International Trade Network

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Publications

1. The International Trade Network: weighted network analysis and modeling,
   K. Bhattacharya, G. Mukherjee, J. Sarämaki, K. Kaski and S. S. Manna
   J. Stat. Mech. P02002, 2008.
2. The International Trade Network,
   K. Bhattacharya, G. Mukherjee and S. S. Manna in
   Econophysics of Markets and Business Networks, Springer 2007.

Key references

1. Ma Á. Serrano and M. Boguňá, Phys. Rev. E 68, 015101(R), (2003).
2. D. Garlaschelli and M. I. Loffredo, Phys. Rev. Lett. 93, 188701 (2004).
3. V. Colizza, A. Flammini, M. A. Serrano and A. Vespignani, Nature Physics, 2, 110 (2006).

International Trade Network (ITN)

- There are N (≈ 210) countries in the world.
- Each country trades with some other countries.
  It exports its surplus products. Imports what it lacks.
- An ITN is constructed on the basis of trade relations among different countries of the world in a specific calendar year.

- In ITN, each country is a node, labeled as $i = 1,N$.
- A link exists between nodes i and j iff there is a non-zero trade between them in a particular year.
- Different year has different ITN.
- The weight w_{ij} of the link is the total volume of trade in M$ between i and j.

International trade data

Kristian Skrede Gleditsch

- Source: http://privatewww.essex.ac.uk/~ksg/exptradegdp.html
- Version 4.1, of Aug. 2005.

Format of the data file, first few lines of a file of 528104 lines

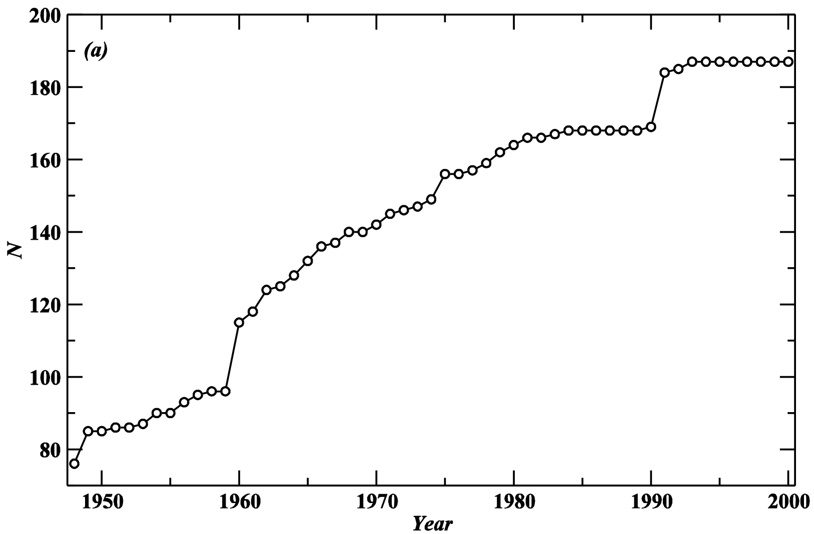
1	2	3	4	5	6	7
USA(2)	CAN(20)	1948	1794	1568	1853	1853
USA(2)	CAN(20)	1949	1743	1475	1910	1910
USA(2)	CAN(20)	1950	2101	1877	1966	1966

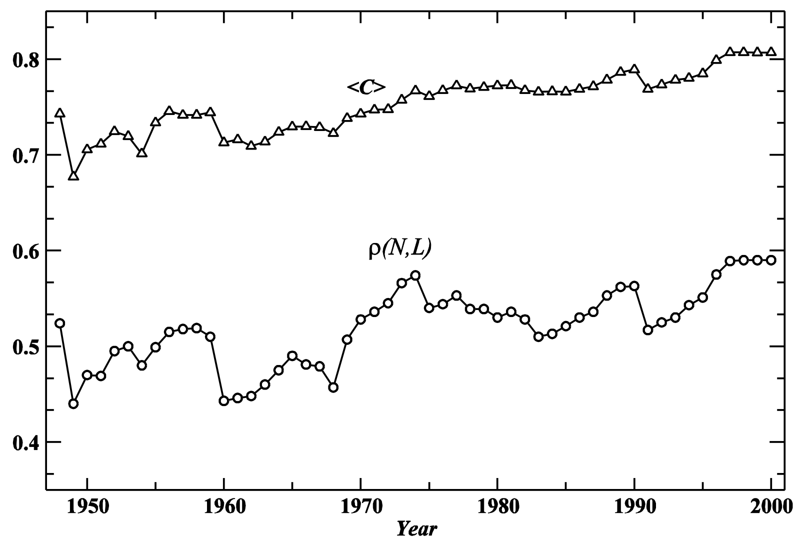
Col 4	exp_{ij} = export from i to j in M$
Col 5	imp_{ij} = import of i from j in M$
Col 6	exp_{ji} = export from j to i in M$
Col 7	imp_{ji} = import of j from i in M$

K. S. Gleditsch: *"Exports from i to j and j's imports from i are reported as different flows in the IMF DOT data. Although these in general will be similar they can differ in many instances since countries have different reporting procedures and duties etc. that give rise to different values."*

General features of the data

- Data available for 53 years, 1948-2000.
- The ITN grows with time, both in size (N) and no. of links (L). N=76 and L= 1494 in 1948 to N=187 and 10252 in 2000.
- Average Link density is around 0.52.





- Distribution of the volumes of trade is highly heterogeneous.
- A large number of economically backward countries trade to few other countries.
- Few rich countries trade to many other countries in the world. They form the large hubs of the network.
- ≈ 8% of the top rich countries trade among themselves 50% of the total world trade

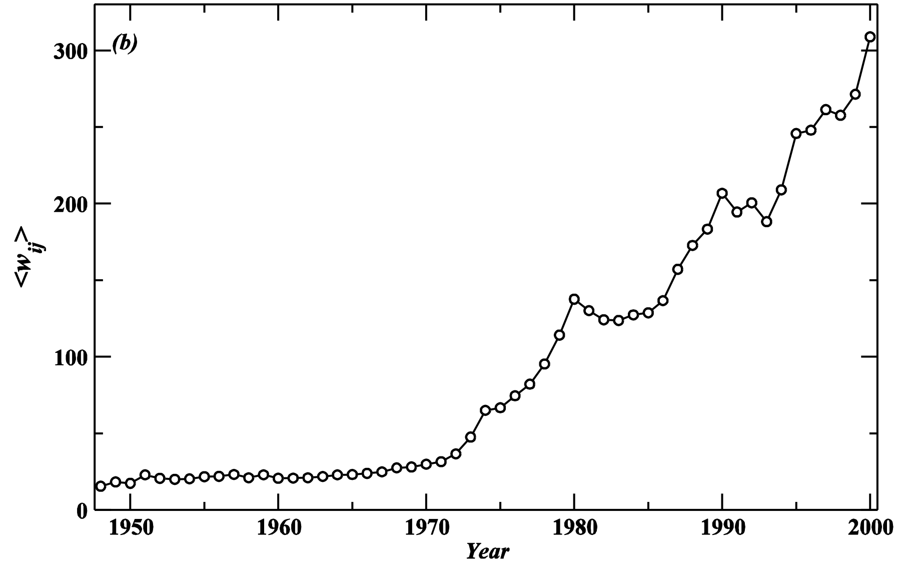
Questions

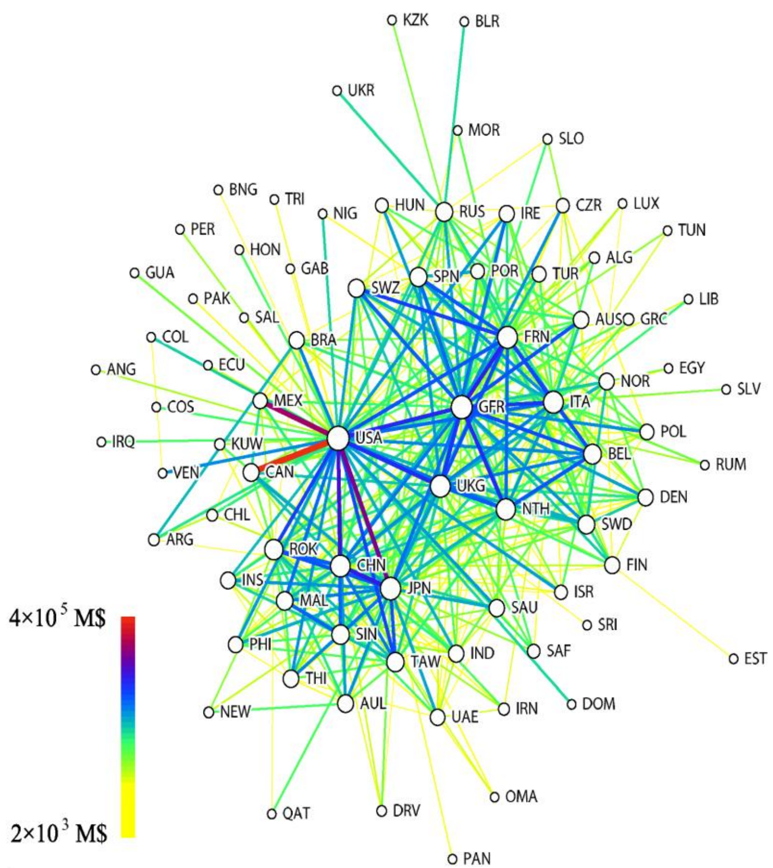
- What is the distribution of link weights?
- What is the nodal Strength vs. GDP relation?
- What is the Rich-Club effect in this problem?

Weight of a link

- Total volume of trade, i.e., export + import between the countries i and j in a year in M$.

$$w_{ij}^{exp} = \frac{1}{2}(\exp_{ij} + \text{imp}_{ji});\ w_{ij}^{imp} = \frac{1}{2}(\exp_{ji} + \text{imp}_{ij});\ w_{ij} = w_{ij}^{exp} + w_{ij}^{imp}$$





- The av. weight of a link has grown from 15.5 M$ in 1948 to 308.8 M$ in 2000.
- A part of the ITN for the year 2000 with top 4 percent of the largest weight links having 80 nodes and 411 links.
- Node size is proportional to its strength and link color is proportional to its weight.

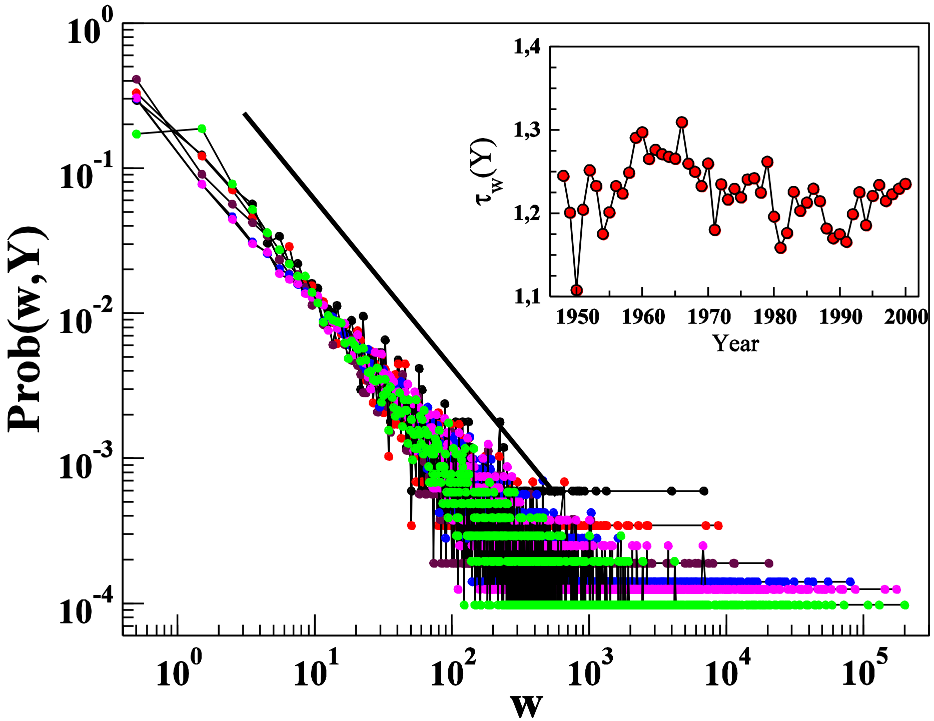
Link weight distribution: Prob(w,Y)

$$\text{Prob}(w,Y) \sim w^{-\tau_w}$$

$\tau_w = 1.22 \pm 0.15.$

- The probability distribution of the link weights in the ITN.
- Six plots for 1950, 1960, 1970, 1980, 1990 and 2000.
- Inset shows the variation of the slope over 53 years.

  Slope variation is robust (within ± 0.15) over 53 years period.



Log-normal distribution

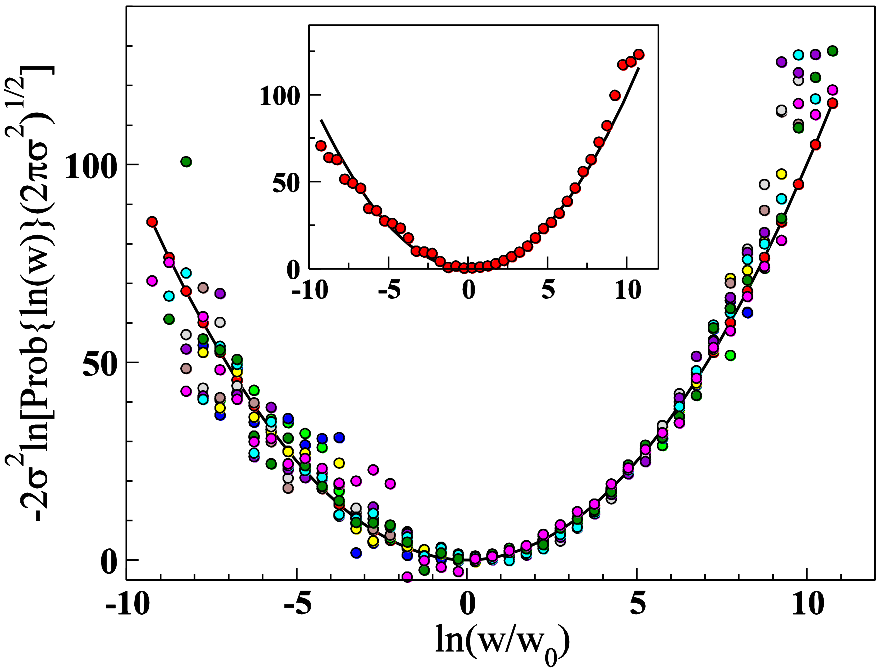
$$\text{Prob}(w) = \frac{1}{\sqrt{2\pi\sigma^2}}\frac{1}{w}\exp\left\{-\frac{\ln^2(w/w_0)}{2\sigma^2}\right\}$$

Plot of $-2\sigma^2 \ln\left\{\text{Prob}\{\ln(w)\}\sqrt{2\pi\sigma^2}\right\}$ Vs. $\ln(w/w_0)$

gives a simple parabola $y = x^2$

$$\sigma = \{\langle(\ln(w))^2\rangle - \langle\ln(w)\rangle^2\}^{1/2}$$

$$w_0 = \exp(\langle\ln(w)\rangle)$$



- ❑ Weight distribution data for ITN has been plotted for five years' periods: 1951-55, 1956-60, 1961-65, …., 1996-2000.
- ❑Ten colored symbols are used.
- ❑The inset shows average over 50 years 1951-2000.

Strength of a node

- Strength S_i of a node is the total mutual trade of a country with all other countries.

$$S_i = \sum_j w_{ij}$$

- How the strength of a node depends on its Gross Domestic Product (GDP)?
- How a small change in GDP is related to a corresponding change in strength?
- Define an elastic constant:

$$\beta = (dS_i / S_i)/(dG_i / G_i)$$

$$S_i(Y) \propto G_i^{\beta}(Y)$$

Our estimate: $\beta = 1.26$.

Variation of nodal strength with GDP

Countries in the Income aggregates, 2003, (Human development reports in http://hdr.undp.org/) data of 22 countries, well mixed in economic strengths, like:

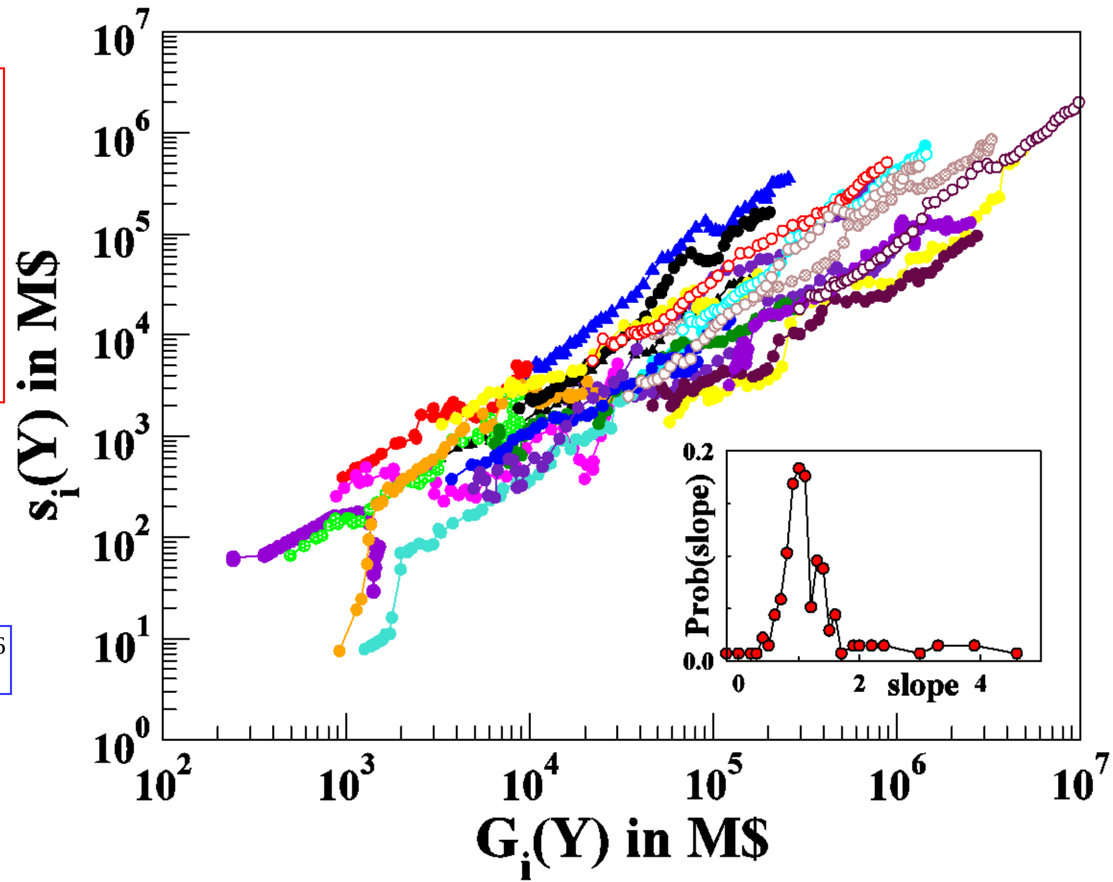
- 10 High income (GNI/capita ≥ $9,386) countries:
  USA, UK, Italy, France, Canada, Denmark, Australia, Switzerland, South Korea, Belgium.
- 5 Middle income ($766 ≤ GNI/capita < $9,386) countries: China, Brazil, Jamaica, Peru, Chile.
- 7 low income (GNI/capita < $ 766) countries:
  India, Pakistan, Bhutan, Jamaica, Myanmar, Nepal, Nicaragua and North Korea.

Inset shows distribution of slopes for 168 countries. Average slope ≈ 1.26.

Long tail has 12 countries from the fragmented Soviet Union, Yugoslavia And Czechoslovakia. Excluding them The average ≈ 1.06. Peak is very close to 1.

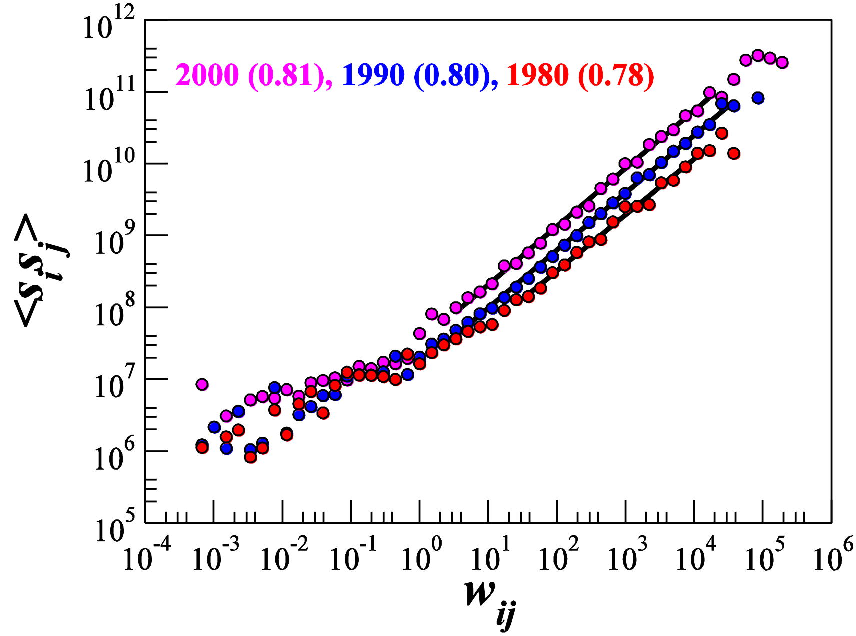
Earlier D. A. Irwin has claimed that:
(World Trade Review, 1, 89, 2002)

$$\textit{Total World Trade} \propto (\textit{Total World GDP})^{1.16}$$



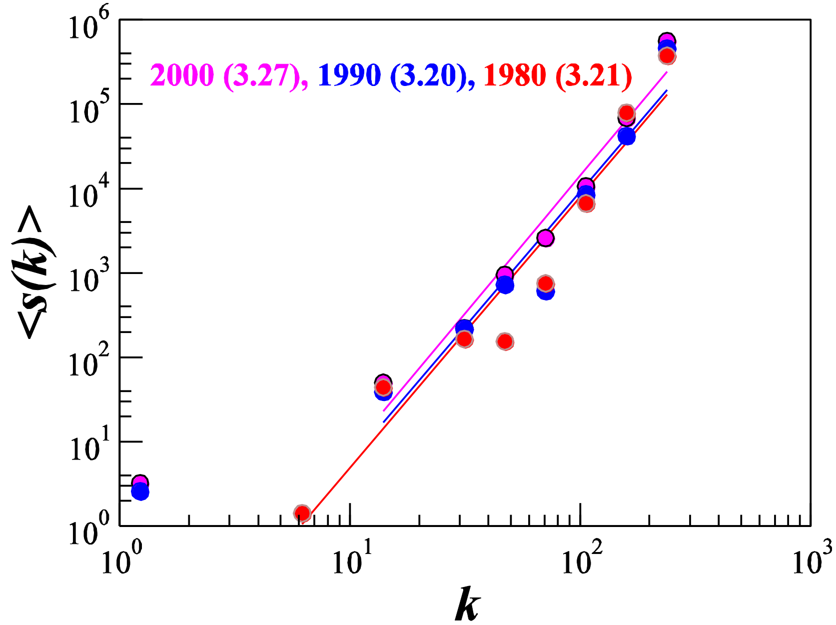
Strength correlation between neighboring nodes

$$\langle s_i s_j \rangle \sim w_{ij}^{\nu}$$



v = 0.81in 2000, 0.80 in 1990 and 0.78 in 1978.

$$\langle s(k) \rangle \sim k^{\mu}$$



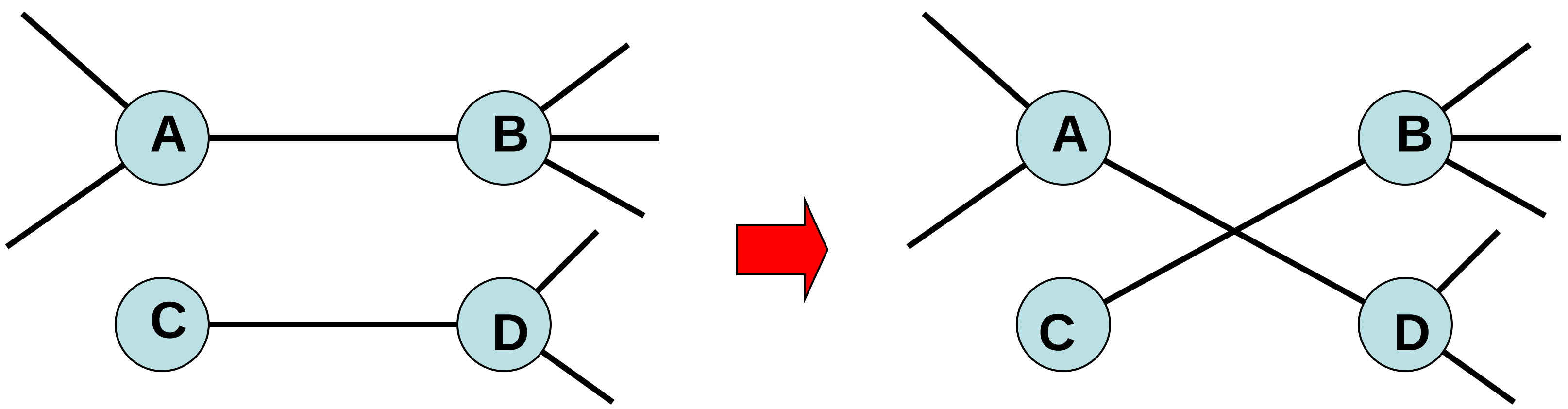
μ ≈ 3.27 during 1996 - 2000
≈ 3.20 during 1986 - 1990
≈ 3.21 during 1976 - 1980

Rich-Club Co-efficient (Un-weighted Network)

Rich-Club: A sub-set of n_k nodes whose degrees are at least k. Rich Club Co-eff.:
Number of links that actually exist in the club / Maximum number of possible links in the club.

$$\phi(k) = 2E_k / [n_k (n_k - 1)]$$

- A high value of $\varphi(k)$ implies that members are indeed tightly connected.
- But even uncorrelated graphs show Rich Club effect. (Colizza et. al.)
- So a Maximally Random Network (MRN) is constructed keeping $\{k_i\}$ same and $\varphi_{ran}(k)$ is studied on that.
- However the value of $\rho(k) = \varphi(k) / \varphi_{ran}(k)$ is found to be nearly unity.



Randomization of links keeping $\{k_i\}$ same

Rich-Club Co-efficient (Weighted Network)

A club of n_s nodes whose strengths are at least s.

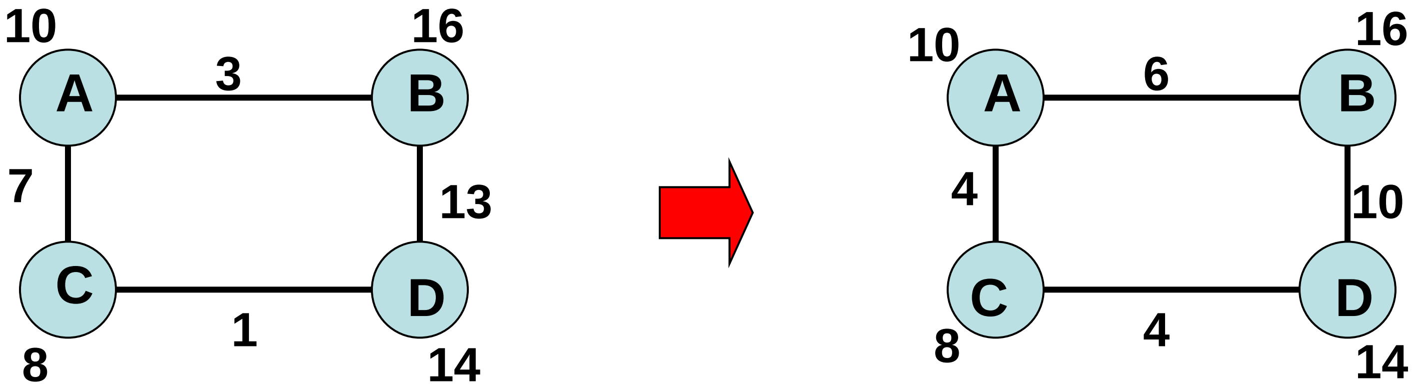
$R_w(s)$ = the total weight among n_s club members / the max.no. of possible links in the club.

$$R_w(s) = 2\sum_{i,j} w_{ij} / [n_s(n_s - 1)]$$

- Nodes are deleted one by one in the sequence of increasing s.
- Low strength countries contribute negligibly to IT and $R_w(s)$ remains un-affected.
- As s increases RC shrinks and the co-eff. grows fast like a power law: $R_w(s) \sim s^{0.85}$.

Randomization of a Weighted Network

- Generation of the Maximally Random Weighted Network (MRWN) from MRN of ITN.
- Given a MRN with a specific $\{k_i\}$ one obtains a weight distribution consistent with $\{s_i\}$



Randomization of link weights keeping $\{k_i\}$ and $\{s_i\}$ same

Self-consistent iteration procedure

- Consider a network whose $\{k_i\}$ and $\{s_i\}$ are fixed.
- We have to randomize $\{w_{ij}\}$ maintaining these two constraints.
- Start assigning random values of $w_{ij.}$ (symmetric).
- For node i calculate

$$\delta_i = s_i - \Sigma_j w_{ij}$$

Weights of all k_i links are then updated as:

$$w_{ij} \rightarrow w_{ij} + \delta_i (w_{ij} / \Sigma_j w_{ij})$$

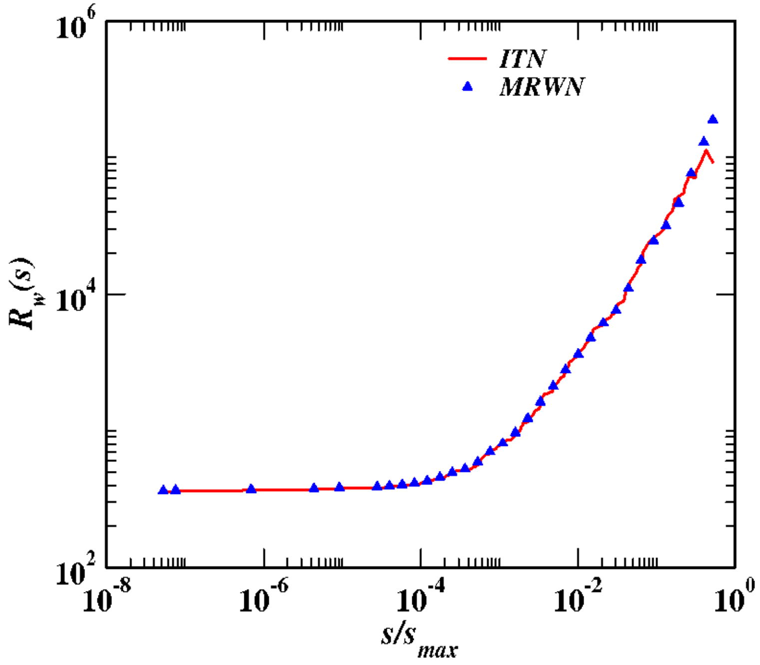
Here j runs over the neighbors of i.

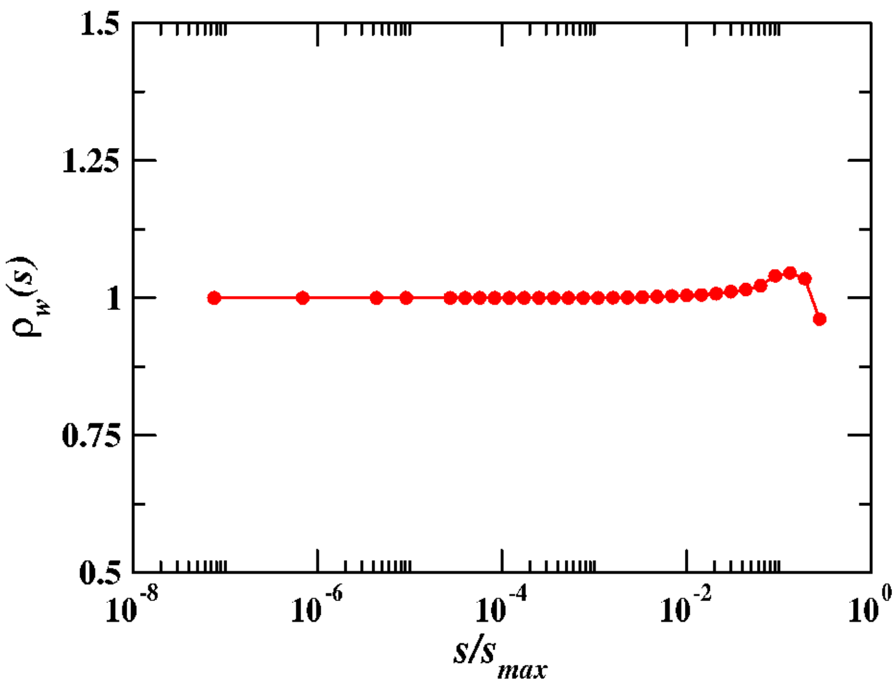
- Find out those nodes for which δ not equal to zero.
- For all such nodes execute this updating process.
- By repeated iterations the link weights quickly converge and become consistent with $\{s_i\}$

IMP.

Check: $\langle s_i s_j \rangle \sim w_{ij}$ relation is well satisfied with MRWN

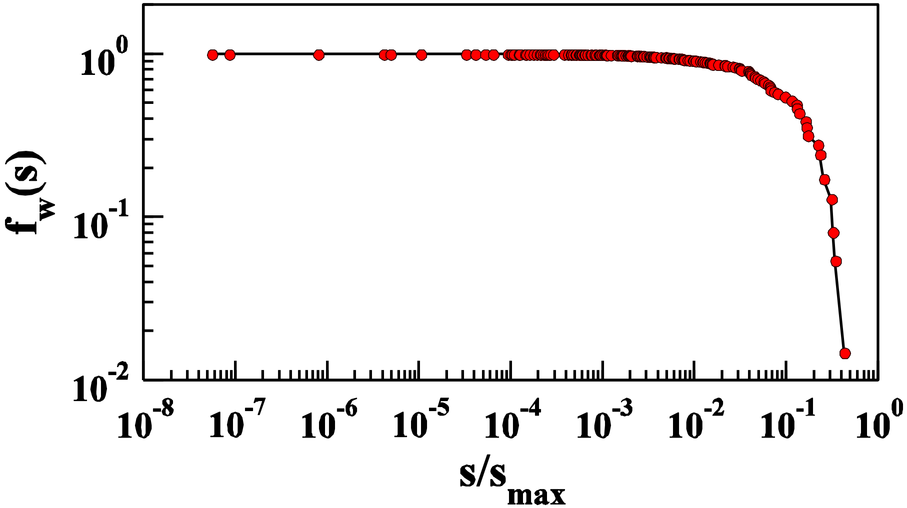
$$\rho_w(s) = R_w^{ITN}(s) / R_w^{MRWN}(s)$$



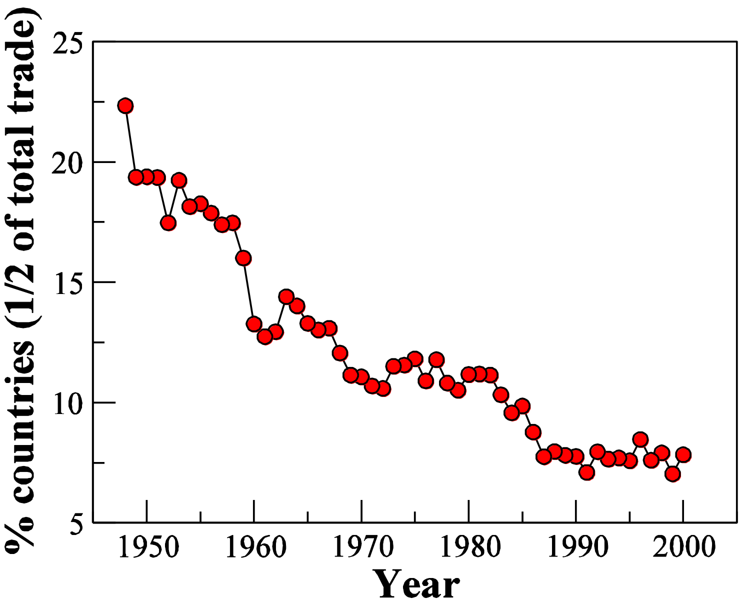


- Only 15% elements of the adjacency matrices of ITN and MRN are different.
- A node retains the links to most of its neighbors even after maximal randomization.
- For the year 2000, link density is ~ 0.59%.
- As a result ρ(k) and $\rho_w(s)$ are nearly equal to 1.

But Rich-Club exists



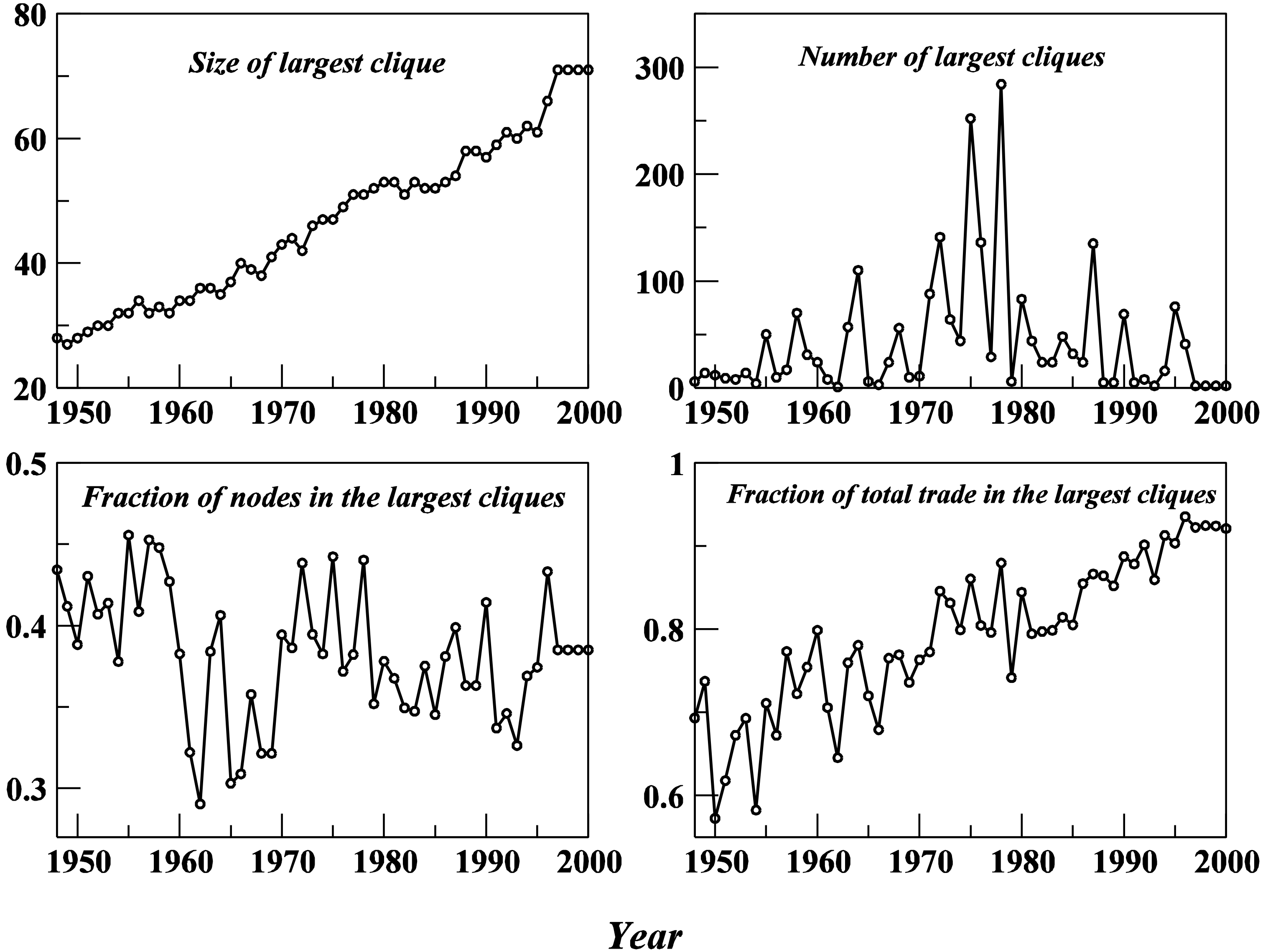
$f_w(s)$ = the fraction of the total volume trade in the rich-club. Remains close to unity until high value of s/s_{max}, decreases to ½ at s/s_{max} ~0.11 for the year 2000.



The size of the RC that trade 50% of the total world trade among themselves shrinks from 22% in 1948 to 8% in 2000.

Community: Largest Clique in ITN

Finding out the largest clique in ITN (i) how its size varies with year (ii) degeneracy (iii) fraction of nodes in the Largest clique (iv) fraction of total trade in this community with year



Gravity Model

Gravity model by J. Timbergen, 1962:

$$F_{i\leftrightarrow j} = G\frac{p_i p_j}{l_{ij}^2}$$

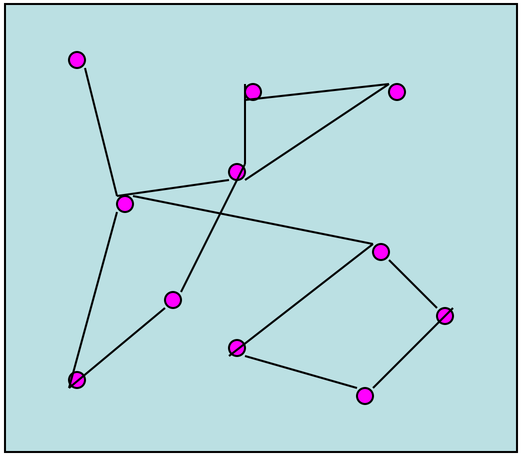
where F_{ij} is the passenger flow between two cities *i* an *j* depending on their populations p_i , p_j and distance l_{ij}. Later this equation had been modified to the parametric form:

$$F_{i\to j} = p_i^{\ \alpha}\left(\frac{p_j^{\beta}}{l_{ij}^{\theta}} / \sum_{k\neq i}\frac{p_k^{\beta}}{l_{ik}^{\theta}}\right)$$

where α, β and θ are the three parameters. $F'_{ij}=F_{ij}+F_{ji}$ denotes the net flow.

Model of ITN using the Gravity Model

- Unit square is the whole world!
- N random points – countries
- Initially GDPs m_i are also random, such that total GDP is unity.



Dynamics is a series of pair-wise transactions

A pair of countries (i,j) is randomly selected (within 1 to N)

- Time t is the number of transactions
- In a transaction the selected countries invest the amounts F_{ij} and F_{ji}. The total investment $F'_{ij}=F_{ij}+F_{ji}$ is randomly shared.

$$m_i(t+1) = m_i(t) - F_{ij} + \varepsilon F'_{ij} + \Delta_i$$

$$m_j(t+1) = m_j(t) - F_{ji} + (1-\varepsilon) F'_{ij} + \Delta_j$$

ε is a random fraction freshly drawn for every transaction.

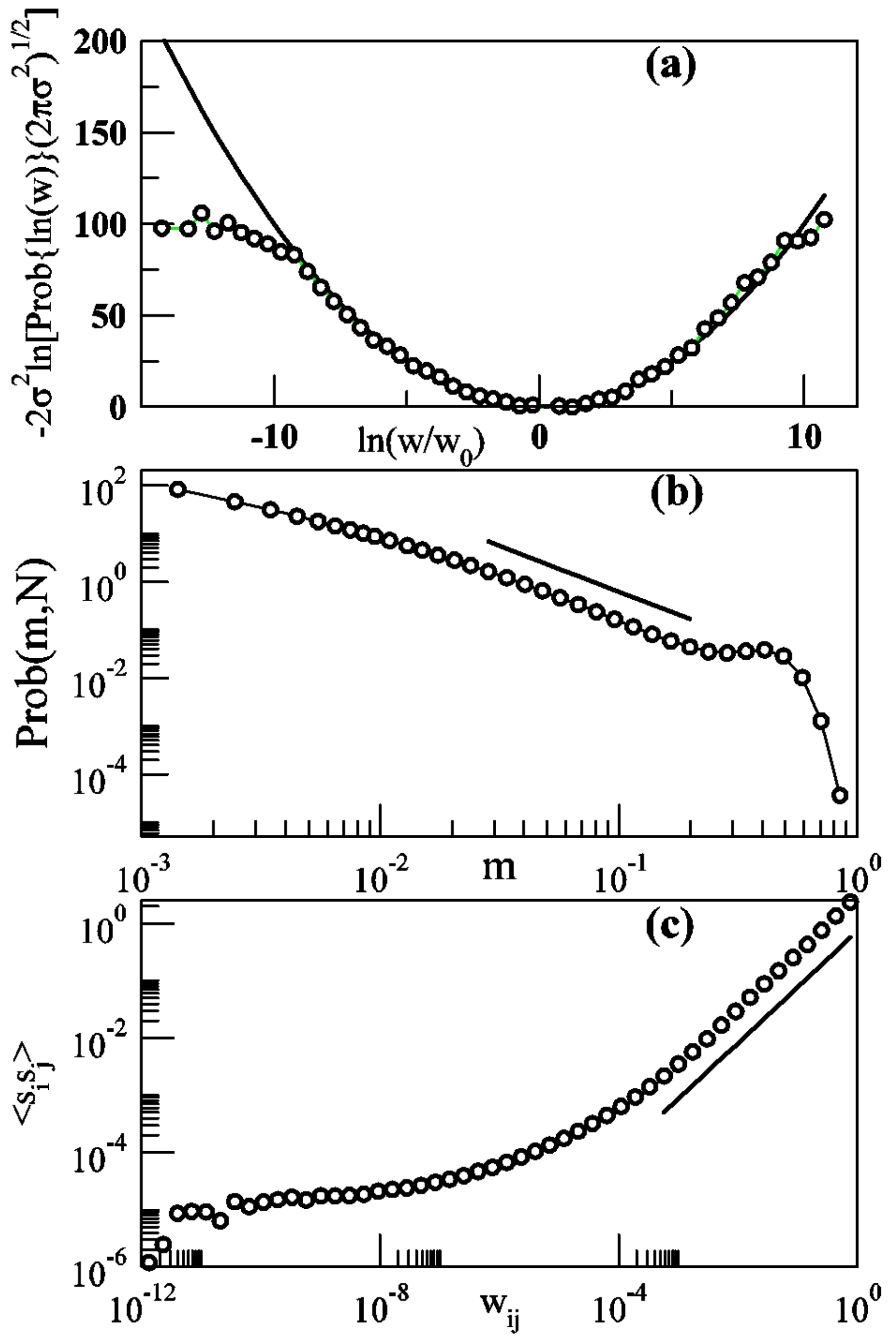
No debt is allowed and $\Delta_i=0$ if $F_{ij} < m_i$.
If $F_{ij} > m_i$ we add $\Delta_i=F_{ij}-m_i$ and similarly for Δ_j makes the model **non-conservative**.

After every transaction m_i is replaced by $m_i/\Sigma m_j$

After reaching the stationary state the **ITN** is constructed such that links are established between countries i and j whenever there is a transaction.

For α=1/2, β=1 and θ=1/2 (tolerance 0.2) the results are:

1. The parabola $y=x^2$ is observed for the scaled weight distribution.
2. The GDP distribution closely fits to a power law with exponent -2 as in the Pareto law.
3. The $\langle s_i s_j \rangle \sim w_{ij}^{\nu}$ shows ≈ 0.98 in the large weight limit.



Summary

- Statistical analysis of the data of the International trade volumes over 53 years have given indications of the following features:

- The weight, i.e., the annual volume of mutual trade between an arbitrary selected pair of countries follow a log-normal distribution. Scaled plots for a period of 53 years collapse well.
- The strength, or the total volume of annual trade made by a country grows non-linearly with its GDP, with an exponent ≈ 1.2.
- A large fraction of the IT is controlled by a club of few rich countries whose size gradually shrinks as time goes on.

- Crucial features of the real-world ITN have been reproduced using a non-conservative dynamical model starting from the well known Gravity Law in economics and social sciences.

Thank You