On Universal Features of Gravity

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1. Universality of gravity

2. Bianchi Derivative $\longrightarrow$ Grav. Dynamics

3. Λ - a constant of spacetime and Vacuum Energy

4. Higher Dimensions

5. Λ - Vacuum in higher dimensions
$\xrightarrow[r \to \infty]{\text{Always}}$ Einstein Vacuum with Λ

6. Gravity inside a uniform density sphere always goes as r in all dimensions

7. Discussion

Universal Force : Links to All including $m=0$

NSL : $m\vec{\ddot{u}} = \vec{F}$ ✗ Not Applicable

PE : $m_i \ddot{u} = -G \frac{m_g M}{r^2}$, $m_i = m_g$? ✗ For $m=0$!

$m=0$ experiences no acceleration $\therefore$ vel. = const. !

Yet it Must feel UF.

Solution: UF must curve spacetime and All, ($m \neq 0$ & $m=0$) fall freely — geodesics of curved spacetime

UF Dynamics $\xrightarrow{\text{Always}}$ Spacetime Curvature R_{abcd}

Bianchi Derivative

Bianchi identity: $\nabla_{[a} R_{bc]de} = 0 \xrightarrow{\text{Trace}} \nabla_b G_a{}^b = 0$

<u>Eqn</u>: $G_{ab} + \Lambda g_{ab} = \kappa T_{ab}$, $\nabla_a T^a_b = 0$

T_{ab}: Universal source for UF = Energy-momentum

UF is thus uniquely Einstein Gravity

Λ on the same footing as T_{ab} !

Two constants: κ = Field strength, Λ = ?

Absence of matter

$G_{ab} + \Lambda g_{ab} = 0 \implies$ Constant curvature: Λ

Maximally symmetric

Space : Homogeneous & Isotropic

Time : Homogeneous

Dynamically "Free" spacetime

Not necessarily Flat !

Homogeneity of Space & Time

$x \leftrightarrow y$ $\qquad\qquad$ $x \leftrightarrow t$?

Required universal 'c': $x \leftrightarrow ct$

What is geometry of spacetime

Constant curvature: Λ – a constant of spacetime!

c: Binds Space & Time into spacetime

Λ: Curves it

They become part of spacetime structure

Most fundamental & liberated constants!

Λ and Vacuum Energy

Matter: T_{ab} produces Vacuum Energy $\sim g_{ab}$!

VE exists as a secondary source like grav. field energy

- has <u>no independent existence</u>

- cannot be put on the same footing as matter T_{ab}

How was grav. field energy taken care of?

By curving the space and not writing T_{ab} on the ~~to~~ right.

Grav. field energy: GFE

$$ds^2 = A\,dt^2 - B\,dr^2 - r^2 d\Omega^2$$

$$\left.\begin{aligned} R^0_0 &\sim \nabla^2 A + ()\left(\frac{A'}{A} + \frac{B'}{B}\right) \\ R^1_1 &\sim R^0_0 + ()\left(\frac{A'}{A} + \frac{B'}{B}\right) \end{aligned}\right\}$$

$R^0_0 = R^1_1 \Rightarrow \frac{A'}{A} + \frac{B'}{B} = 0 \Rightarrow AB = 1$

$R^0_0 \sim \nabla^2 A = 0$: Laplace Eqn.

What happened to GFE? $\nabla^2 A \sim A'^2$!

This would be so if $B = 1$.

GFE goes into B – curving the space leaving $\nabla^2 A = 0$!

Similarly VE should be accounted for by enlarging the spacetime framework for QG.

Does VE not gravitate? (Weinberg, Paddy, Ellis et al)

Against the Principle of universality of Gravity.

$$(\rho+p)\dot{u}^a = p_{,b}\left(g^{ab} - u^a u^b\right) \sim m\dot{u}^a = F^a$$

For VE, $\underline{\rho+p=0}$ like $m=0$ in NSL.

Like GFE, VE must also gravitate but it would require tapping some new property of spacetime — what could that be?

- Higher dimensions
- Non-commutivity
- New Quantum relation — Self conjugate variable!

Higher Dimensions?

* For high energy effects — R^2_{abcd}

  yet Eqn to be second order Quasilinear

$\Longrightarrow$ Lovelock/Gauss-Bonnet

$\Downarrow$

Makes non-zero contribution only in

$\underline{D > 4}$

Hence higher D.

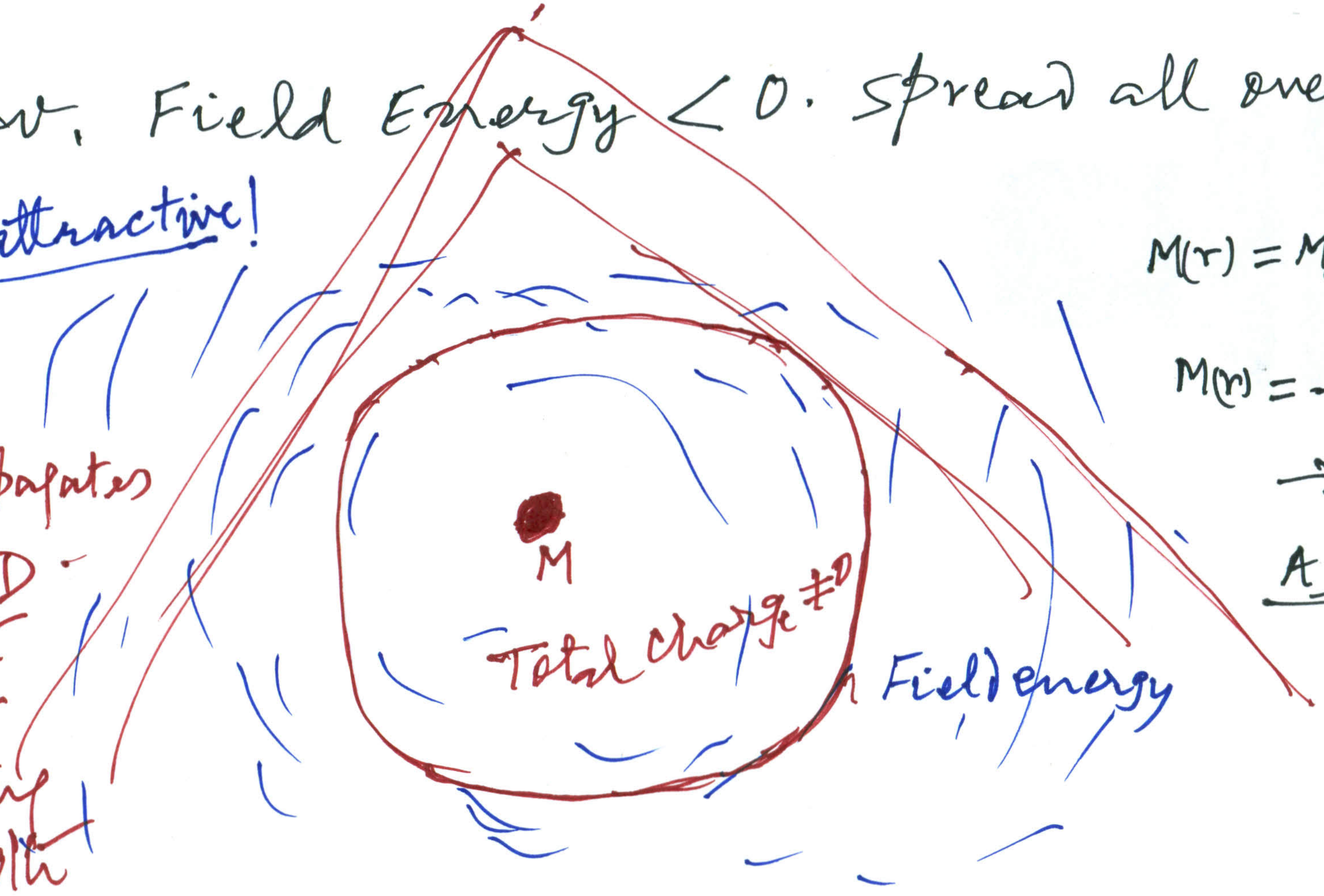
Total Charge = 0

For Gravity, Charge : T_{ab} Energy/Momentum > 0

How to make it zero ?

Grav. Field Energy < 0. spread all over space!

Always attractive!



Field propagates in extra D. but not freely + diminishing strength

$$M(r) = M_\infty - \frac{M(r)^2}{2r}$$

$$M(r) = -r + \sqrt{r^2 + 2M_\infty r}$$

$\rightarrow 0$ as $r \rightarrow 0$!

ADM

Analogue of R_{abcd} in higher orders

$$F_{abcd} = \mathbb{R}_{abcd} - \frac{n-1}{n(D-1)(D-2)} \mathbb{R}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$$

n : order

D : Dimension

$$\mathbb{R}_{abcd} = Q_{ab}{}^{mn} R_{mncd}$$

$$Q_{abcd} = \delta^{aba_1b_1 \ldots a_nb_n}_{cdc_1d_1 \ldots c_nd_n} R_{a_1b_1}{}^{c_1d_1} \ldots R_{a_nb_n}{}^{c_nd_n}$$

$$Q_{abcd}{}^{;d} = 0$$

Quadratic

$$Q_{abcd} = R_{abcd} - 2R_{a[c}g_{d]b} + 2R_{b[c}g_{d]a} + Rg_{a[c}g_{d]b}$$

$$F_{ab[cd;e]} \neq 0 \quad \text{But its trace} = 0$$

$$g^{ac} g^{bd} F_{ab[cd;e]} = 0 = H_e{}^c{}_{;c} = 0$$

$$H_{ab} = n\left(F_{ab} - \frac{1}{2} F g_{ab}\right)$$

For quadratic GB

$$H_{ab} = 2\left(R R_{ab} - 2 R_a{}^m R_{mb} - 2 R^{mn} R_{ambn} + R_a{}^{mnl} R_{bmnl}\right) - \frac{1}{2} L_{GB}\, g_{ab}$$

$$L_{GB} = R^2_{abcd} - 4 R^2_{ab} + R^2$$

For each term in Lovelock polynomial there exists F_{abcd} and

Trace $F_{ab(cd;e)} = 0 \Longrightarrow$ Corresponding $H_{a;b}^{b} = 0$

Bianchi Derivative characterizes Lovelock gravity — Gravitational dynamics in general.

Lovelock Electrovac Solution

$$G^{(n)}_{ab} = -\Lambda g_{ab} + E_{ab} \text{ for a given } n.$$

NEC: $G^{(n)}_{ab} k^a k^b = 0, \; k_a k^a = 0 \Longrightarrow \nu = -\lambda$

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 d\Omega^2_{d-2}$$

$$G^{(n)}_{ti} = 0 \Longrightarrow \nu = -\lambda(r) : \text{Static}$$

Then

$$\left(G^t_t = -\Lambda + E^t_t\right)' \Longrightarrow G^\theta_\theta = -\Lambda + E^\theta_\theta$$

Reduced to a Single Equation

And that Equation is

$$\left(r^{d-2n-1} f^n\right)' = \frac{2\Lambda}{d-2} r^{d-2} + \frac{2q^2}{r^{2(d-n-2)}}$$

$$\sum_{n=1}^{n} \left(\alpha_n r^{d-2n-1} f^n\right)' =$$

The solution is: $f^n = \Lambda_1 r^{2n} + \dfrac{k}{r^{d-2n-1}} - \dfrac{Q^2}{r^{2(d-n-2)}}$

where $e^{-\lambda} = F = 1-f$, $\Lambda_1 = 2\Lambda/(d-1)(d-2)$, $Q^2 = \dfrac{2q^2}{(d-2)(d-3)}$

For large r,

$$F = 1 - \Lambda_1^{1/n} r^2 - \frac{K}{r^{d-3}} + \frac{e^2}{r^{2(d-3)}}$$

Einstein solution

$$K = k/n\Lambda_1^{1/n}, \quad e^2 = Q^2/n\Lambda_1^{1-1/n}$$

Einstein - ~~Gauss-B~~ Gauss-Bonnet Solution : $n=2$, ~~$d=5$~~, $Q=0$

$$F = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + 4\alpha\left(\frac{M}{r^{d-1}} + \Lambda\right)}\right)$$

$$\simeq 1 + \Lambda r^2 - \frac{M}{r^{d-3}}, \quad \text{S-AdS in } d\text{-dimension}$$

Asymptotically Lovelock for a fixed n or Σ_n with Λ

$\Longrightarrow$ Einstein solution with Λ in the given dimension

Asymptotic limit is universal

Universality of constant density sphere

[ND, A Molina & A Khugaev, PRD 81, 104026 (2010)]
arxiv:1001.3922

$$\phi_N \sim \frac{M(r)}{r^{D-3}} \sim \frac{\rho r^{D-1}}{r^{D-3}} \sim \rho r^2, \ \underline{\text{free of } D}$$

This must be true for any gravity theory in general.

Einstein - Lovelock / GB

despite their non-linearity.

$\phi \sim r^2$
$\phi' \sim r$
$\rho = \text{const.}$

Einstein Gravity

Density Eqn.

$$\frac{2}{D-2}\rho = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{D-3}{r^2}\right) + \frac{D-3}{r^2} \longrightarrow \underline{e^{-\lambda} = 1-\rho_0 r^2} \qquad (1)$$

$$\rho_0 = 2\rho/(D-1)(D-2)$$

Sufficient

Pressure isotropy

$$e^{-\lambda}\left(2\nu'' + \nu'^2 - \lambda'\nu' - 2\frac{\nu'+\lambda'}{r} - \frac{4}{r^2}\right) + \frac{4}{r^2}$$

$$-2(D-4)\underbrace{\left((D-1)\left(\frac{e^{-\lambda}-1}{r^2}\right) + \frac{2\rho}{D-2}\right)}_{=\,0\ \Longrightarrow\ (1)} = 0$$

Universality

Rest $= 0 \Longrightarrow \underline{e^{\nu/2} = A + B\, e^{-\lambda/2}}$

Necessary

Schwarzschild's interior Solution.

GB gravity

$$G_{ab} + \alpha H_{ab} = -T_{ab}$$

Density

$$\frac{2\rho}{D-4} r^4 = r^2\left(r\lambda' - (D-3)(1-e^{\lambda})\right) + \tilde{\alpha} e^{-2\lambda}(1-e^{\lambda})\left(-2r\lambda' + (D-5)(1-e^{\lambda})\right)$$

Integrates to give the same

$$\therefore \quad e^{-\lambda} = 1 - \rho_{GB} r^2 , \qquad \rho_{GB} = \frac{\sqrt{1+4\tilde{\alpha}\rho_0} - 1}{2\tilde{\alpha}}$$

$$\tilde{\alpha} = (D-4)(D-5)\alpha$$

Sufficient

Pressure isotropy

$$I_{GB} = \left(1 + \frac{2\tilde{\alpha}}{r^2} f\right) I_E + \frac{2\tilde{\alpha}}{r}\left(\frac{f}{r^2}\right)'\left(r\psi' + \frac{f}{1-f}\psi\right) = 0$$

$$I_E = \frac{1-f}{\psi}\left(\psi'' - \left(\frac{f'}{2(1-f)} + \frac{1}{r}\right)\psi' - \frac{D-3}{2r^2(1-f)}\left(rf' - 2f\right)\psi\right)$$

$$e^{-\lambda} = 1 - f, \qquad e^{\nu/2} = \psi$$

$e^{-\lambda} = 1 - \rho_{GB} r^2 \Longrightarrow I_E = 0$ which integrates to give the same

$$e^{\nu} = A + B e^{-\lambda/2}$$

Sufficient

Schwarzschild interior solution.

Necessary condition

$I_{GB} = 0$ which implies

either $\left(f/r^2\right)' = 0 \Longrightarrow$ Sch. Soln

or $r\psi' + \frac{f}{1-f}\psi = 0 \Longrightarrow \rho = -p = \text{Const}$

dS/AdS: Special case $A = 0$ in Sch. Soln.

$\rho = \text{Const.}$ is necessary & Sufficient for Universality
true for all $D \geq 4$

Schwarzschild interior is unique universal solution for constant density.

Discussion

* Explored Universal Features : Gravity/Spacetime

* High Energy $\longrightarrow$ GB/Lovelock $\longrightarrow$ Higher D

* QG should include High Energy effects of CG $\rightarrow$ Higher D

$$CG/GR \xrightarrow[\text{Higher D}]{\text{GB/Lovelock}} QG$$

* Vacuum Energy

$$NG \xrightarrow[\text{Grav. Field Energy}]{\text{Curved Spacetime}} GR$$

$$GR \xrightarrow[\text{Vacuum Energy}]{\text{??? HD/Non-Comm./Q.Relation}} QG$$