that is, hold) at time t, respectively. It is assumed here that the evolution of price in a free market is governed only by the relation between supply and demand for the asset. Thus, the price of the asset at any time t, p_t, will rise if the number of agents wishing to buy it (that is, the demand) exceeds the number wishing to sell it (that is, supply). Conversely, it will fall when supply outstrips demand. A possible relation between prices at two successive time instants that satisfies the above criterion is:

$$p_{t+1} = p_t (1 + M_t)/(1 - M_t),$$

where, $M_t = \Sigma_i S_i(t)/N$ is the net demand for the asset, as the state of agents who do not trade is represented by 0 and do not contribute to the sum. This functional form has the desirable feature that when everyone wants to sell the asset ($M_t = -1$), its price goes to zero, whereas if everyone wants to buy it ($M_t = 1$), the price diverges. When the demand equals supply, the price remains unchanged from its preceding value, indicating an equilibrium situation. The multiplicative form of the function not only ensures that price can never be negative, but also captures the empirical feature of the magnitude of stock price fluctuations in actual markets being proportional to the price. If the ratio of demand to supply is equivalent to an uncorrelated stochastic process, the price will follow a geometric random walk, as originally suggested by Bachelier (1900). However, other functions for the price evolution that have the same properties can also be written and the exact form does not critically affect the results obtained from the model.

Trading in the Model Market

Once the price of the asset is determined based on the activity of traders, the process by which individual agents decide to buy, sell or hold has to be specified. As direct interactions between agents have not been assumed, nor is information external to the market included in the basic version of the model, the only factor governing such decisions is the asset price (the current value at a given time as well as the record of all previous values up to that time). First, one decides whether an agent will trade at a particular time (that is, $S_i = \pm 1$) rather than hold ($S_i = 0$). This decision is based on the deviation of the current price at which the asset is being traded from an individual agent's perception of the "fundamental value" of the asset. Potters and Bouchaud (2003) have reported, based on observations of market order book dynamics, that the lifetime of a limit order is longer the further it is from the current bid-ask. In analogy to this, the probability of an agent to trade at a particular price is taken to be a decreasing function of the difference between that price and the "fundamental" value of the asset. The fundamental or "true" asset price is estimated from the price history (as the agents do not have access to any other knowledge) and thus can vary with time. The simplest proxy for this estimate is a long-time moving average of the price time-series, $< p_t >$, with the averaging window size, τ, being a parameter of the model. The use of moving average is validated by a study of Alfi et al (2006) that the long-time moving average of prices defines an effective potential which is a determining factor for empirical market dynamics. In light of the above information, one possible formulation for the probability of an agent i to trade at time t is simply:

$$P(|S_i(t)| = 1) = 1 - P(S_i(t) = 0) = \exp(-\mu\, |\{p_t - < p_t >\}/< p_t >|),$$

where μ is a parameter that controls the sensitivity of the agent to the magnitude of the deviation from the fundamental value. This deviation is expressed in terms of a ratio so that there is no dependence on the scale of measurement. For the limiting case of $\mu=0$, a binary-state model is obtained where each agent trades at every instant of time. Again, the exact form for determining the probability is not critical for the results obtained from the model.

To Buy or To Sell

Once an agent decides to trade based on the above dynamics, it has to choose between buying and selling a unit quantity of the asset. If we assume that the decision is made at random (for example, by tossing a coin), the theoretical treatment of the model is considerably simplified. However, in reality this choice will be based on information about the price variation in the recent past. So, for example, one can assume that agents sell (buy) if there is an excess of demand (supply) resulting in an increase (decrease) of the price in the previous instant. Using the logarithmic return as the measure for price movement, the following simple form can be used for calculating the probability that an agent will sell at a given time t:

$$P(S_i(t) = -1) = 1/[1 + \exp(-\beta \log\{p_t / p_{t-1}\})].$$

The form of this probability function is similar to the well-known Fermi function used in statistical physics, where it describes the transition probability between states in a system at thermal equilibrium. The parameter β, that corresponds to inverse "temperature" in the context of the Fermi function, is a measure of how strongly the information about price variation influences the decision of a trader to sell. Operationally speaking, it controls the slope of the function at the transition region where the probability increases from 0 to 1, with the transition getting sharper as β increases. In the limit $\beta \to \infty$, the probability function becomes step-like, with every agent who has decided to trade selling (buying) if the price has risen (fallen) in the previous instant. In the other limiting case of $\beta = 0$, a trader buys or sells with equal probability regardless of whether the price rises or falls, indicating an insensitivity to the price movement.

The Inverse Cubic Law and Other Stylised Facts

The variation of the asset price generated by the model dynamics is found to be qualitatively similar to price (or index) time-series obtained from real markets. The moving average of the price, that is considered to be the notional fundamental price for agents in the model, is seen to track a smoothed pattern of price variations, coarse-grained at the time-scale of the averaging window, τ. The price fluctuations, as measured by the normalised log returns show large deviations that are significantly greater than that expected from a Gaussian distribution. They also exhibit volatility clustering, with periods marked by large fluctuations following each other in rapid succession. The probability distribution of returns is seen to follow a power-law having exponent –3 over an intermediate range with an exponential cut-off at the tail (as is expected for a simulation with a small finite number of agents). The quantitative value of the exponent is seen to be unchanged over a large range of variation in the parameter μ and does not appear to depend sensitively on β. For very low values of μ (for example, $\mu < 10$) the return distribution is seen to become exponential.

The dynamics leading to an agent choosing whether to trade or not is the crucial component of the model that is necessary for generating the non-Gaussian fluctuation distribution. This is explicitly shown by observing the absence of power-law in the return distribution for the special case when $\mu = 0$, where, as already mentioned, every agent trades (that is, either buys or sells) at all times. Thus, the overall dynamics of the model can be described by a difference equation in a single variable, the net demand (M_t). Analysis of the map reveals that the system has two classes of equilibria, with the transition occurring at the critical value of $\beta = 1$. For $\beta < 1$, the mean value of M is 0, and the price fluctuations calculated over long time intervals follow a Gaussian distribution. When β exceeds 1, the net demand goes to 1, implying that price diverges. This prompts every agent to sell at the next instant, pushing the price to zero, analogous to a market crash (see Sinha and Raghavendra 2006). It is a stable equilibrium of the system, corresponding to market failure.

As each trader can buy/sell only a unit quantity of asset at a time in the model, the number of trading agents at time t, $V_t = \Sigma_i |S_i(t)|$, is equivalent to the trading volume at that instant. The cumulative distribution of this variable has a power-law decay characterised by an exponent $\zeta_V \cong 1$, indicating a Zipf's law distribution for the trading volume at a given instant. As in the case of the return distribution exponent, the quantitative value of the exponent is unchanged over a large range of values for the parameter μ. In fact, the power-law nature of this distribution is seen to be more robust than that for the return distribution.

In real markets the parameter values for different agents need not be identical, as they can have different responses to the same market signal, for example, in terms of their decisions to trade in a high-risk situation. This heterogeneity in agent behaviour can be captured by using a random distribution of parameter values. For example, a low value of the parameter μ represents an agent who is relatively indifferent to the deviation between current price and "fundamental value". On the other hand, an agent who is extremely sensitive to this difference and refuses to trade when the price goes outside a certain range around the "fundamental value" is a relatively conservative (or risk-averse) market player having a higher value of μ. In the model, when μ for the agents is distributed uniformly over a large interval (for example, between [10,200]), the power-law nature of the return and volume distributions is similar to that for the case with constant parameters. However, the exponent values characterising these power-laws now appear to be quantitatively identical to those seen in real markets. In particular, the return distribution is seen to accurately reproduce the inverse cubic law of price fluctuations. Thus, the model suggests that heterogeneity in agent behaviour is the key factor behind the distributions observed in real markets. It predicts that when the behaviour of market players become more homogeneous, as, for example, during a market crash event, the return distribution exponent will tend to decrease. Indeed, earlier work by Kaizoji (2006) has found that during crashes, the exponent for the power-law tail of the distribution of relative prices has a significantly different value from that seen at other times. From the results of the model simulations, it has been predicted that for real markets the return distribution exponent during a crash will be close to 2, the value obtained in the model when every agent behaves identically.

It may be pertinent here to discuss the relevance of the observation of an exponential return distribution in the model at lower values of the parameter μ. Although the inverse cubic law is seen to be valid for most markets, it turns out that there are a few cases, such as the Korean market index KOSPI, for which the return distribution is reported to have an exponential form (Yang et al 2006). Analysis of the model suggests that these deviations from the universal behaviour can be due to the existence of a high proportion of traders in these markets who are relatively indifferent to large deviations of the price of stocks from their "fundamental values". In other words, the presence of a large number of risk takers in the market can cause the return distribution to have exponentially decaying tails. The fact that for the same set of parameter values, the cumulative distribution of number of traders still shows a power-law decay with exponent – 1, leads to the prediction that, despite deviating from the universal form of the return distribution, the trading volume distribution of these markets will follow a power-law form with exponent close to – 1.

The Kolkata Paise Restaurant Problem

Introduction

The Kolkata Paise Restaurant (KPR) problem (Chakrabarti et al 2009: Ghosh and Chakrabarti 2009; Ghosh et al 2010) is a repeated game, played between a large number N of agents having no interaction or discussion among themselves. In the KPR problem, prospective customers (agents) choose from n ($\leq N$) restaurants each evening simultaneously (in parallel decision mode); N and n are both large and fixed (typically $n = N$). Each restaurant has the same price for a meal (hence no budget constraint for the agents). We assume that each can serve only one customer any evening (generalisation to a larger value is trivial). Information regarding the customer distributions for earlier evenings is available to everyone. If more than one customer arrives at any restaurant on any evening, one of them is randomly chosen (each of them are anonymously treated) and is served. The rest do not get dinner that evening. Each agent develops his own (parallel) algorithm to choose the restaurant every evening such that he is alone there. Also, the times required to settle to such a solution (if it exists) should be low (less than, say, log N). Additional complications arise when the restaurants have different ranks which are agreed upon by all the agents.

In Kolkata, there were very cheap and fixed rate "Paise Restaurants" that were popular among the daily labourers in the city. During lunch hours, the labourers used to walk (to save the transport costs) to one of these restaurants and would miss lunch if they arrived at a restaurant where there were too many customers. Walking down to the next restaurant would mean failing to report back to work on time. Paise is the smallest Indian coin and there were indeed some well-known rankings of these restaurants, as some of them would offer tastier items compared to the others. A more general example of such a problem would be when society provides hospitals (and beds) in every locality but the local patients go to hospitals of better rank (commonly perceived) elsewhere, thereby competing with the local patients of those hospitals. Unavailability of treatment in time may be considered as lack of the service for those people and consequently as (social) wastage of service by those unattended hospitals.

A dictator's solution to the KPR problem is the following. The dictator asks everyone to form a queue and then assigns each one a restaurant with rank matching the sequence of the person in the queue on the first evening. Then each person is told to go to the next ranked restaurant the followingevening (for the person in the last ranked restaurant this means going to the first ranked restaurant). This shift then proceeds continuously for successive evenings. This is clearly one of the most efficient solutions (with utilisation fraction f of the services by the restaurants is at its maximum and equal to unity) and the system arrives at this solution immediately (from the first evening itself). However, in reality this cannot be the true solution of the KPR problem, where each agent decides on its own (in parallel or democratically) every evening, based on complete information about past events. In this game, the customers try to evolve a learning strategy to eventually get dinners at the best possible ranked restaurant, avoiding the crowd. It is seen that the evolution of these strategies take considerable time to converge (growing with N) and even then the eventual utilisation fraction f is far below unity.

KPR Game Strategies

In the KPR problem, all agents are trying to get meals in best-ranked restaurants. But a restaurant can serve only one customer. If more than one customer arrives any evening in a restaurant, one will be served and rest will miss their dinner for that evening. The aim of the KPR game strategy is to find a strategy which gives maximum utilisation of the services such that all agents will be getting food from every restaurant on an average. This problem is a repeated game and agents learn their strategies by analysing the previous history. Let us now consider some simple strategies.

Random Strategies

For a random choice, where there is no ranking of the restaurants, on each day an agent selects any one restaurant with equal probability (Chakrabarti et al 2009). There is no memory or learning and each day the same process is repeated. When there are n restaurants (choices) and gn ($= N$) agents ($g = 1$ in the typical KPR problem) choosing randomly every evening from the set of n restaurants (choices), the probability $D(m)$ that any particular restaurant is chosen simultaneously by m agents is given by the Poisson distribution

$$D(m) = (g^m/m!) \exp(-g) \text{ as } n \to \infty.$$

In other words, the fraction of restaurants not chosen by anyone in any evening is given by $D\ (m=0) = exp(-g)$, giving the average fraction of restaurants occupied on any evening

$$f = 1 - \exp(-g) \approx 0.63 \text{ for } g=1.$$

The distribution of the fraction utilised any day will be Gaussian around the average given above and the time required to reach steady state is zero (random choosers/traders do not have any memory). Obviously, for rank-dependent choices, this utilisation fraction decreases further.

Stochastic Crowd Avoiding Choice

We again consider first the case where there exists no ranking of the restaurants. Let us proceed with the following stochastic strategy (Ghosh et al 2010). If an agent has chosen the restaurant number k the previous evening (t-1), then that agent goes to the same restaurant next evening (t) with probability $p_k(t) = 1/N_k(t-1)$, where N_k denotes the number of agents who had chosen the same k-th restaurant on that (t-1)-th evening ($\Sigma_k N_k = N$). This agent goes (in the next t-th evening) to any other restaurant $k'(\neq k)$ with probability $p_{k'}(t) = (1-p_k(t))/(N-1)$. In this process the average utilisation fraction f becomes close to 0.8, and the time required to reach this steady value is bounded by log N (Ghosh et al 2010). When the ranking of the restaurants are considered (and k denotes the rank of the restaurant), the strategy to follow is very similar. Any of the N_k agents who had chosen the k-th ranked restaurant last evening goes to (k-1)-th ranked restaurant with probability $p_k(t)=1/N_k(t-1)$ this evening and with probability $p_{k'}(t)=(1-p_k(t))/(N-1)$ to any other restaurant with rank $k'\neq k-1$. For periodic boundary condition we assume $k'=1$ for $k=N$. One can show (Ghosh et al 2010) that full occupation solution, with rotation of the population along the periodic chain of restaurants, is achieved in about N evenings (trials).

KPR and Minority Game

The Minority Game (Challet et al 2005) is a two choice game ($n = 2$) and an agent will win (pay off 1) if she is in a less crowded restaurant, or else lose (pay off 0). Now, anyone can apply a "stochastic crowd avoiding" strategy for this two choice case (Dhar et al 2011). The strategy of the two choice game will be as follows. Consider a city with exactly two restaurants and let us assume there are N persons in the city, each of whom goes for dinner every evening to one of the two restaurants. Assume that N is odd, and write $N = 2M + 1$. A restaurant is said to be crowded any evening if the number of persons turning up for dinner there exceeds $M + 1$ that evening. A person is happy if she goes to a restaurant that is not crowded, and will get a payoff 1. If she turns up at a crowded restaurant, her payoff is 0. Once the choice of which restaurant to go to is made, an agent cannot change it for that day/evening. The previous history of how many agents chose to go to both the restaurants is available to everyone and the agents do not communicate with each other for any decision.

The strategy is defined as follows. At t=0, each agent chooses one of the two restaurants with probability 1/2, independently of others. Suppose any instance of time t, one restaurant has M-$\Delta(t)$ persons (minority group) and other has $M + \Delta\ (t) + 1$ (majority group). At any subsequent evening (time t+1), each person follows the following strategy: If at time t, she found herself in the minority, she chooses the same restaurant next time or evening (t+1). If she finds herself in the majority, she changes next evening her choice with a probability p_+, given by

$$p_+ = \Delta(t)/[M+\Delta(t)+1].$$

(As discussed already $p_- = 0$ in minority side). For large M, the number of agents changing their choice is distributed according to the Poisson distribution, with mean approximately equal to Δ, and width varying as $\sqrt{\Delta(t)}$. Thus we have the approximate recursion $\Delta(t+1) \approx \sqrt{\Delta(t)}$, for $\Delta(t) >> 1$. This shows that within a time of order log log N, the magnitude of Δ will become of $O(1)$.

If all agents follow the above strategy, the dynamics of the system will stop when $\Delta = 0$ (call it absorbing state)

(Dhar et al 2011). But one can avoid this absorbing state by having only two random agents in the whole game (for details, see Biswas et al 2012). The reason is as follows. A population of N agents try to evolve a strategy such that two of them will flip randomly among the two choices (restaurants), while the rest (N-2) of them will follow the stochastic strategy given by $p_+ = \Delta(t)/[M+\Delta(t)+1]$. This will ensure (see Biswas et al 2012 for details) that the fluctuation will become arbitrarily small in value (giving maximum social efficiency) and that too achieved in log N time or evenings. The absorbing state will never appear while everyone will have an average period of 2 in the minority/majority. A decrease in the number of random traders will enforce the indefinite stay in the majority for the random trader. Increasing the number of such traders above 2 will increase fluctuation, eventually converging to its square root of N value (with least social efficiency).

Summary

A repetitive game has been considered where N agents choose every time (in parallel) one among the $n(\leq N)$ choices, such that each agent can be in minority: no one else made the same choice in the KPR case (typically $n = N$) and $N_k < N/2$ for the Minority Game ($n = 2$; $k = 1, 2$). The strategies to achieve this objective evolve with time bounded by N. Acceptable strategies are those which evolve quickly (say within log N time). Also the effectiveness of a strategy is measured by the resulting utilisation factor f giving the (steady state) number of occupied restaurants in any evening for the KPR, by the value of fluctuation Δ in the Minority Game case ($\Delta = 0$ corresponds to maximum efficiency).

The study of the KPR problem shows that a dictated solution leads to one of the best possible solutions to the problem, with each agent getting his dinner at the best-ranked restaurant with a period of N evenings, and with best possible value of f (=1) starting from the first evening on itself. For a democratic situation (for parallel decision strategies), the agents employ stochastic algorithms based on past occupation information (for example, of $N_k(t)$). These strategies are of course less efficient (f<<1; the best one discussed in Ghosh et al 2010, giving $f \cong 0.8$ only). Here the time required is very weakly dependent on N, if at all. We also note that most of the "smarter" strategies lead to much lower efficiency.

We note that the stochastic strategy Minority Game (Dhar et al 2011), is a very efficient one: The strategy is described by $p_+ = \Delta(t)/[M+\Delta(t)+1]$, where the agents very quickly (in log log N time; $N = 2M + 1$) get divided almost equally (M and $M + 1$) between the two choices. This strategy guarantees that a single cheater, who does not follow this strategy, will always be a loser. However, the dynamics in the system stops very quickly (leading to the absorbing state), making the resource distribution highly asymmetric (people in the majority stay there forever), thereby making this strategy socially unacceptable. To rectify for this, note that the presence of a single random trader (who picks between the two choices completely randomly) will avoid this absorbing state and the asymmetric distribution. However, this will always make that random trader a loser. But the presence of more than one random trader will avoid that situation too, making the average time period of switching between majority and minority for all the traders (irrespective of whether they are chartists or random traders) to be 2. Hence, the system will always evolve collectively such that only two agents will make random choices between the binary choices, while the rest N-2 will follow the probabilities p_+.

In brief, in the Minority Game N (=$2M + 1$) players develop their respective strategies, based on past experiences, to choose every evening between the two equally acceptable restaurants. Successful players are those who are in the less crowded restaurants (with a population $M - \Delta$, $\Delta \geq 0$) on any evening. The system is most efficient when the players play such that Δ=0, and they stay on average half of the times in the minority. As discussed above, this can be achieved with a stochastic strategy and in log log N time (or evenings). For the KPR problem N players typically chose among N restaurants, again by employing parallel decision strategies by each player every evening such that any one is alone in choosing that restaurant any evening and this has to occur very quickly (not more than log N trials or evenings). This is again achieved by following the stochastic strategy described above, with an average success of 80% approximately.

Concluding Remarks

In the above sections we have tried to provide a glimpse of the variety of questions that are being addressed by practitioners of the new discipline of econophysics. As mentioned earlier, it is marked by a desire to accurately describe real economic phenomena by careful observation and reproducing the empirical features with models inspired by statistical physics. A common point to most of these works is a desire to identify universal features that are independent of system-specific details, and these features are often manifested as scaling relations (or power laws, in the language of physics). While several attempts at describing phenomena such as inequality distribution and scale-invariant statistical properties of financial markets have met with reasonable success, there is still some way to go before econophysics can replace mainstream economics as the dominant paradigm for theoretically explaining the entire range of economic activities.

Most importantly, we should not forget that economic phenomena form just one aspect of the entire set of processes that make up the human social organisation. For example, the process by which certain products or ideas achieve extreme popularity while other competing products and ideas (that are often indistinguishable from the eventual winner in terms of intrinsic quality) fall on the wayside is as much a subject of study for economists (or econophysicists) as it is for sociologists (Sinha and Pan 2007; Pan and Sinha 2010). Thus, econophysics should strive to be a theory for the entire spectrum of human social behaviour. As John Maynard Keynes (1932), one of the greatest economists, had once said "do not let us overestimate the importance of the economic problem, or sacrifice to its supposed necessities other matters of greater and more permanent significance".

REFERENCES

Abul-Magd, A Y (2002): "Wealth Distribution in an Ancient Egyptian Society", *Physical Review E*, 66, 057104.

Alfi, V, F Coccetti, M Marotta, L Pietronero and M Takayasu (2006): "Hidden Forces and Fluctuations from Moving Averages: A Test Study", *Physica A*, 370, pp 30-37.

Anderson, P W, K J Arrow and D Pines, ed. (1988): *The Economy as an Evolving Complex System* (Redwood City, California: Addison-Wesley).

Aoyama, H, Y Nagahara, M P Okazaki, W Souma, H Takayasu and M Takayasu (2000): "Pareto's Law for Income of Individuals and Debt of Bankrupt Companies", *Fractals*, 8, pp 293-300.

Arrow, K J and L Hurwicz (1958): "On the Stability of Competitive Equilibrium I", *Econometrica*, 26, pp 522-52.

Arrow, K J, H D Block and L Hurwicz (1959): "On the Stability of Competitive Equilibrium II", *Econometrica*, 27, pp 82-109.

Axtell, R L (2001): "Zipf Distribution of US Firm Sizes", *Science*, 293, pp 1818-20.

Bachelier, L (1900): "Theorie de la speculation", *Annales scientifiques de l Ecole Normale Superieure*, Ser 3, 17, pp 21-86.

Bacry, E, J Delour and J F Muzy (2001a): "Modelling Financial Time Series Using Multifractal Random Walks", *Physica A*, 299, pp 84-92.

– (2001b): "Multifractal Random Walk", *Physical Review E*, 64, 026103.

Banerjee, A and V M Yakovenko (2010): "Universal Patterns of Inequality", *New Journal of Physics*, 12, 075032.

Bernstein, J (2005): "Bachelier", *American Journal of Physics*, 73, pp 395-98.

Beyeler, W E, R J Glass, M Bech and K Soramaki (2007): "Congestion and Cascades in Payment Systems", *Physica A*, 384, 693-718.

Biswas, S, A Ghosh, A Chatterjee, T Naskar and B K Chakrabarti (2012): "Continuous Transition of Social Efficiencies in the Stochastic Strategy Minority Game", *Physical Review E*, 85, 031104.

Bouchaud, J-P (2008): "Economics Needs a New Scientific Revolution", *Nature*, 455, p 1181.

Brede, M and S Sinha (2005): "Assortative Mixing by Degree Makes a Network More Unstable", arxiv preprint, available at cond-mat/0507710.

Brown, G (2007): *Budget Statement to the House of Commons*, available at http://www.direct.gov.uk/en/Nl1/Newsroom/DG_067074.

Brown, J H, W R Burnside, A D Davidson, J R Delong, W C Dunn, M J Hamilton, N Mercado-Silva, J C Nekola, J G Okie, W H Woodruff and W Zuo (2011): "Energetic Limits to Economic Growth", *Bioscience*, 61, pp 19-26.

Chakrabarti, A S and B K Chakrabarti (2010): "Statistical Theories of Income and Wealth Distributions", *Economics E Journal*, 4, 2010-4.

Chakrabarti, A S, B K Chakrabarti, A Chatterjee and M Mitra (2009): "The Kolkata Paise Restaurant Problem and Resource Utilisation", *Physica A*, 388, pp 2420-26.

Chakrabarti, B K, A Chakraborti, S R Chakravorty and A Chatterjee (2012): *Econophysics of Income and Wealth Distributions* (Cambridge: Cambridge University Press).

Chakrabarti, B K and S Marjit (1995): "Self-Organisation in Game of Life and Economics", *Indian Journal of Physics*, B69, pp 681-98.

Chakraborti, A and B K Chakrabarti (2000): "Statistical Mechanics of Money: How Saving Propensity Affects Its Distributions", *European Physical Journal B*, 17, pp 167-70.

Challet, D, M Marsili and Y C Zhang (2005): *Minority Games* (Oxford: Oxford University Press).

Chancellor, E (1999): *Devil Take the Hindmost: A History of Financial Speculation* (London: Macmillan).

Chatterjee, A and B K Chakrabarti (2007): "Kinetic Exchange Models of Income and Wealth Distributions", *European Physical Journal B*, 60, pp 135-79.

Chatterjee, A, B K Chakrabarti and S S Manna (2004): "Pareto Law in a Kinetic Model of Market with Random Saving Propensity", *Physica A*, 335, pp 155-63.

Chatterjee, A, S Sinha and B K Chakrabarti (2007): "Economic Inequality: Is It Natural?", *Current Science*, 92, pp 1383-89.

Dhar, D, V Sasidevan and B K Chakrabarti (2011): "Emergent Cooperation Amongst Competing Agents in Minority Games", *Physica A*, 390, pp 3477-85.

Dragulescu, A A and V M Yakovenko (2000): "Statistical Mechanics of Money", *European Physical Journal B*, 17, pp 723-29.

– (2001): "Exponential and Power-Law Probability Distributions of Wealth and Income in the United Kingdom and United States", *Physica A*, 299, pp 213-21.

Elton, C S (1958): *The Ecology of Invasions by Animals and Plants* (London: Methuen).

Fama, E (1970): "Efficient Capital Markets: A Review of Theory and Empirical Work", *Journal of Finance*, 25, pp 383-417.

Farmer, J D, M Shubik and D E Smith (2005): "Is Economics the Next Physical Science?", *Physics Today*, 58 (9), pp 37-42.

Feynman, R P (1964): "The Principle of Least Action", *Feynman Lectures in Physics*, Vol 2, Chapter 19 (Reading, MA: Addison-Wesley).

Forrester, J W (1961): *Industrial Dynamics* (Cambridge, MA: MIT Press).

Fujiwara, Y, W Souma, H Aoyama, T Kaizoji and M Aoki (2003): "Growth and Fluctuations of Personal Income", *Physica A*, 321, pp 598-604.

Garber, P M (1990): "Famous First Bubbles", *Journal of Economic Perspectives*, 4, pp 35-54.

Ghosh, A and B K Chakrabarti (2009): *Kolkata Paise Restaurant Problem*, http://demonstrations.wolfram.com/KolkataPaiseRestaurantKPRProblem

Ghosh, A, A Chatterjee, M Mitra and B K Chakrabarti (2010): "Statistics of the Kolkata Paise Restaurant Problem", *New Journal of Physics*, 12, 075033.

Gopikrishnan, P, M Meyer, L A N Amaral and H E Stanley (1998): "Inverse Cubic Law for the Probability Distribution of Stock Price Variations", *European Physical Journal B*, 3, pp 139-40.

Hayes, B (2002): "Follow the Money", *American Scientist*, 90, pp 400-05.

Jansen, D W and C G de Vries (1991): "On the Frequency of Large Stock Returns: Putting Booms and Busts into Perspective", *Review of Economics and Statistics*, 73, pp 18-24.

Kaizoji, T (2006): "A Precursor of Market Crashes: Empirical Laws of Japan's Internet Bubble", *European Physical Journal B*, 50, pp 123-27.

Kalecki, M (1935): "A Macrodynamic Theory of Business Cycles", *Econometrica*, 3, pp 327-44.

Keynes, J M (1932): *Essays in Persuasion* (New York: Harcourt Brace).

Laloux, L, P Cizeau, J-P Bouchaud and M Potters (1999): "Noise Dressing of Financial Correlation Matrices", *Physical Review E*, 83, pp 1467-70.

Levitt, S D and S J Dubner (2005): *Freakonomics* (New York: Harper Collins).

Levy, M, and S Solomon (1997): "New Evidence for the Power-Law Distribution of Wealth", *Physica A*, 242, pp 90-94.

Lucas, R E (2003): "Macroeconomic Priorities", *American Economic Review*, 93, pp 1-14.

Lux, T (1996): "The Stable Paretian Hypothesis and the Frequency of Large Returns: An Examination of Major German Stocks", *Applied Financial Economics*, 6, pp 463-75.

Lux, T and M Marchesi (1999): "Scaling and Criticality in a Stochastic Multi-Agent Model of a Financial Market", *Nature*, 397, pp 498-500.

Lux, T and F Westerhoff (2009): "Economics Crisis", *Nature Physics*, 5, pp 2-3.

McCann, K S (2000): "The Diversity-Stability Debate", *Nature*, 405, pp 228-33.

Mandelbrot, B B and R L Hudson (2004): *The Misbehaviour of Markets* (New York: Basic Books).

Mantegna, R N (1999): "Hierarchical Structure in Financial Markets", *European Physical Journal B*, 11, pp 193-97.

Mantegna, R N and H E Stanley (1995): "Scaling Behaviour in the Dynamics of an Economic Index", *Nature*, 376, pp 46-49.

Mantegna, R N and H E Stanley (1999): *An Introduction to Econophysics* (Cambridge: Cambridge University Press).

May, R M (1972): "Will a Large Complex System Be Stable?", *Nature*, 238, pp 413-14.

– (1973): *Stability and Complexity in Model Ecosystems* (Princeton: Princeton University Press).

– (1976): "Simple Mathematical Models with Very Complicated Dynamics", *Nature*, 261, pp 459-67.

May, R M, S A Levin and G Sugihara (2008): "Ecology for Bankers", *Nature*, 451, pp 893-95.

Mirowski, P (1989): *More Heat Than Light: Economics as Social Physics, Physics as Nature's Economics* (Cambridge: Cambridge University Press).

Mohanty, P K (2006): "Generic Features of the Wealth Distribution in Ideal-Gas-Like Markets", *Physical Review E*, 74, 011117.

Montroll, E W and M F Shlesinger (1982): "On 1/*f* Noise and Other Distributions with Long Tails", *Proceedings of the National Academy of Sciences USA*, 79, pp 3380-83.

Negishi, T (1962): "The Stability of a Competitive Economy: A Survey Article", *Econometrica*, 30, pp 635-69.

Newman, M E J (2010): *Networks: An Introduction* (Oxford: Oxford University Press).

Okuyama, K, M Takayasu and H Takayasu (1999): "Zipf's Law in Income Distribution of Companies", *Physica A*, 269, pp 125-31.

Onnela, J-P, A Chakraborti, K Kaski and J Kertesz (2002): "Dynamic Asset Trees and Portfolio Analysis", *European Physical Journal B*, 30, pp 285-88.

Pan, R K and S Sinha (2007a): "Self-organisation of Price Fluctuation Distribution in Evolving Markets", *Europhysics Letters*, 77, 58004.

– (2007b): "Collective Behaviour of Stock Price Movements in an Emerging Market", *Physical Review E*, 76, 046116.

– (2008): "Inverse-Cubic Law of Index Fluctuation Distribution in Indian Markets", *Physica A*, 387, pp 2055-65.

– (2009): "Modularity Produces Small-World Networks with Dynamical Time-Scale Separation", *Europhysics Letters*, 85, 68006.

– (2010): "The Statistical Laws of Popularity: Universal Properties of the Box Office Dynamics of Motion Pictures", *New Journal of Physics*, 12, 115004.

Pareto, V (1906): *Manual of Political Economy* (trans A S Schwier, 1971) (London: Macmillan).

Plerou, V, P Gopikrishnan, L A N Amaral, M Meyer and H E Stanley (1999a): "Scaling of the Distribution of Price Fluctuations of Individual Companies", *Physical Review E*, 60, pp 6519-29.

Plerou, V, P Gopikrishnan, B Rosenow, L A N Amaral and H E Stanley (1999b): "Universal and Non-universal Properties of Cross Correlations in Financial Time Series", *Physical Review Letters*, 83, pp 1471-74.

Posner, R A (2009): *A Failure of Capitalism: The Crisis of '08 and the Descent into Depression* (Cambridge, Massachusetts: Harvard University Press).

Potters, M and J-P Bouchaud (2003): "More Statistical Properties of Order Books and Price Impact", *Physica A*, 324, pp 133-40.

Pradhan, N, S Dasgupta and S Sinha (2011): "Modular Organisation Enhances the Robustness of Attractor Network Dynamics", *Europhysics Letters*, 94, 38004.

Prigogine, I and I Stengers (1984): *Order Out of Chaos* (London: Flamingo).

Rosser, J B (2008): "Econophysics" in Durlauf and Blume (ed.), *The New Palgrave Dictionary of Economics* (London: Palgrave-Macmillan).

Saha, M N and B N Srivastava (1931): *A Treatise on Heat* (Allahabad: Indian Press).

Samuelson, P A (1947): *Foundations of Economic Analysis* (Cambridge, Massachusetts: Harvard University Press).

Schumpeter, J A (1975): *Capitalism, Socialism and Democracy* (New York: Harper).

Sen, A (2009): "Capitalism Beyond the Crisis", *New York Review of Books*, 56 (5), available at http://www.nybooks.com/articles/22490.

Shiller, R (2005): *Irrational Exuberance* (Princeton: Princeton University Press).

Sinha, S (2003): "Stochastic Maps, Wealth Distribution in Random Asset Exchange Models and the Marginal Utility of Relative Wealth", *Physica Scripta T*, 106, pp 59-64.

– (2005a): "Complexity vs Stability in Small-World Networks", *Physica A*, 346, pp 147-53.

– (2005b): "The Rich Are Different! Pareto Law from Asymmetric Interactions in Asset Exchange Models" in Chatterjee, Yarlagadda and Chakrabarti (ed.), *Econophysics of Wealth Distributions* (Milan: Springer-Verlag), pp 177-83.

– (2006): "Evidence for Power-Law Tail of the Wealth Distribution in India", *Physica A*, 359, pp 555-62.

– (2010a): "Are Large Complex Economic Systems Unstable?", *Science and Culture*, 76, 454-58.

– (2010b) "Why Econophysics" in Basu, Chakrabarti, Chakravarty and Gangopadhyay (ed.), *Econophysics and Economics of Games, Social Choices and Quantitative Techniques* (Milan: Springer-Verlag), pp 156-61.

Sinha, S, A Chatterjee, A Chakraborti and B K Chakrabarti (2011): *Econophysics: An Introduction* (Weinheim: Wiley-VCH).

Sinha, S and R K Pan (2007): "How a Hit Is Born: The Emergence of Popularity from the Dynamics of Collective Choice" in Chakrabarti, Chakraborti and Chatterjee (ed.), *Econophysics and Sociophysics: Trends and Perspectives* (Weinheim: Wiley-VCH), pp 417-47.

Sinha, S and S Poria (2011): "Multiple Dynamical Time-Scales in Networks with Hierarchically Nested Modular Organisation", *Pramana – Journal of Physics*, 77, pp 833-42.

Sinha, S and S Raghavendra (2004): "Hollywood Blockbusters and Long-Tailed Distributions: An Empirical Study of the Popularity of Movies", *European Physical Journal B*, 42, pp 293-96.

– (2006): "Market Polarisation in Presence of Individual Choice Volatility" in Bruun (ed.), *Advances in Artificial Economics* (Berlin: Springer), pp 177-90.

Sinha, S and S Sinha (2005): "Evidence of Universality for the May-Wigner Stability Theorem for Random Networks with Local Dynamics", *Physical Review E*, 71, 020902.

– (2006): "Robust Emergent Activity in Dynamical Networks", *Physical Review E*, 74, 066117.

Sinha, S and N Srivastava (2007): "Is Inequality Inevitable in Society?: Income Distribution as a Consequence of Resource Flow in Hierarchical Organisations" in Chatterjee and Chakrabarti (ed.), *Econophysics of Markets and Business Networks* (Milan: Springer-Verlag), pp 215-26.

Sinha, S, M Thess and S Markose (2012): "How Unstable Are Complex Financial Systems? Analysing an Inter-Bank Network of Credit Relations", forthcoming.

Solow, R M (1985): "Economic History and Economics", *American Economic Review*, 75, pp 328-31.

Soramaki, K, M L Bech, J Arnold, R J Glass and W E Beyeler (2007): "The Topology of Interbank Payment Flows", *Physica A*, 379, pp 317-33.

Souma, W (2000): "Universal Structure of the Personal Income Distribution", *Fractals*, 9, pp 463-70.

Sterman, J D (1989): "Modelling Managerial Behaviour: Misperceptions of Feedback in a Dynamic Decision Making Experiment", *Management Science*, 35, pp 321-39.

Szabo, G and G Fath (2007): "Evolutionary Games on Graphs", *Physics Reports*, 446, pp 97-216.

Trigger, B G (1998): *Sociocultural Evolution* (Oxford: Blackwell).

Vijayraghavan, V S and S Sinha (2011): "Are the Trading Volume and the Number of Trades Distributions Universal?" in Abergel, Chakrabarti, Chakraborti and Mitra (ed.), *Econophysics of Order-Driven Markets* (Milan: Springer-Verlag), pp 17-30.

Vikram, S V and S Sinha (2011): "Emergence of Universal Scaling in Financial Markets from Mean-Field Dynamics", *Physical Review E*, 83, 016101.

Wilmers, C C, S Sinha and M Brede (2002): "Examining the Effects of Species Richness on Community Stability: An Assembly Model Approach", *Oikos*, 99, pp 363-67.

Yakovenko, V M and J B Rosser (2009): "Statistical Mechanics of Money, Wealth, and Income", *Reviews of Modern Physics*, 81, pp 1703-25.

Yang, J-S, S Chae, W-S Jung and H-T Moon (2006): "Microscopic Spin Model for the Dynamics of the Return Distribution of the Korean Stock Market Index", *Physica A*, 363, pp 377-82.