

Contents

Preface	XI
1 Introduction	1
1.1 A Brief History of Economics from the Physicist's Perspective	5
1.2 Outline of the Book	10
2 The Random Walk	13
2.1 What is a Random Walk?	13
2.1.1 Definition of Random Walk	13
2.1.2 The Random Walk Formalism and Derivation of the Gaussian Distribution	17
2.1.3 The Gaussian or Normal Distribution	21
2.1.4 Wiener Process	23
2.1.5 Langevin Equation and Brownian Motion	24
2.2 Do Markets Follow a Random Walk?	27
2.2.1 What if the Time-Series Were Similar to a Random Walk?	28
2.2.2 What are the "Stylized" Facts?	31
2.2.3 Short Note on Multiplicative Stochastic Processes ARCH/GARCH	33
2.2.4 Is the Market Efficient?	34
2.3 Are there any Long-Time Correlations?	36
2.3.1 Detrended Fluctuation Analysis (DFA)	36
2.3.2 Power Spectral Density Analysis	37
2.3.3 DFA and PSD Analyses Of the Autocorrelation Function Of Absolute Returns	38
3 Beyond the Simple Random Walk	41
3.1 Deviations from Brownian Motion	43
3.2 Multifractal Random Walk	46
3.3 Rescaled Range (R/S) Analysis and the Hurst Exponent	47
3.4 Is there Long-Range Memory in the Market?	48
3.4.1 Mandelbrot and the Joseph Effect	49
3.4.2 Cycles in Economics	49
3.4.3 Log-Normal Oscillations	50

4	Understanding Interactions through Cross-Correlations	53
4.1	The Return Cross-Correlation Matrix	54
4.1.1	Eigenvalue Spectrum of Correlation Matrix	55
4.1.2	Properties of the “Deviating” Eigenvalues	58
4.1.3	Filtering the Correlation Matrix	60
4.2	Time-Evolution of the Correlation Structure	62
4.3	Relating Correlation with Market Evolution	64
4.4	Eigenvalue Spacing Distributions	67
4.4.1	Unfolding of Eigenvalues for the Market Correlation Matrix	69
4.4.2	Distribution of Eigenvalue Spacings	69
4.4.3	Distribution of Next Nearest Spacings between Eigenvalues	70
4.4.4	The Number Variance Statistic	70
4.5	Visualizing the Network Obtained from Cross-Correlations	72
4.6	Application to Portfolio Optimization	76
4.7	Model of Market Dynamics	77
4.8	So what did we Learn?	79
5	Why Care about a Power Law?	83
5.1	Power Laws in Finance	83
5.1.1	The Return Distribution	84
5.1.2	Stock Price Return Distribution	86
5.1.3	Market Index Return Distribution	92
5.1.3.1	TP Statistic	94
5.1.3.2	TE Statistic	95
5.1.3.3	Hill Estimation of Tail Exponent	97
5.1.3.4	Temporal Variations in the Return Distribution	98
5.2	Distribution of Trading Volume and Number of Trades	103
5.3	A Model for Reproducing the Power Law Tails of Returns and Activity	104
5.3.1	Reproducing the Inverse Cubic Law	110
6	The Log-Normal and Extreme-Value Distributions	115
6.1	The Log-Normal Distribution	115
6.2	The Law of Proportionate Effect	115
6.3	Extreme Value Distributions	119
6.3.1	Value at Risk	121
7	When a Single Distribution is not Enough?	125
7.1	Empirical Data on Income and Wealth Distribution	125
8	Explaining Complex Distributions with Simple Models	131
8.1	Kinetic Theory of Gases	131
8.1.1	Derivation of Maxwell–Boltzmann Distribution	131
8.1.2	Maxwell–Boltzmann Distribution in D Dimensions	135
8.2	The Asset Exchange Model	136
8.3	Gas-Like Models	137
8.3.1	Model with Uniform Savings	140
8.3.2	Model with Distributed Savings	142

9	But Individuals are not Gas Molecules ...	147
9.1	Agent-Based Models: Going beyond the Simple Statistical Mechanics of Colliding Particles	147
9.2	Explaining the Hidden Hand of Economy: Self-Organization in a Collection of Interacting “Selfish” Agents	149
9.2.1	Hidden Hand of Economy	149
9.2.2	A Minimal Model	150
9.2.2.1	Unlimited Money Supply and Limited Supply of Commodity	151
9.2.2.2	Limited Money Supply and Limited Supply Of Commodity	153
9.3	Game Theory Models	154
9.3.1	Minority Game and its Variants (Evolutionary, Adaptive and so on)	159
9.3.1.1	El Farol Bar Problem	159
9.3.1.2	Basic Minority Game	161
9.3.1.3	Evolutionary Minority Games	161
9.3.1.4	Adaptive Minority Games	164
9.4	The Kolkata Paise Restaurant Problem	168
9.4.1	One-Shot KPR Game	169
9.4.2	Simple Stochastic Strategies and Utilization Statistics	172
9.4.2.1	No Learning (NL) Strategy	173
9.4.2.2	Limited Learning (LL) Strategy	173
9.4.2.3	One Period Repetition (OPR) Strategy	175
9.4.2.4	Follow the Crowd (FC) Strategy	176
9.4.3	Limited Queue Length and Modified KPR Problem	176
9.4.4	Some Uniform Learning Strategy Limits	178
9.4.4.1	Numerical Analysis	179
9.4.4.2	Analytical Results	180
9.4.5	Statistics of the KPR Problem: A Summary	181
9.5	Agent-Based Models for Explaining the Power Law for Price Fluctuations, and so on	184
9.5.1	Herding Model: Cont–Bouchaud	184
9.5.2	Strategy Groups Model: Lux–Marchesi	187
9.6	Spin-Based Model of Agent Interaction	190
9.6.1	Random Network of Agents and the Mean Field Model	194
9.6.2	Agents on a Spatial Lattice	195
10	... and Individuals don’t Interact Randomly: Complex Networks	203
10.1	What are Networks?	204
10.2	Fundamental Network Concepts	206
10.2.1	Measures for Complex Networks	207
10.3	Models of Complex Networks	210
10.3.1	Erdős–Rényi Random Network	210
10.3.2	Watts–Strogatz Small-World Network	212
10.3.3	Modular Small-World Network	213
10.3.4	Barabasi–Albert Scale-Free Network	216
10.4	The World Trade Web	220

- 10.5 The Product Space of World Economy 230
- 10.6 Hierarchical Network within an Organization:
Connection to Power-Law Income Distribution 234
- 10.6.1 Income as Flow along Hierarchical Structure: The Tribute Model 236
- 10.7 The Dynamical Stability of Economic Networks 237

11 Outlook and Concluding Thoughts 245

- 11.1 The Promise and Perils of Economic Growth 246
- 11.2 Jay Forrester's World Model 247

Appendix A Thermodynamics and Free Particle Statistics 251

- A.1 A Brief Introduction to Thermodynamics
and Statistical Mechanics 251
- A.1.1 Preliminary Concepts of Thermodynamics 251
- A.1.2 Laws of Thermodynamics 253
- A.2 Free Particle Statistics 256
- A.2.1 Classical Ideal Gas:
Maxwell–Boltzmann Distribution and Equation of State 257
- A.2.1.1 Ideal Gas: Equation of State 258
- A.2.2 Quantum Ideal Gas 260
- A.2.2.1 Bose Gas: Bose–Einstein (BE) Distribution 261
- A.2.2.2 Fermi Gas: Fermi–Dirac Distribution 263

Appendix B Interacting Systems: Mean Field Models, Fluctuations and Scaling Theories 265

- B.1 Interacting Systems: Magnetism 265
- B.1.1 Heisenberg and Ising Models 265
- B.1.2 Mean Field Approximation (MFA) 266
- B.1.2.1 Critical Exponents in MFA 269
- B.1.2.2 Free Energy in MFA 272
- B.1.3 Landau Theory of Phase Transition 273
- B.1.4 When is MFA Exact? 275
- B.1.5 Transverse Ising Model (TIM) 276
- B.1.5.1 MFA for TIM 278
- B.1.5.2 Dynamical Mode-Softening Picture 280
- B.2 Quantum Systems with Interactions 281
- B.2.1 Superfluidity and Superconductivity 281
- B.2.2 MFA: BCS Theory of Superconductivity 282
- B.3 Effect of Fluctuations: Peierls' Argument 286
- B.3.1 For Discrete Excitations 286
- B.3.2 For Continuous Excitations 289
- B.4 Effect of Disorder 290
- B.4.1 Annealed Disorder: Fisher Renormalization 290
- B.4.2 Quenched Disorder: Harris Criterion 291
- B.5 Flory Theory for Self-Avoiding Walk (SAW) Statistics 292
- B.5.1 Random Walk Statistics 292

- B.5.2 SAW Statistics 292
- B.6 Percolation Theory 293
- B.6.1 Critical Exponents 295
- B.6.2 Scaling Theory 296
- B.7 Fractals 297

Appendix C Renormalization Group Technique 301

- C.1 Renormalization Group Technique 301
- C.1.1 Widom Scaling 301
- C.1.2 Formalism 303
- C.1.3 RG for One-Dimension Ising Model 305
- C.1.4 Momentum Space RG for $4 - \epsilon$ Dimensional Ising Model 307
- C.1.5 Real Space RG for Transverse Field Ising Chain 316
- C.1.6 RG Method for Percolation 319
- C.1.6.1 Site Percolation in One Dimension 319
- C.1.6.2 Site Percolation in Two Dimension Triangular Lattice 321
- C.1.6.3 Bond Percolation in Two Dimension Square Lattice 322

Appendix D Spin Glasses and Optimization Problems: Annealing 325

- D.1 Spin Glasses 325
- D.1.1 Models 325
- D.1.2 Critical Behavior 326
- D.1.3 Replica Symmetric Solution of the S-K Model 327
- D.2 Optimization and Simulated Annealing 329
- D.2.1 Some Combinatorial Optimization Problems 330
- D.2.1.1 The Traveling Salesman Problem (TSP) 330
- D.2.2 Details of a few Optimization Techniques 333
- D.3 Modeling Neural Networks 336
- D.3.1 Hopfield Model of Associative Memory [20] 337

Appendix E Nonequilibrium Phenomena 339

- E.1 Nonequilibrium Phenomena 339
- E.1.1 Fluctuation Dissipation Theorem 339
- E.1.2 Fokker-Planck Equation and Condition of Detailed Balance 340
- E.1.3 Self-Organized Criticality (SOC) 340
- E.1.3.1 The BTW Model and Manna Model 341
- E.1.3.2 Subcritical Response: Precursors 342
- E.1.4 Dynamical Hysteresis 345
- E.1.5 Dynamical Transition in Fiber Bundle Models 346

Some Extensively Used Notations in Appendices 351

Index 353

