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Controlled transition from chaos to periodic oscillations in a neural network model

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Abstract

Neurobiological studies have indicated that rapid transitions between chaotic and relatively more ordered states may be the key towards understanding how the brain performs cognitive tasks. This immediately suggests that methods of controlling chaos put forward by Ott et al. and Hunt may be used to study similar phenomena in neural network models. In the work described in this paper, an oscillatory neural network that can exhibit chaos under certain conditions was used for controlling purpose. On imposition of control, transition of the network behavior from chaos to periodicity was observed. This has implications for both the explanation of observed neurobiological phenomena (e.g., during epileptic seizures) as well as a more dynamic interpretation of associative recall performed by neural network models.

1. Introduction

Neural network models of computation have been enjoying a period of revival for quite some time now, from the perspective of both theory and applications [1]. These models comprise of networks of large numbers of simple processing elements, usually having continuously varying activation values and stochastic threshold dynamics. The activity of these elements, x_i $(i = 1, 2, \ldots, N)$ are determined by the temporal evolution equation:

$$x_i = \mathrm{F}(\Sigma_j W_{ij} x_j - \theta_i), \qquad (1)$$

where θ_i is an internal threshold (usually taken as zero), W_{ij} is the connection weight from element j to element i, and F is a nonlinear function. Often, this function has the following sigmoid form:

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$$F(z) = \frac{2}{\pi}\tan^{-1}\left(\frac{z}{a}\right), \tag{2}$$

a being the slope. For $a = 0$, F is a "hard limiting" or step function,

$$x_i = \mathrm{sgn}(\Sigma_j W_{ij}x_j - \theta_i).$$

Different neural network models are specified by

- network topology, i.e. the pattern of connections between the elements comprising the network;
- characteristics of the processing element, e.g. the explicit form of the nonlinear function F, and the value of the threshold, θ;
- learning rule, i.e. the rules for computing the connection weights W_{ij} appropriate for a given task.

Some networks have been shown to be capable of associative memory and learning. However, one of the limitations of most models of associative memory at present is that they basically store a small number of single patterns, i.e. to say, once a pattern has been recalled, the network remains in that state, until the arrival of new external input. In contrast, real neural networks show a preponderance of sequential memories. Once we recall a memory, our minds are not stuck to it, but also recall other associated memories without being prompted by any additional external stimulus. This ability to 'jump' from one memory to another in the absence of appropriate stimuli is one of the hallmarks of the brain. It is an ability which one should try to recreate in a network model if it is ever to come close to human-like performance in intellectual tasks.

In this paper it is argued that the insights of chaos theory into the behavior of dynamical systems can help us in approaching the above problem of storing a large number of dynamic spatio-temporal patterns in a network and making it spontaneously recall associated patterns in a sequence on the arrival of some external stimulus. Section 2 discusses the identification of chaotic activity in neurobiological systems and its relevance to brain function. Section 3 reviews the technique of chaos control which can drive a system from chaotic to more periodic behavior. Section 4 describes a model for simulating the chaotic dynamics observed in the brain to which control is applied, while in Section 5 the relevance of this type of research to cognitive theory, clinical practice and practical neural net applications is discussed. Finally, some observations on the connection of this work with research on coupled lattice maps and possible directions of future research are made in the conclusion.

2. Chaos in neurobiology

Evidence of deterministic chaos in the brain and associated neuronal apparatus were found within a very short time of the emergence of the field of chaotic dynamics. Hayashi et al. [2] investigated the nonperiodic behavior in self-sustained oscillation of the internodal cell of Nitella under sinusoidal stimulation. The analysis of the nonperiodic oscillations of the excitable biological membrane revealed chaotic behavior.

Another group of researchers [3] studied the self-sustained oscillation of action potentials in an axon immersed in calcium-deficient sea water. The oscillations were analyzed by stroboscopic plots revealing both periodic and chaotic behavior, determined by the amplitude and the frequency of the stimulating current. The results corroborated similar studies carried out previously in squid giant axons. The group of Glass and Guevara [4] showed that recurrent inhibition and periodic forcing of neural oscillators can produce chaos and explored its implications in modeling normal and abnormal function in neurophysiology. However, these experiments were concerned solely with the microstructure of the nervous system and did not answer the issue of whether chaos played any role in the overall cognitive functioning of the brain.

The empirical evidence indicating the possible relevance of chaos to brain function was first obtained by Walter J. Freeman, through his work on the large-scale collective behaviour of neurons in the perception of olfactory stimuli [5,6]. Olfactory stimuli are detected by receptor neurons in the nasal passage. The number of receptors excited by a smell is a measure of the intensity of the stimulus, while the spatial pattern of activated receptors is dependent on the nature of the scent. On trapping molecules carrying specific odors these receptors fire action potentials which are transmitted to the olfactory bulb in the cortex. The bulb then transmits signals to the olfactory cortex which, in its turn, sends information to many regions of the brain. The test animals used by Freeman were trained to recognize several different odors and were then subjected to smells, both familiar and unfamiliar. EEG data was recorded by a gridlike array of 64 electrodes placed over the surface of the olfactory bulb. Most of the time, the EEGs showed irregular oscillations. However, when an animal inhaled, a "burst" occurred in each EEG tracing as all the waves in the array became more regular for a brief period until the animal exhaled. These waves, named gamma waves, had a higher amplitude and frequency than usual and varied in frequency from 20 to 90 hertz, mostly occurring in the neighborhood of 40 hertz. Each set of burst recordings had a common carrier waveform, although the average amplitude of the different recordings varied widely. It was inferred that, as the carrier waveform changed during each inhalation, even for the same stimulus, the information about a particular scent was not encoded in the shape of the waveform but rather in the spatial pattern of the carrier-wave amplitudes across the bulb, which remained invariant over trials. On plotting the different amplitudes of the carrier waves in different regions of a surface representing the locations in the gridlike array over the olfactory bulb from which they were obtained, a contour diagram was produced. This remained the same for a specific scent throughout the testing period. However, if the reinforcement associated with a scent was altered, then the amplitude contour map representing it also changed. This indicated that the olfactory bulb is involved in the assigning of meaning to stimuli. One of the early pointers to chaotic activity was the aperiodicity of the common carrier wave in the bulb both during and between bursts. Another clue was the sudden transitions of neuronal networks in the bulb and the cortex from a nonburst to an active, bursting state. These factors prompted Freeman to develop a model for the olfactory system having cells in a network connected by both excitatory and inhibitory synapses. Computer simulations of the model showed that it recreated all the observed behavior of the olfactory system and, thus, was an accurate representation of it. The network was then made to produce EEGs of extended bursts and of interburst activity for a longer period than is possible in actual EEGs. The attractor of the underlying dynamics, reconstructed from the EEG data using the delay-coordinate technique was found to be chaotic in nature. The primary findings suggested that a separate chaotic attractor is maintained for each stimulus and the act of perception consists of a transition of the system from the domain of influence of one attractor to another. Later findings led Freeman to hypothesize that each brain area (rather than each stimulus) has a chaotic attractor. A specific sensory stimulus drives the system into a localized region within the attractor, which can be identified with the appearance of specific spatial patterns of carrier waveform amplitudes, associated with a specific stimulus. Further, the attractors themselves would have to change as a result of new experience and continued development of the brain. Thus, according to this picture, chaotic activity is fundamental to the general process of perception.

3. Chaos control

One of the distinguishing features of chaotic attractors is that they have an infinite number of periodic attractors embedded within them. If the system state exists at any time on a region belonging to one of the periodic attractors then it will remain within it. However, as the attractors are unstable, when perturbed by noise, the system diverges away from the periodic orbit at an exponential rate. In a chaotic attractor, the system state may be, at any given time, infinitesimally close to any one of the infinite periodic attractors but due to the highly unstable nature of the periodic orbits, the periodicity is never manifested over a measurable period of time.

Edward Ott, Celso Grebogi and James A. Yorke have used this feature of chaotic attractors to construct a general method of controlling chaos, i.e. to convert the chaotic behavior of a system to a time-periodic one [7]. Their method (referred to as OGY method henceforth) achieves this control by making small, carefully chosen time-dependent perturbations of one of the parameters of the system. To obtain a periodic orbit, a local map around the desired attractor is constructed by the method of delay-coordinate embedding. In this method, the experimentally obtained time-series of some variable $x(t)$ is obtained, with a delay T. The measured values are then used to construct an $(n+1)$-dimensional delay-coordinate vector

$$X(t) = [x(t),\ x(t-T),\ x(t-2T),\ \ldots,\ x(t-nT)], \tag{3}$$

whose time-evolution can be plotted to give a 3-dimensional projection of the trajectory. It has been mathematically demonstrated that such a projection is a good approximation of the dynamical attractor of the relevant system in $(n+1)$-dimensional space [8,9]. The OGY control method relies upon the identification of saddle instabilities, i.e., unstable periodic points located on a surface having both stable and unstable directions. The system approaches the periodic point along a stable direction and diverges away from it along an unstable one. When the chaotic system state is in the neighborhood of the desired attractor, a perturbation is applied to a system parameter such that on the next iteration the system state falls on the stable direction. The state will then move towards the attractor in successive iterations. Using this principle, many physical systems exhibiting chaotic behavior have been subjected to control. However, one drawback of this method is that only low-period orbits can be stabilized, since, because of the exponential error magnification in chaotic systems, high-period orbits are impossible to achieve by making only one correction in the long period.

The OGY method has been used to control chaos in a biological neural network prepared from a hippocampal slice of a rat brain [10]. The brain slice was induced to exhibit chaos by immersing it in an artificial K^+-enriched cerebrospinal fluid. Synchronous bursts were recorded and the interburst interval times, T, were plotted as points on a first-return map (i.e., T_n vs. T_{n+1} plot). A period-1 interburst interval was manifested as a single point lying on the diagonal of the return map. The neighborhood of these points showed both a stable direction along which the distance of the point from the diagonal decreased with iterations, and an unstable one, along which the distance increased. Using precisely timed electrical stimulation of the brain slice, the system was perturbed such that a trajectory coincided with a stable direction in the neighborhood of a periodic attractor. Thereafter the point moved towards the attractor with successive iterations and the resulting trajectories became periodic. In a similar way, anticontrol was achieved by perturbing a stable orbit, so that the next iteration falls close to the unstable direction, with a corresponding transition from periodic to chaotic behavior.

A modified method of controlling chaos has been proposed by E.R. Hunt [11], which allows stabilization of orbits of periods longer than that is possible through the OGY method. As higher period orbits visit most regions of the attractor, which correspond to different physical states of the system, it is desirable to have high-period orbits if one wishes to sample as many of the states as possible. In the Hunt method, occasional proportional feedback is used to amplitude modulate the driving agency of the system. Specifically, deviations of a chaotic variable within a specified window centered about a specified value of a chosen system variable are fed back to perturb the controlling drive. By varying the specified variable value, the window and the amplitude of the feedback, different periodic orbits may be made to lock in (i.e., stabilized). This method was used by Hunt to control the chaotic behavior found in a diode resonator driven with sinusoidal voltage, where the chaotic variable was the peak forward current through the diode. Orbits with periods up to 23 drive cycles could be stabilized. In the work described later in this paper a variation of the Hunt method, suitably adapted for the given purpose, was used to stabilize periodic attractors in a neural network model exhibiting chaotic behavior.

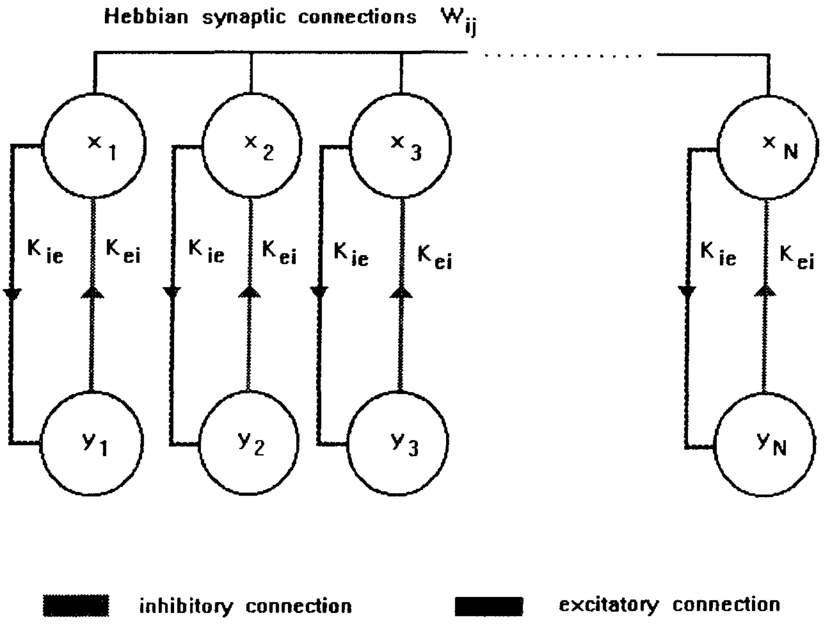

Fig. 1. The oscillatory neural network model. Excitatory and inhibitory cells are labeled as x and y, respectively. W_{ij} represents the connection weights between the excitatory cells. K_{ei} is the strength of inhibitory connection from y to x and K_{ie} is the strength of excitatory connection from x to y.

4. Chaotic neural network model

One of the ways of studying the relevance of chaos to brain activity is through the construction of specific neural network models that can give rise to chaotic activity. One such model, utilizing mutual excitatory-inhibitory synaptic connections giving rise to oscillatory behavior has been recently proposed by Hayashi [12]. The present work was done on a modified version of the Hayashi network. The model consists of N excitatory and N inhibitory elements, denoted by x_i and y_i, respectively (Fig. 1). The cells update their states in the $(n+1)$th iteration, according to the following transformations:

$$x_i(n+1) = G\left(\sum_{j=1}^{N} W_{ij}x_j(n) - K_{EI}y_i(n) + I_i(\omega)\right) \tag{4}$$

and

$$y_i(n+1) = G(K_{IE}x_i(n) + I_i(\omega+\delta)), \tag{5}$$

where $I(\omega)$ is an external periodic input or bias, δ is a phase difference between the inputs to x_i and y_i and the function G is defined as

$$G(z) = \frac{2}{\pi}\arctan(z/a), \tag{6}$$

a being the slope of the function. This parameter is analogous to 'temperature' in a physical system.

Thus, while excitatory cells are all connected to each other, inhibitory cells are connected only to the corresponding excitatory cells – K_{IE} being the weightage of the excitatory connection from x_i to y_i and K_{EI} being the inhibitory connection weightage from y_i to x_i. It is evident that x_i and y_i vary between +1 and −1. The activation values of the excitatory cells at time t, $x_i(t)$, are taken as the output of the network at that instant of time.

The synaptic connection weights W_{ij} between the excitatory cells, x_i, are evaluated by a modification of the standard Hebb rule,

$$W_{ij} = \frac{1}{N} \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu} + \delta_{ij}, \tag{7}$$

where ξ_i^{μ} is the ith component of the μth pattern vector being stored in the network and δ_{ij} is the Kronecker delta function.

It is easily seen that the pair of first-order difference equations (4),(5) is equivalent to the second-order difference equation:

$$x_i(n+1) = G\left(\sum_{j=1}^{N} W_{ij} x_j(n) - K_{EI} G\left(K_{IE} x_i(n-1) + I_i(\omega + \delta)\right) + I_i(\omega) \right), \tag{8}$$

which resembles a Hopfield-like model with delay-dynamics.

4.1. Stability analysis

The stability of the fixed points of a single excitatory–inhibitory pair is investigated in this section. Let z^* denote a fixed point of the pair of evolution equations for x_i and y_i. The Jacobian in the neighborhood of the fixed point in the absence of any external input is given by

$$\mathbf{J} = \begin{vmatrix} G'(z)|_{z=z^*} W_{ii} & G'(z)|_{z=z^*} K_{EI} \\ G'(z)|_{z=z^*} K_{IE} & 0 \end{vmatrix}, \tag{9}$$

where $G'(z)$ is the derivative of $G(z)$ w.r.t. z.

The eigenvalues are given by the solution of the equation

$$\mathrm{Det}\,|\mathbf{J} - \lambda \mathbf{I}| = \mathbf{0}, \tag{10}$$

where $\mathbf{I}$ is the identity matrix. This equation can be reduced to

$$\lambda^2 - \mathrm{Tr}\,\mathbf{J}\,\lambda + \mathrm{Det}\,\mathbf{J}. \tag{11}$$

Thus,

$$\lambda_{1,2} = \frac{\mathrm{Tr}\,\mathbf{J} \pm \sqrt{(\mathrm{Tr}\,\mathbf{J})^2 - 4\,\mathrm{Det}\,\mathbf{J}}}{2}. \tag{12}$$

For $(\mathrm{Tr}\,\mathbf{J})^2 - 4\,\mathrm{Det}\,\mathbf{J} \geq 0$ the solutions eventually converge to the fixed point, which is thus stable. Evaluating $\mathbf{J}$, $\mathrm{Tr}\,\mathbf{J} = G'(z)|_{z=z^*}W_{ii}$ and $\mathrm{Det}\,\mathbf{J} = (G'(z)|_{z=z^*})^2 K_{IE}K_{EI}$. As $G'(z) = 2a/\pi(a^2+z^2)$, the eigenvalues are given by

$$\lambda_{1,2} = \frac{2a}{\pi(a^2+z^{*2})}\left(W_{ii} \pm \sqrt{W_{ii}^2 - 4K_{EI}K_{IE}}\right). \tag{13}$$

According to (7), $W_{ii} = 2$. Therefore, the condition for stability of the fixed point is $K_{EI}K_{IE} \leq 1$. If this condition is not satisfied, all the fixed points of the second-order difference equation become unstable giving rise to periodic orbits.

4.2. Simulation results

For the present work, a network with $N = 3$ was used for carrying out computer simulations. The number of cells was kept low for ease of graphical analysis. The pattern vectors chosen for storage in the network were

$$\xi^1 = (\,1 \quad 1 \quad 1\,), \qquad \xi^2 = (\,1 \quad -1 \quad -1\,), \qquad \xi^3 = (\,-1 \quad -1 \quad 1\,).$$

The memory patterns are, therefore, vertices of a cube, $-1 \leq x_i \leq 1$ $(i = 1,2,3)$. After evaluating the weight matrix, the network was made to evolve from a randomly chosen initial value. The initial states were taken as $x_i(0) = I_i$ and $y_i(0) = 0$. As pointed out above the condition for oscillations in a single excitatory-inhibitory pair is $K_{EI}K_{IE} \geq 1$. The system thus corresponds to three oscillators weakly coupled together through connection strengths $\sim \mathcal{O}(1/N)$. If these three oscillators have independent periods of oscillations then, according to the results of Li and Yorke [13], the system is capable of chaotic behaviour.

The simulation was carried out for the following set of parameters: $K_{EI} = 2.0$, $K_{IE} = 2.0$ and $a = 0.1$. An external sinusoidal input which completed one period every 300 iterations of the network with $\delta = 0$ was used to stimulate the network. The system showed chaotic activity when the input pattern vector was far from any of the stored pattern vectors. The 3-dimensional plot of the trajectory of the network state in state space is shown in Fig. 2, while Fig. 3 shows the variation of the average activation of the excitatory cells with iteration number along with the corresponding logarithmic spectral density. Another performance parameter to be noted is the overlap of the output of the network at a given time with each of the stored patterns. This is defined for the μth pattern vector at the nth iteration as

$$m^\mu(n) = \frac{1}{N}\sum_{i=1}^{N} x_i^\mu(n)\xi_i^\mu. \tag{14}$$

In Fig. 4 the overlap values are plotted against the number of iterations, for each of the three ξ^μs.

This chaotic system was then subjected to control by modulating the amplitude of the external periodic input in accordance with Hunt's method described above. The resulting

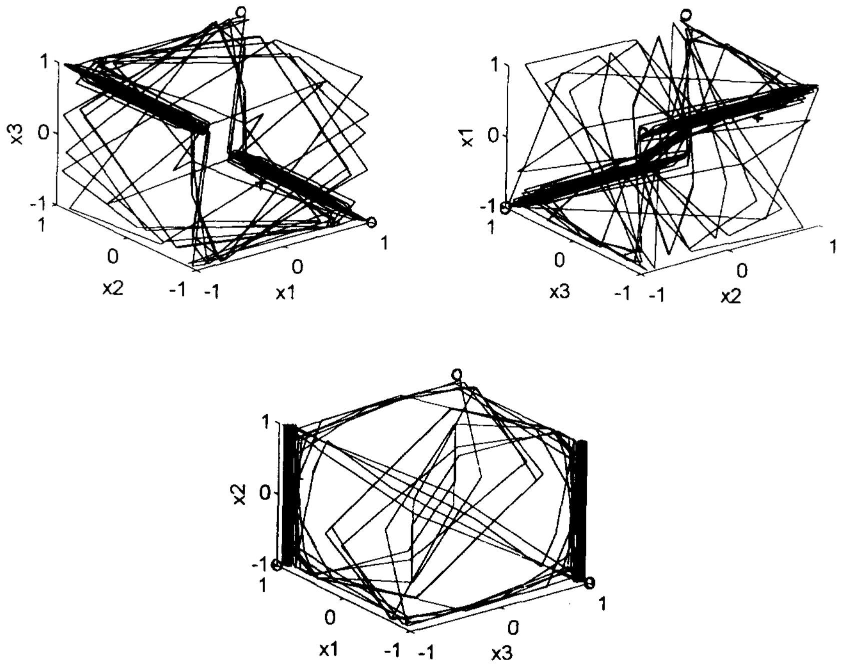

Fig. 2. Trajectory of the network output in the chaotic state ($K_{ei} = 2.0$, $K_{ie} = 2.0$). The activation values for the cells $x[1]$, $x[2]$ and $x[3]$ are plotted along the three axes. The circles represent the locations of the stored patterns and the plus sign indicates the location of the input pattern.

system starting from the same initial conditions as in the previous chaotic case and with the network parameters unchanged, showed a time-series repeating every 15 iterations, after the initial transients had died away. The trajectory of the system state over time in Fig. 5 shows a simple periodic orbit. Regular periodic behavior is clearly manifested in the plot of the average excitatory cell activation and the corresponding logarithmic spectral density curve in Fig. 6. The overlap vs. iterations diagram in Fig. 7 shows that the network periodically comes close to each of the three stored patterns. Thus, on imposing control, the system state exhibits periodicity quite clearly.

5. Implications

The modeling of neurobiological chaos and its subsequent control to produce periodic behavior, points out several possible avenues for research aimed at understanding how the brain works. As Freeman has already pointed out [5,6], Chaos is omnipresent in the brain - demonstrably so in the somatosensory and the olfactory cortices. It has been suggested that the quiescent state of the brain is chaos, while during perception, i.e. when attention is focused on any sensory stimulus, brain activity becomes more periodic. From this perspective, the periodic orbits observed in the controlled state of the network model can be interpreted as specific memories. If the different spatio-temporal patterns stored

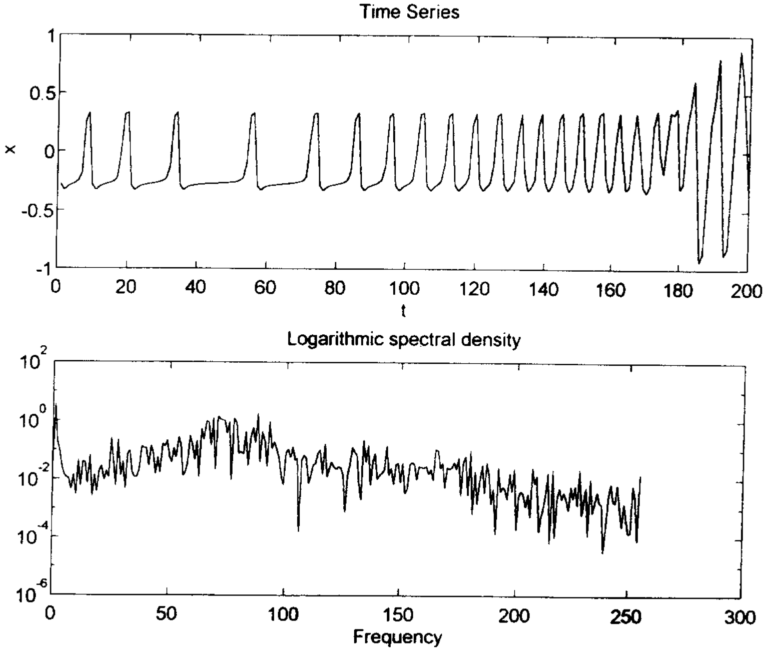

Fig. 3. Temporal evolution of the average activation value of the excitatory cells in the chaotic state and the corresponding logarithmic spectral density.

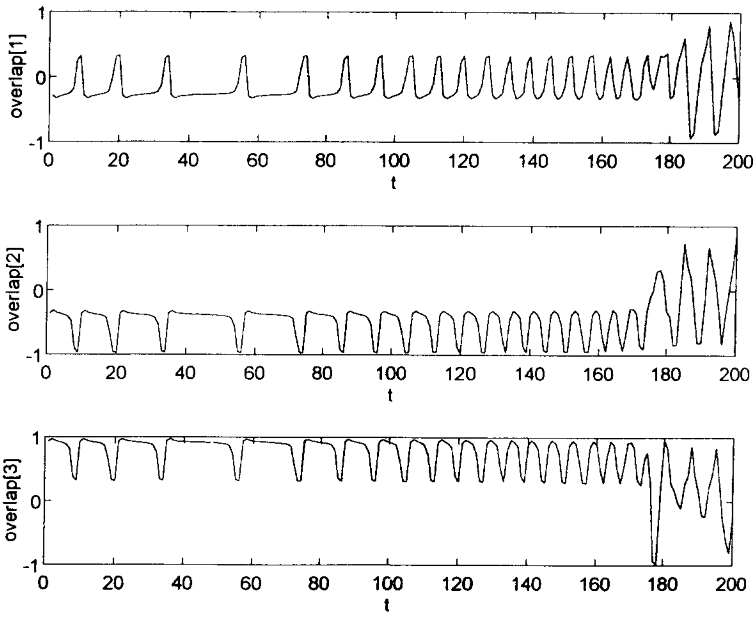

Fig. 4. Temporal evolution of the overlap of the network output with the stored patterns in the chaotic state.

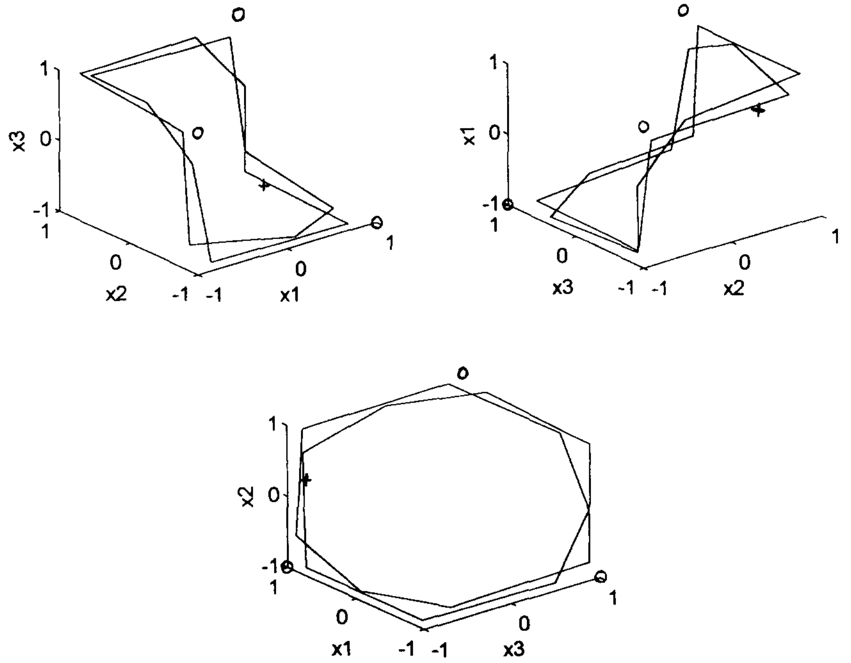

Fig. 5. Trajectory of the network output in the controlled state for the same set of network parameters and input pattern as in Fig. 2.

in memory are identified with the infinite number of unstable periodic attractors that are embedded in the attractor, then the transition from quiescence to attention can be interpreted as the controlling of chaos to give rise to periodic behavior, culminating in the identification of the sensory stimulus that has been received. This control, of course, is not imposed by any external agency, but is an emergent property of the brain.

This identification is also indirectly supported by the clinical observations of hallucinations that are prevalent during sensory epileptic seizures [14]. Complex visual hallucinations are usually due to an epileptic focus in the posterior part of the temporal lobe, near its junction with the occipital lobe. Often the hallucinatory visual images are distorted, being too small or too large or unnaturally arranged. More striking is the case of olfactory hallucinations, that are often associated with disease of the inferior and medial parts of the temporal lobe, usually in the region of the hippocampal convolution. Usually the hallucinatory smell is thought by the subjects to come from some place in the environment and is described as disagreeable or foul, although otherwise unidentifiable. This can be explained in terms of the proposed picture of brain function by noting that during epilepsy, undesired control of the chaotic activity of the brain occurs as a result of the highly synchronized electrical stimulation of large parts of the brain. The stabilizing of an undesired periodic attractor results in the erroneous recognition of a sensory stimulus even when such a stimulus is absent from the immediate environment of the epileptic subject.

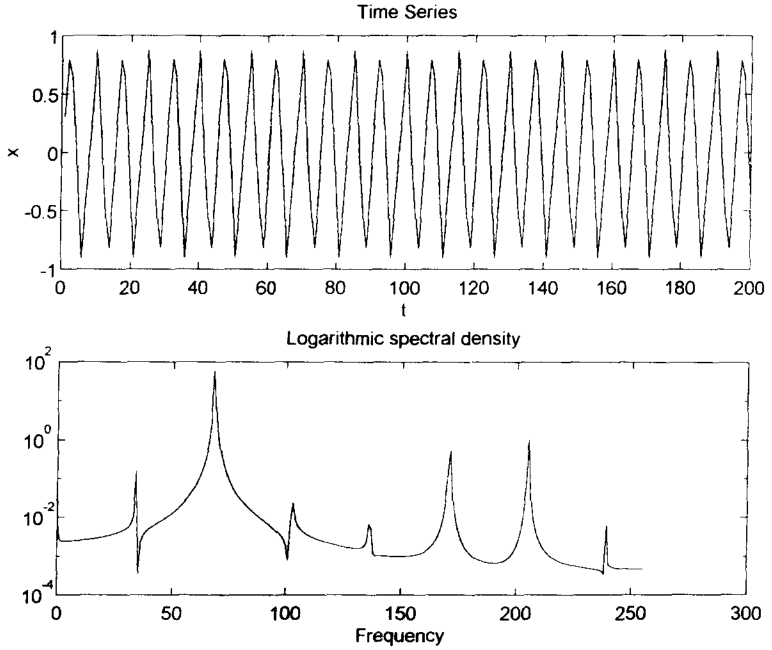

Fig. 6. Temporal evolution of the average activation value of the excitatory cells in the controlled state and the corresponding logarithmic spectral density.

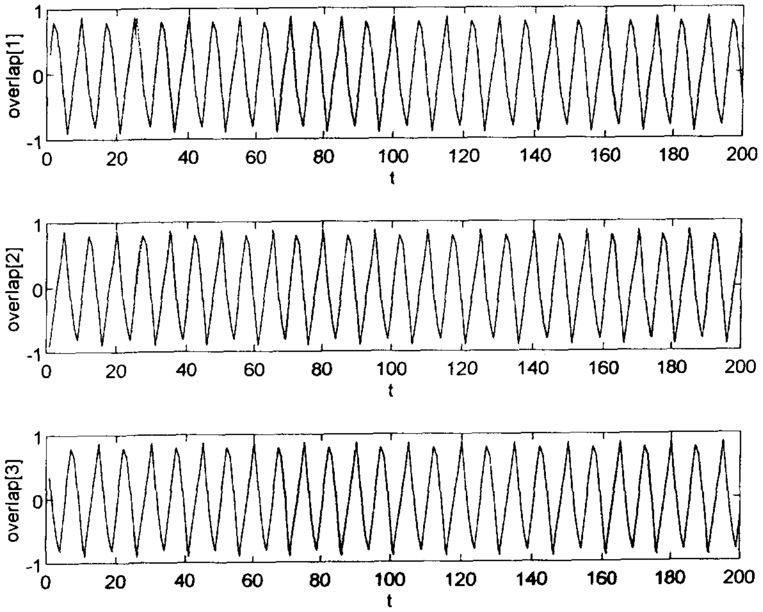

Fig. 7. Temporal evolution of the overlap of the network output with the stored patterns in the controlled state.

The use of chaos control can also benefit fields which require the practical implementation of neural network models. The possibility of embedding an enormous amount of dynamic patterns (e.g., moving images, pieces of music, etc.) in a chaotic neural network and their systematic recognition will widen the scope of applications of neurocomputers. Temporal sequences can be associated with the unstable periodic attractors by suitable learning rules and then recalled on presentation of suitable stimuli as input to the network. The convergence to the stored patterns will be extremely rapid owing to the chaotic dynamics governing the network. Neural networks implementing delay dynamics, which closely resembles the above model (as mentioned above), have already been shown to perform better than standard models in tasks such as associative memory (in terms of storage capacity) and combinatorial optimisation [15]. However, further work on finding suitable architectures and learning algorithms will be necessary before this promise is fully realized.

6. Conclusion

The network used here has simple Boolean processing elements with a nonlinear transition function. Biological neurons however are far more complicated and are better modeled by the Bonhoeffer–Van der Pol (BVP) equations. Single BVP oscillators undergoing forced oscillations have been shown to exhibit chaotic behaviour [16]. Future work is therefore planned on studying the large-scale collective behaviour of a network of BVP oscillators coupled to each other through Hebbian connections.

Another possible avenue of further work has been opened by the relation of the above model to coupled lattice maps, i.e. networks where the evolution of each element is governed by a chaotic difference equation (e.g., the logistic map). The elements are coupled to their nearest neighbors. Depending on the strength and nature of these couplings, such systems show a remarkable variety of behaviour, including synchronization. It is hoped that the theoretical apparatus developed for studying such lattices can be used fruitfully for the detailed analysis of the chaotic neural network model. In this context, it is interesting to note that Pecora and Carroll [17] who have worked on synchronizing certain subsystems of chaotic systems by linking them with common signals have suggested that the concept of synchronization can be extended to some neural processes. According to them, the process of synchronization can be viewed as a response system that "knows" what brain state (attractor) to go to when driven (stimulated) by a particular stimulus.

The traditional neural network paradigm has always seen the phenomena of associative memory as being due to a multitude of fixed-point attractors [18]. Each such attractor was identified with a specific pattern which was desired to be stored and recalled associatively. But limitations of neural networks designed according to this prescription as well as recent neurobiological research is now forcing this "fixed-point" approach to be replaced by a more dynamic interpretation of how associative recall may actually be performed in the brain. It is hoped that the work presented in this paper shows to some extent the promise of "chaotic-attractor" neural networks.

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