TRIPLET HIGGS BOSON AND MAJORANA NEUTRINOS

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ν OSCILLATION $\Rightarrow$ ν HAS MASS

BEST FIT

SOLAR OSC. : $m_2^2 - m_1^2 \simeq 7 \times 10^{-5}\, eV^2$

$\tan^2\theta_\odot \simeq 0.4$

ATMOSPHERIC OSC : $|m_3^2 - m_2^2| \simeq 2.5 \times 10^{-3}\, eV^2$

$\tan^2\theta_{Atm} \simeq 1$

QUESTIONS

WHAT IS THE SCALE OF m_ν

IS IT NORMAL, INVERTED OR DEGENERATE

IS THERE $\not{CP}$ IN THE LEPTON SECTOR

(WHAT IS THE SIZE OF θ_{13})

IS ν DIRAC OR MAJORANA

(IS $L_\#$ GOOD)

WHAT ABOUT LSND

(IS THERE A 4^{th} ν)

DIRAC ν

$$\mathcal{L} \ni -\bar{L}_\alpha \cdot H^\dagger \lambda^\nu_{\alpha\beta} \nu_{R\beta} + h.c. \qquad L = \begin{pmatrix} \nu_L \\ \ell \end{pmatrix}$$

$$\Rightarrow \mathcal{L}_{mass} \ni -\bar{\nu}_{L\alpha} m_{\alpha\beta} \nu_{R\beta} + h.c$$

$\alpha, \beta \to$ GEN INDEX

$\alpha, \beta \to (1, 2, 3)$

$$m_{\alpha\beta} \equiv \frac{v}{\sqrt{2}} \lambda^\nu_{\alpha\beta}$$

$A \cdot B \to$ ANTI-SYMM PRODUCT

$$\text{DIRAC } \nu_D \equiv \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

$L_\#$ GOOD, SIMILAR TO QUARK SECTOR

BUT (1) WHY $\lambda^\nu \sim 10^{-12}$?

(2) WHY NOT ADD MAJORANA MASS FOR ν_R?

S.M. + NON RENORMALIZABLE TERM

$$\mathcal{L}_{EFF} \ni -\frac{\lambda}{M} (\bar{L}^c \cdot H)(L \cdot H) \qquad L^c \equiv C L^*$$

$$\Rightarrow \mathcal{L}_{MASS} \ni -\frac{\lambda v^2}{2M} \overline{\nu_L^c} \nu_L$$

MAJORANA ν

$L_\#$ BROKEN

WHY m_ν SMALL ? → SEESAW

TYPE I SEESAW

ADD LARGE MAJ. MASS FOR ν_R

$$\mathcal{L} \ni -\overline{L}_\alpha \cdot H^\dagger \lambda^\nu_{\alpha\beta} \nu_{R\beta} - \overline{\nu^c_{R\alpha}} M_{\alpha\beta} \nu_{R\beta} + h.c.$$

$$\Rightarrow \mathcal{L}_{MASS} \ni -\begin{pmatrix} \overline{\nu_L} & \overline{\nu^c_R} \end{pmatrix} \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu^c_L \\ \nu_R \end{pmatrix}$$

DIAGONALIZE

$$\nu_1 \qquad m_1 \simeq \frac{m^2}{M}$$

$$\nu_2 \qquad m_2 \simeq M$$

IF $M \sim 10^{14}$ GeV, $m_1 \sim 0.1$ eV

NATURAL EXPLANATION FOR LIGHT ν

TYPE II SEESAW

COUPLE ν_L TO ϕ

$$\mathcal{L} \ni -\lambda \phi \overline{\nu^c_L} \nu_L + h.c + V(\phi)$$

$$\frac{\partial V(\phi)}{\partial \phi} = 0 \Rightarrow \langle \phi \rangle \equiv u \sim 0.1\,eV$$

$$m_\nu = \lambda u \sim 0.1\,eV$$

NATURAL FOR APPROPRIATE $V(\phi)$

TRIPLET HIGGS

$SU(2)_L \times U(1)_Y$: $\quad \psi \to U \psi$

$SU(2)_L$: $\quad U = e^{-ig\alpha^a T^a} \qquad T^a \equiv \frac{\sigma^a}{2}$

$U(1)_Y$: $\quad U = e^{-ig'\alpha Y}$

$L \to UL \qquad H \to UL \qquad Y(L) = -\frac{1}{2} \quad Y(H) = \frac{1}{2}$

INTRODUCE $\xi \to$ COMPLEX SU(2) TRIPLET

$$\xi^a = \begin{pmatrix} \xi^{++} \\ \xi^{+} \\ \xi^{0} \end{pmatrix}$$ $\longrightarrow$ WRITE AS 2×2 MATRIX

$$\xi \equiv \xi^a T^a$$

$$\xi = \begin{pmatrix} \xi^+/2 & \xi^{++}/\sqrt{2} \\ \xi^0/\sqrt{2} & -\xi^+/2 \end{pmatrix}$$

$SU(2)_L$: $\quad \xi \to U \xi U^\dagger$

$U(1)_Y$: $\qquad Y(\xi) = +1$

GELMINI, RONCADELLI
SECHECHTER, VALLE

$\xi \leftrightarrow L$ COUPLING

($SU(2)_L \times U(1)_Y$, LORENTZ INVARIANT)

$$\mathcal{L} \ni -\sqrt{2}\, f_{\alpha\beta}\, L_\alpha^T \cdot \xi L_\beta + h.c$$

$$L_\#(\xi) = -2$$

EXPAND ABOVE $\quad \kappa_{\alpha\beta} \equiv \frac{f_{\alpha\beta} + f_{\beta\alpha}}{2}$

$$\mathcal{L} \ni \kappa_{\alpha\beta}\left(\sqrt{2}\,\xi^+ e^\alpha \nu^\beta + \xi^{++} e^\alpha e^\beta - \xi^0 \nu^\alpha \nu^\beta\right) + h.c.$$

ν MASS $\quad \langle \xi^0 \rangle = u \neq 0 \quad$ ($L_\#$ BROKEN)

$$\mathcal{L} \ni -\kappa_{\alpha\beta}\, u\, \nu^\alpha \nu^\beta + h.c.$$

IN 4-COMP: $-\kappa_{\alpha\beta} u\, \overline{\nu^{c}}^{\alpha} \nu^\beta + h.c.$

MAJORANA MASS $\quad m_{\alpha\beta} \equiv \kappa_{\alpha\beta} u$

NEED $u \sim 0.1\,\mathrm{eV}$

ASIDE

FLAVOR BASIS (ν_α) $\rightarrow$ MASS BASIS (ν_i)

$$\nu_\alpha = V_{\alpha i}^{MNS}\, \nu_i$$

$V^{MNS} \rightarrow$ 1 "DIRAC" PHASE
+
2 "MAJORANA" PHASES

IS THERE $\cancel{CP}$ IN LEPTON SECTOR ?

WHY $u \sim 0.1\,\mathrm{eV}$?

MA, SARKAR

$$\mathcal{L} \ni -\sqrt{2}\,\mu\, H^T \cdot \xi^\dagger H + h.c.$$

(EXPLICITLY BREAKS $\cancel{L}$)

FULL POTENTIAL $\quad \mathcal{L} \ni -V$

$$
\begin{aligned}
V(\phi) = \; & m^2 H^\dagger H + \tfrac{1}{2}\lambda_1 (H^\dagger H)^2 && \leftarrow \text{SM} \\
& + 2M^2\, \mathrm{Tr}[\xi^\dagger \xi] && \leftarrow \xi \text{ MASS} \\
& - \sqrt{2}\,\mu\, H^T \cdot \xi^\dagger H + h.c. && \leftarrow \cancel{L} \\
& + 2\lambda_2 \{\mathrm{Tr}[\xi^\dagger \xi]\}^2 \\
& + 2\lambda_3 (H^\dagger H)\, \mathrm{Tr}[\xi^\dagger \xi] && \leftarrow \text{QUARTIC TERMS} \\
& + 2\bar{\lambda}_3 (H^\dagger T^a H)\, \mathrm{Tr}[\xi^\dagger T^a \xi]
\end{aligned}
$$

$$\frac{\partial V(\phi)}{\partial \phi} = 0 \Rightarrow \quad u \simeq -\frac{\mu v^2}{M^2} \quad \text{FOR } \begin{array}{l} M \gg v \\ u \ll v \end{array}$$

$$v \simeq -\frac{m^2}{\lambda_1} - \frac{2\mu u}{\lambda_1}$$

(i) IF $M \gg v \Rightarrow u \ll v$ $\quad$ (TYPE II SEE SAW)

EG: FOR $\mu = M$, $u = -\dfrac{v^2}{M}$

(ii) IF $\mu \ll v$ $(M \simeq v) \Rightarrow u = -\mu \ll v$

t'HOOFT NATURAL

ASIDE

$$\phi^+ = \Phi^+ - s\,\Sigma^+$$
$$\xi^+ = s\,\Phi^+ + \Sigma^+$$

Φ^+ EATEN BY W_μ^+

$\Sigma^+ \rightarrow$ PHYSICAL

$$s \simeq -\sqrt{2}\,\mu \Big/ \left(1 - \frac{\mu v}{u}\right)\left(1 + \bar{\lambda}_3 \frac{u}{\mu}\right)$$

FEYNMAN RULES

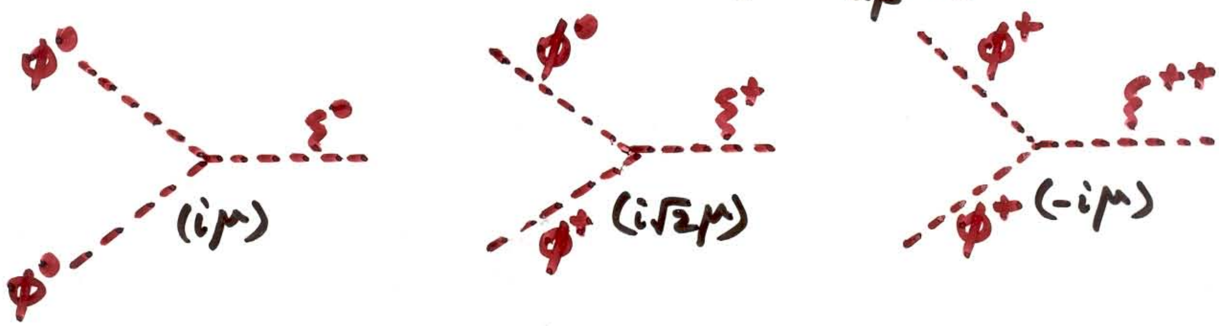
SM:

$\left(\frac{ig}{\sqrt{2}}\gamma^\mu P_L\right)$ $(-ie\gamma^\mu)$

TRIPLET ξ:

$(i\sqrt{2}K_{\alpha\beta}P_L)$ $(-iK_{\alpha\beta}P_L)$

$(iK_{\alpha\beta}P_L)$ $(-iM_{\alpha\beta}P_L)$



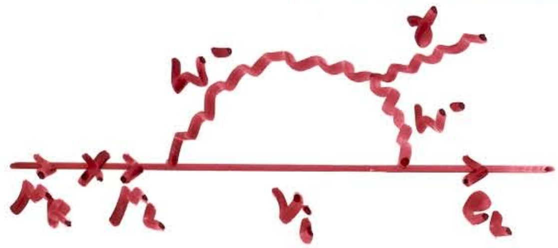
$\mu e \gamma$ FORM FACTOR

$$\mathcal{L} \ni \bar{\Psi}_e \left[q^2 \gamma^\alpha (A_1^L P_L + A_1^R P_R) + m_\mu i \sigma^{\alpha\beta} q_\beta (A_2^L P_L + A_2^R P_R) \right] \Psi_\mu A_\alpha$$

$\mu \to e \gamma$

EXPT LIMIT : $B.R.(\mu \to e\gamma) \lesssim 10^{-11}$ (PDG 04)

SM:



$$m_\mu A_2^R = e \frac{g^2}{16\pi^2} \frac{m_\mu}{m_W^2} \left(\frac{m_{\nu_i}^2 - m_{\nu_j}^2}{m_W^2} \right) \rightarrow \text{GIM SUPPR}$$

$$B.R.^{SM}(\mu \to e\gamma) = \frac{\Gamma_{\mu \to e\gamma}}{\Gamma_{TOTAL}} \sim \frac{\alpha_{em}}{(16\pi^2)^2} \left(\frac{m_{\nu_i}^2 - m_{\nu_j}^2}{m_W^2} \right)^2$$

$$\sim 10^{-50} \quad \text{TINY!}$$

TRIPLET ξ

$$m_\mu A_2^R = e \frac{\kappa_{\mu\alpha} \kappa_{e\alpha}^*}{16\pi^2} \frac{m_\mu}{M_{\xi^-}^2}$$

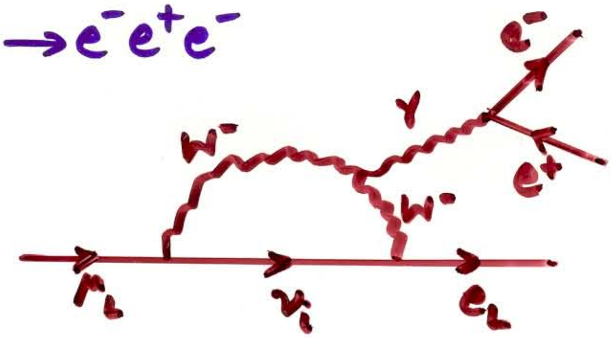
$$B.R.^{\xi}(\mu \to e\gamma) \sim \frac{\alpha_{em}}{(16\pi^2)^2} \frac{(\kappa_{\mu\alpha} \kappa_{e\alpha}^*)^2}{g^4} \left(\frac{M_W}{M_{\xi^-}} \right)^4$$

$$\sim 10^{-9} \left[(\kappa^\dagger \kappa)_{e\mu} \right]^2 \sim 10^{-13}$$

[KAKIZAKI, OGURA, SHIMA]

$\mu^- \to e^- e^+ e^-$

SM



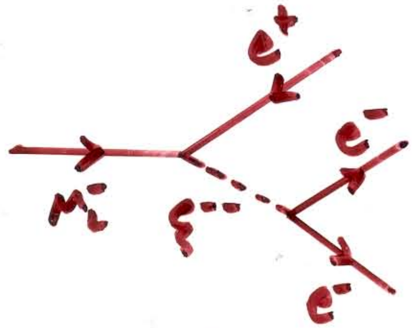
EXPT. LIMIT. $B.R(\mu \to eee) \lesssim 10^{-12}$ (PDG 04)

$$B.R.^{SM}(\mu \to eee) = \frac{3\alpha^2}{16\pi^2}\left(\frac{m_i^2}{M_W^2} \ln \frac{m_i^2}{M_W^2}\right)^2$$

[SEE BOOK BY MOHAPATRA, PAL]

$\sim 10^{-46}$ (TINY!)

TRIPLET ξ



$$A \sim \frac{\kappa_{e\mu}\kappa_{ee}^*}{M^2_{\xi^{--}}}$$

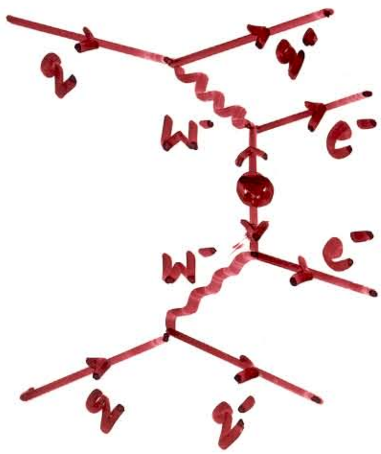
$$B.R.(\mu \to eee) \sim \frac{(\kappa_{e\mu}\kappa_{ee}^*)^2}{g^4}\left(\frac{M_W}{M_{\xi^-}}\right)^4$$

$$\sim (\kappa_{e\mu}\kappa_{ee}^*)^2\, 10^{-4} \sim 10^{-12}$$

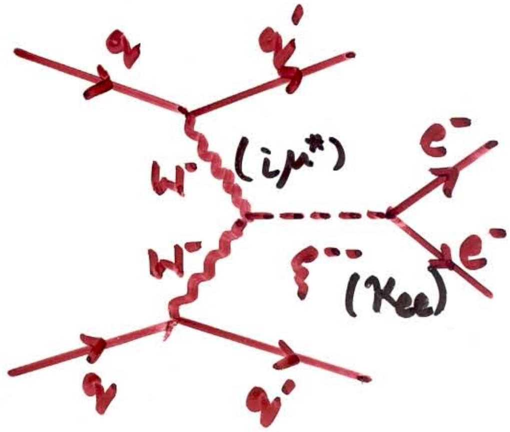
[KAKIZAKI ET AL.]

$0\nu\beta\beta$ ν-LESS DOUBLE β DECAY

SM



ξ TRIPLET



$$A \sim \frac{1}{M_W^4}\frac{m_{ee}}{q^2}$$

FOR $q^2 \sim (100\,\mathrm{MeV})^2$

EXPT: $m_{ee} < 0.35\,\mathrm{eV}$ (^{76}Ge)

SEE EG. [ATRE, BARGER, HAN]

$$A \sim \frac{\mu}{M^2}$$

COMPARE W/ SM

$$\kappa\frac{\mu}{M^2} \leftrightarrow 10^{-8}\,(\mathrm{GeV})^{-1}$$

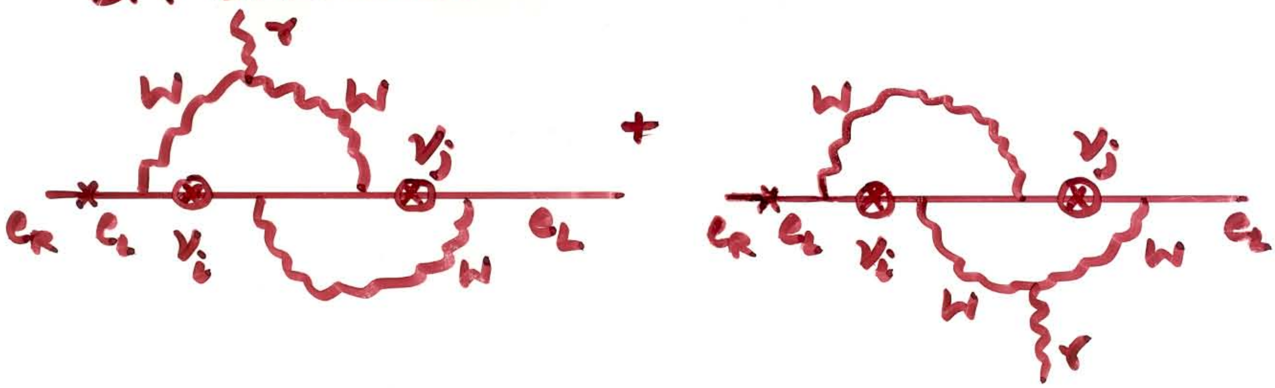
$$m_\nu \sim 0.1\,\mathrm{eV} \sim \kappa u = \kappa\left(\frac{\mu v^2}{M^2}\right) \Rightarrow \frac{\kappa\mu}{M^2} \sim \frac{m_\nu}{v^2}$$

$$\frac{m_\nu}{v^2} \sim 10^{-14}\,(\mathrm{GeV})^{-1}$$

$\therefore$ $0\nu\beta\beta$ UNOBSERVABLE FOR ξ^{--}

e^- EDM

SM EXTENSION



$$\mathcal{L} \ni e\bar{\Psi}\gamma^{\mu}\Psi A_{\mu} + \theta_f\,\bar{\Psi}\sigma^{\mu\nu}P_R\Psi F_{\mu\nu} + \text{h.c.}$$

EDM $d_e \equiv -2\,\mathrm{Im}(\theta_f)$

$$\propto \frac{g^4}{(16\pi^2)^2}\,\frac{m_e m_i m_j (m_i^2 - m_j^2)}{M_W^6} \sim 10^{-68} \text{ (TINY!)}$$

[IF 2 FLAVOR, NO DIRAC PHASE. SO $d_e \neq 0 \Rightarrow$ MAJ ν]

EXPT LIMIT: $d_e \lesssim 10^{-27}$ e-cm

NOTE: AT 3-LOOP QUARK $\gamma^{CKM} \Rightarrow d_e \sim 10^{-37}$ e-cm

[KHRIPLOVICH, POSPELOV]

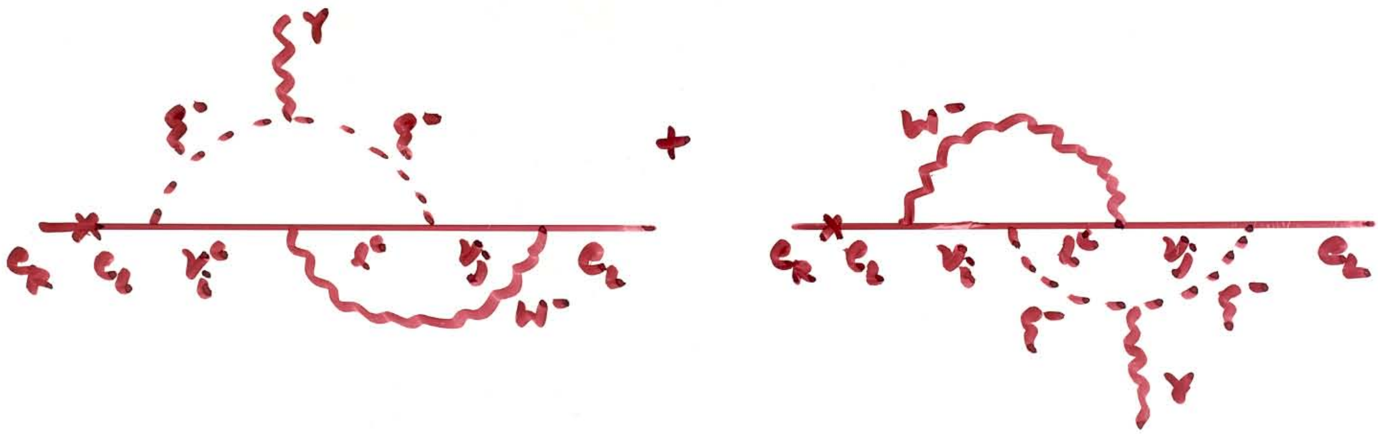
SEE EG. [BERNREUTHER, SUZUKI]

NOTE: W/ HEAVY ν_R (DIRAC ν)

$d_e \sim 10^{-46}$ e-cm

[ARCHAMBAULT, CZARNECKI, POSPELOV]

e^- EDM DUE TO TRIPLET ξ



NO ν MASS INSERTIONS

IF ν MASS IGNORED, SUM IS REAL

$$d_e \propto m_e \frac{g^2}{(16\pi^2)^2} \frac{(m_i^2 - m_j^2)}{M^2 M_W^2}$$

$\sim 10^{-50}$ e-cm FOR $M \sim 1$ TeV

10^{18} TIMES d_e^{SM} (LEPTONIC)

(BUT STILL TOO SMALL)

ARE THERE OTHER DIAGRAMS ?

CONCLUSIONS

ν DIRAC ? OR MAJORANA ?

WHY IS m_ν SO SMALL ?

TYPE I SEESAW: ADD ν_R DIRAC ν

$L_\#$ GOOD

TYPE II SEESAW: HERE W/ TRIPLET HIGGS

ADD $\xi \Rightarrow$ MAJORANA ν

$L_\#$ BROKEN

PROBES

$0\nu\beta\beta$

e^- EDM

$\mu \to e\gamma$

$\mu \to eee$

$\nu_\alpha \to \nu_\beta$ TRANSITION MOMENT