The Universe as a Soap Film

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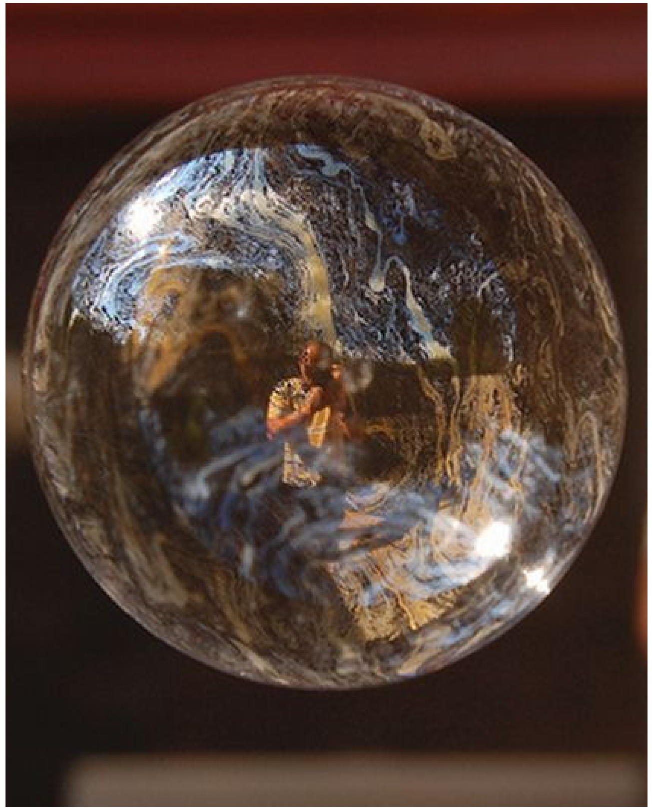


Introduction: motivation

- The Sand Reckoner: How many grains of sand are there in the Universe? Archimedes 10^{63} circa 250 BC. Grappling with the large numbers needed to describe our Universe.
- Modern Version: replace the *spatial* volume of the Universe by its *spacetime* four volume, which is around $10^{112}\mathrm{cm}^4$. The modern analogue of "a grain of sand" is the smallest element of spacetime, which from our theories of relativity (c) , gravitation (G) and quantum mechanics $(\hbar)$ is around the Planck four volume $(\hbar G/c^3)^2$ of $10^{-132}\mathrm{cm}^4$.
- Answer is 10^{244} Archimedes' Number $\mathcal{N}_{Arch}$.
- Dirac's Large Number Hypothesis: large numbers are unnatural in Cosmology. One should try to minimise the number of independent ones.
- In the last decade, there have been detailed observations of dim, distant supernovae, which clearly indicate the presence of a tiny (in natural Planck units $\hbar = c = G = 1$) but non zero cosmological constant λ.
- Inverse of a small number is a large one
- Can one relate these two large numbers $\mathcal{N}_{Arch}$ and λ^{-1}?

Introduction:Avogadro and Brown

- This is precisely what was done by Sorkin. Sorkin argued that quantum gravity effects would predict an order of magnitude for fluctuations in the cosmological constant which in natural units is $1/\sqrt{\mathcal{N}_{Arch}}$. This is precisely the order of magnitude of the observed value of the cosmological constant. Sorkin's proposal was made in the context of Causal Sets, which is one of several approaches to quantum gravity. We find using an analogy between GR and Soft Condensed matter that this is a generic prediction of quantum gravity.
- Approaches to quantum gravity. Some have Violation of Local Lorentz Invariance. Discreteness Black Hole Entropy.
- Avogadro's Number and Brownian motion. $\mathcal{N}_{Avo} \approx 10^{23}$
  $1/\mathcal{N}_{Avo}$ is small
  $1/\sqrt{\mathcal{N}_{Avo}}$ is not quite as small
  Brownian motion is visible under an optical microscope. (Jan Ingen-Hausz)
- Can the Cosmological Constant be today's Brownian Motion?

Summary

- We will show that Sorkin's suggestion can be better understood using an analogy from Soft Condensed Matter: the physics of fluid membranes.
- Develop an analogy between the Cosmological Constant and the Surface tension of membranes Bring the subject down to earth and into the laboratory.
- Find that a fluctuating cosmological constant is far more general than the context of Causets in which Sorkin proposed it. *Generic Prediction of Quantum Gravity Models*
- This talk develops the analogy and its consequences.
- For more see PRL **97**, 161302 (2006)(arXiv:cond-mat/0603804) Class.Quant.Grav.26:135018,2009 (arXiv:0904.1057))

The cosmological constant problem: dynamics of General Relativity

- **Cosmological constant problem in GR**:
  Space-time is a pair $(\mathcal{M},\ g)$
  $\mathcal{M}$ = Four dimensional manifold; set of all events; four dimensional continuum
  g = Lorentzian metric
  $(\mathcal{M},\ g)$ is a history $\mathcal{H}$
  Dynamics of pure gravity is described by the Einstein-Hilbert Action

$$I_2 = c_2 \int d^4x \sqrt{-g}\ \ R$$

  modified by the addition of a cosmological term

$$I_0 = c_0 \int d^4x \sqrt{-g}\ .$$

- Classical equations of motion emerge by extremising the action.

The cosmological constant problem: the dilemma

Standard notation $c_2 = \frac{1}{16\pi G}$ $c_0 = \lambda$. Usually, higher derivative terms like

$$I_4 = c_4 \int d^4x \sqrt{-g} \; R^2$$

are dropped as being negligible at low Energies Entirely in the spirit of Effective Field theory or Landau theory in condensed matter. Identify basic fields (order parameter) Identify symmetries Expand energy functional in derivatives of the fields Low energy physics dominated by the low derivative terms.

- Consistently applying this logic we expect the low energy physics of gravity to be dominated by I_0.
  Crude dimensional analysis $\rightarrow \lambda \sim 1$ in Planck units $(c = G = \hbar = 1)$.
  Observed value $\lambda = 0$.
- But not exactly!
  We have

$$\lambda_{\rm obs} = 10^{-122} l_P^{-4}$$

tiny but non-zero!

The cosmological constant problem: Sorkin´s prediction

- Cosmological constant problem dilemma with two horns
  (a) why is the cosmological constant nearly zero?
  (b) why is it not *exactly* zero?
- Hard to come up with a natural explanation for both these facts
- Symmetry could imply $\lambda = 0$. But why $\lambda \sim 0$?
- Gulliver and the Learned men of Brobdingnag
- Beautiful idea due to Sorkin: Quantum gravity may provide a natural explanation stemming from fundamental discreteness of space-time
- Sorkin´s proposal is in the framework of causal sets and unimodular gravity
- Causal sets: Space-time replaced by a discrete structure.
  Points with causal relations
  N number of points volume of space-time

$$\mathcal{V} = \int d^4x \sqrt{-g} \; = N \, l_P^4$$

- The rest of the metrical information is captured in causal relations.
  Space-time is an emergent notion as N gets large

The cosmological constant problem: the runaway Universe

- $\mathcal{V}$ also plays a role in unimodular gravity. The metric field is subject to $\det g = 1$. (Einstein, Weinberg, Unruh-Wald)
- Unimodular gravity: GR with the constraint of fixed $\mathcal{V}$. Classically equivalent to GR with cosmological constant. But λ is a dynamical variable unlike in GR, where it is a coupling constant
- Sorkin addresses part (b) of the Cosmological Constant problem, suppose (a) has been solved: $< \lambda >= 0$. There will be fluctuations about this mean value which will give a small Cosmological Constant.
- Sorkin (1990) predicted the right order of magnitude.
  From uncertainly principle $\Delta\lambda \quad \Delta\mathcal{V} \sim 1$ $\mathcal{V} = N{l_P}^4$ $\mathcal{V}$ has Poisson fluctuations $\Delta N \approx \sqrt{N}$

$$\rightarrow \Delta\lambda \approx \frac{l_{\mathrm{P}}^{-4}}{\sqrt{N}}.$$

- Prediction consistent with Astronomical data (1998-present)
  Redshift Luminosity relations for type Ia supernovae
  Acoustic Peak of the microwave background
  Age of the Universe vs the age of the globular clusters
- Universe is accelerating at the present epoch indicating $\lambda > 0$.
  Correct order of magnitude predicted. Either sign.

The Analogy: membranes

- Membranes in soft matter physics.
  $\hbar = 0$ A configuration $\mathcal{C}$ is described as a two dimensional surface Σ embedded in flat three dimensional space. Σ has extrinsic curvature H and intrinsic curvature K. $H \sim 1/L \quad K \sim 1/L^2$.
  Need to write an energy $\mathcal{E}(\mathcal{C})$.
- Assume Σ 2-sided and symmetric in its sides. Terms you can write down consistent with this symmetry are:

$$\mathcal{E}_0 = a_0 \int d^2x \sqrt{\gamma}$$

$$\mathcal{E}_2 = a_2 \int_\Sigma d^2x \sqrt{\gamma} \;\; (H)^2 + a_2' \int_\Sigma d^2x \sqrt{\gamma} \;\; K$$

  γ = pulled back metric.
  Leading term is the surface tension. Conventionally $a_0 = \sigma$
- Higher derivative terms negligible in the long wavelength limit

$$\mathcal{E}_4 = \int_\Sigma d^2x \sqrt{\gamma} \;\; H^4$$

The Analogy: Histories and Configurations

- Physics of membranes captured in the partition function

$$Z = \sum_{\mathcal{C}} \exp[-\frac{\mathcal{E}(\mathcal{C})}{k_B T}]$$

  where $\mathcal{E}(\mathcal{C}) = \mathcal{E}_0(\mathcal{C}) + \mathcal{E}_2(\mathcal{C}) + \mathcal{E}_4(\mathcal{C}) \ldots$
  Expansion of energy in inverse powers of length.

- If you give up symmetry you can have

$$\mathcal{E}_1 = a_1 \int_{\Sigma} d^2x \sqrt{\gamma} \ \ (H)$$

  gives spontaneous curvature. Assume symmetric membranes

- Mathematical model of a membrane realised physically as an interface between fluids Clear analogy between the GR and soft matter situations Usual correspondence between quantum physics and statistical physics.
- History replaced by a configuration sum over histories replaced by a sum over configurations Action by the energy Planck's constant temperature.
- Energy cost for making unit area of surface (mechanical work) Action cost per unit 4-volume of spacetime

The Analogy: in tabular form

Table of Analogy

Membranes	Universe
Configuration $\mathcal{C}$	History $\mathcal{H}$
Area of a configuration	Four volume of a history
Sum over configurations	Sum over histories
Energy $\mathcal{E}(\mathcal{C})$	Classical Action $\mathcal{I}(\mathcal{H})$
$\mathcal{E}_0 = a_0 \int d^2x \sqrt{\gamma}$	$I_0 = c_0 \int d^4x \sqrt{-g}$
$\mathcal{E}_2 = a_2 \int d^2x \sqrt{\gamma} H^2$	$I_2 = c_2 \int d^4x \sqrt{-g} R$
$Z = \sum_{\mathcal{C}} \exp[-\frac{\mathcal{E}(\mathcal{C})}{k_B T}]$	$Z = \sum_{\mathcal{H}} \exp[\frac{i\mathcal{I}(\mathcal{H})}{\hbar}]$
Minimum energy configuration	Classical Path of Least Action
Temperature T	Planck's constant $\hbar$
Thermal Fluctuations	Quantum Fluctuations
Surface Tension σ	Cosmological Constant Λ
Free Energy	Effective Action

The Analogy: discreteness in membranes

- Geometric description of a membrane as a smooth 2-manifold is only an idealisation
  Real membrane is composed of molecules.
- Similar to the break down of the smooth manifold picture of space-time at the Planck scale.
  Planck length 10^{-33}cm $\sim l_{\rm mol} \approx .3$nm.
- At Mesoscopic scales of microns the membrane appears smooth and in a statistical sense locally homogeneous and isotropic. Just as spacetime appears locally Lorentz invariant, even though it may be grainy at Planck scales.
- Probability of a micron sized void (crude estimate assuming Poisson distribution)

$$P_{\rm void} \sim \frac{\mathcal{A}}{\mathcal{A}_{\rm void}} \exp - \frac{\mathcal{A}_{\rm void}}{l_{\rm mol}^2} \approx \frac{\mathcal{A}}{\mathcal{A}_{\rm void}} \exp -10^7.$$

- Similar in spirit to Causet estimates of a nuclear sized void

$$P_{\rm void} \sim \exp -10^{80}.$$

The Analogy: extended table

Table of Analogy

Membranes	**Universe**
Configuration $\mathcal{C}$	History $\mathcal{H}$
Area of a configuration	Four volume of a history
Sum over configurations	Sum over histories
Energy $\mathcal{E}(\mathcal{C})$	Classical Action $\mathcal{I}(\mathcal{H})$
$\mathcal{E}_0 = a_0 \int d^2x \sqrt{\gamma}$	$I_0 = c_0 \int d^4x \sqrt{-g}$
$\mathcal{E}_2 = a_2 \int d^2x \sqrt{\gamma} H^2$	$I_2 = c_2 \int d^4x \sqrt{-g} R$
Minimum energy configuration	Classical Path of Least Action
Temperature T	Planck's constant $\hbar$
Thermal Fluctuations	Quantum Fluctuations
Surface Tension σ	Cosmological Constant Λ
Free Energy	Effective Action
Molecular Length $l_{\mathrm{mol}} = .3\mathrm{nm}$	Planck Length $l_P = 10^{-33}\mathrm{cm}$
Molecules	Causet elements
Molecular level discreteness of space	Planck scale level discreteness of space-time

Limitations of Analogy

Membranes	**Universe**
dimension two	dimension four
Euclidean geometries	Lorentzian geometries
Postive Surface tension minimises area	Positive λ causes accelerated expansion
No Causal Structure	Causal Structure
Ambient Space and Extrinsic geometry	Purely Intrinsic geometry
Exponentially damped sum over configurations	Oscillatory phase sum over histories
Non-Poissonian distribution of molecules	Poissonian distribution of Causet elements

The Analogy: surface tension in natural units

- Using analogy, expect $\sigma \sim 1$ in dimensionless units

$$\sigma = \sigma_0 = \frac{k_B T}{l_{\rm mol}^2}.$$

Indeed even if we set $\sigma = 0$ by hand in the microscopic energy, such a term is generated by thermal fluctuations.
Flat membrane will vibrate about equilibrium configuration like a drum
Equipartition gives us that $< E >$ is $k_B T$ from each mode.
Sum over modes is divergent
Regulate by $k_{\rm max} = \frac{2\pi}{l_{\rm mol}}$

$$k_B T \int_0^{k_{\rm max}} \frac{d^2x \;\; d^2k}{(2\pi)^2} = \frac{k_B T}{l_{\rm mol}^2} \;\; \mathcal{A}$$

surface tension is generated by thermal fluctuations.

-

$$\sigma = \frac{k_B T}{l_{\rm mol}^2} \qquad\qquad k_B T = \frac{1}{40 \; eV} \;\; (300^{\rm o} K)$$

$l_{\rm mol} = .3$nm. We would naively expect $\sigma \sim 40$ milli Joules/m^2
Expectation turns out to be correct!

The Analogy:typical surface tensions

Table of Typical Interfacial Tension Values

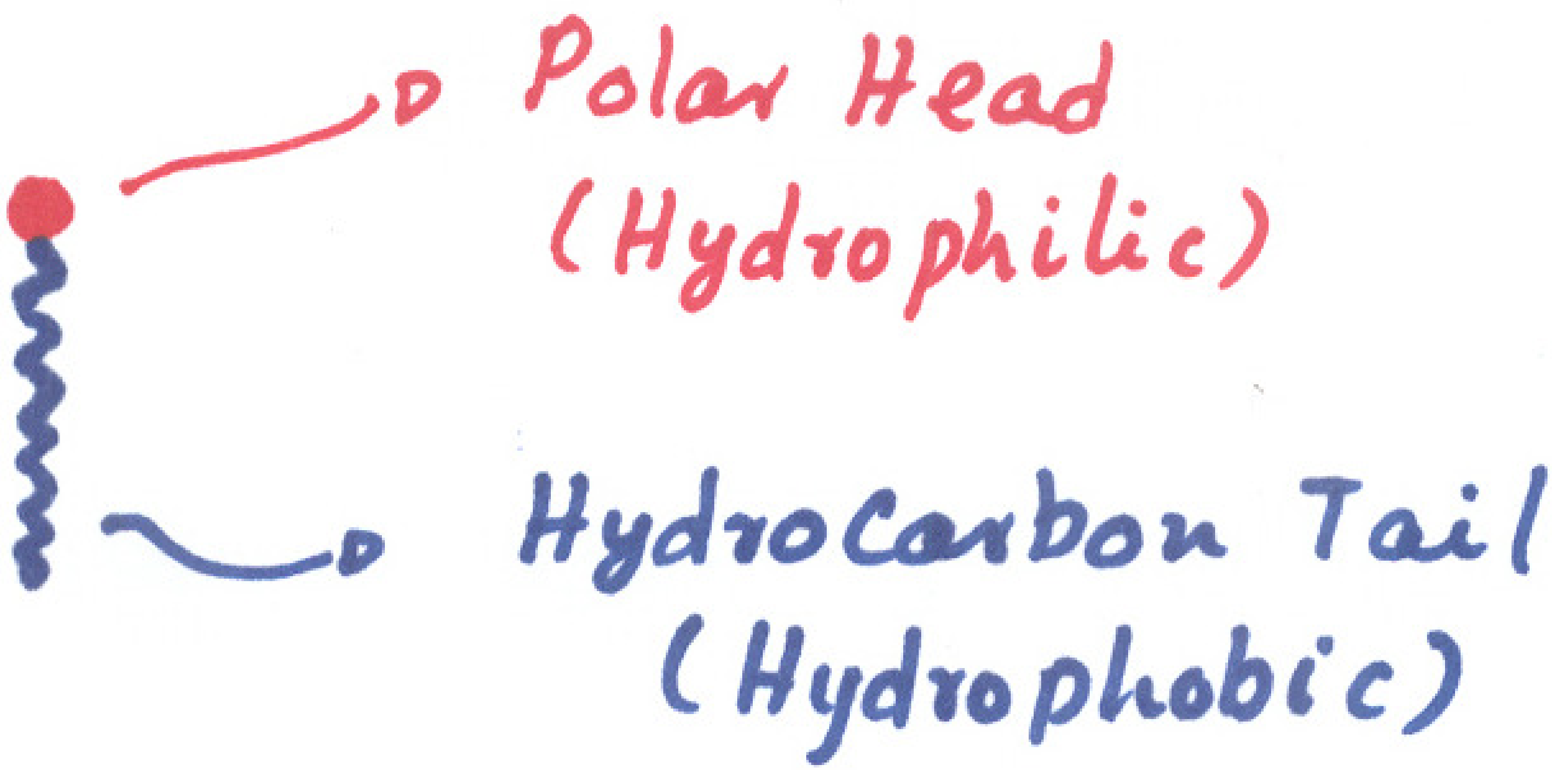
	Interfaces	**Surface Tensions in milli Joules per meter squared**
(i)	Water-Vapour	72.6
(ii)	Water-Oil	57
(iii)	Mercury-Water	415
(iv)	Glycerol-Air	63.4
(v)	Decane-Air	23.9
(vi)	Hexadecane-Air	27.6
(vii)	Octane-Air	21.8
(viii)	Water-Air	40

- soap, big molecules No "cosmological constant" problem here!
- Reinforces our faith in the naive dimensional argument.

Fluid Membranes:tensionless membranes

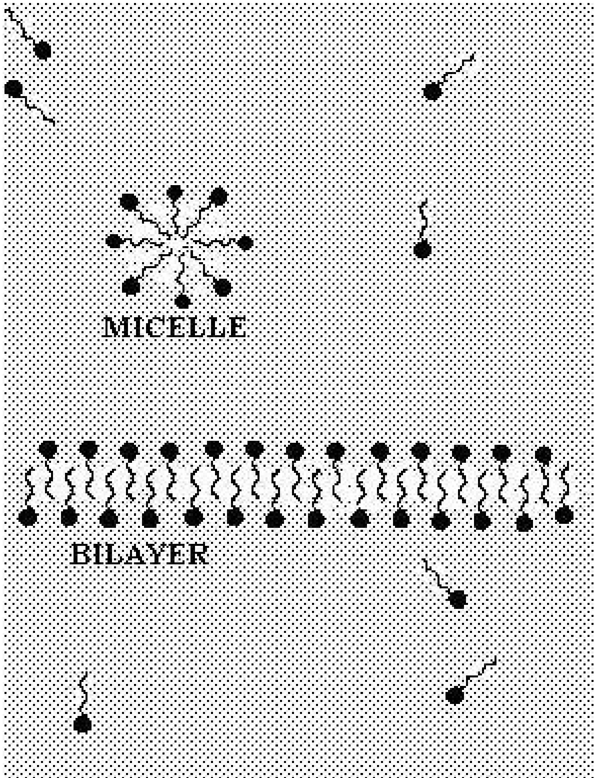
- However, there is an important exception: FLUID MEMBRANES
  Characterised by a negligibly small surface tension. Orders of magnitude below the dimensional expectation.
- Statistical mechanics of Tensionless Membranes is dominated by $\mathcal{E}_2$ rather than $\mathcal{E}_0$ exact counterpart of the cosmological constant problem.
- Example where part (a) is naturally solved
  Something to understand for cosmology
  Why do fluid membranes have vanishing surface tension?
- Fluid Membrane composed of amphiphilic molecules.

Fluid Membranes: amphiphilic molecules



- When you add amphiphilic molecules to water, they cluster to hide their tails from water micelles, vescicles, symmetric bilayers.
- Lipid bilayers, rich phase diagram
- Cell membrane is composed of phospholipids

Fluid Membranes: Bilayers



- area per molecule $\alpha = \mathcal{A}/N$. Optimal value is $\alpha = \alpha_0$. Free energy per molecule $f(\alpha)$ has a minimum at $\alpha = \alpha_0$.

Fluid Membranes: Cosmological Constant part a

- A membrane will adjust its area (or N) so that optimal density is achieved. Consider a membrane at optimal density

$$\frac{\partial f(\alpha)}{\partial \alpha}|_{\alpha=\alpha_0} = 0.$$

  Saturated membrane with $\mathcal{A}$ fixed $N = \mathcal{A}/\alpha$ molecules

$$F(\mathcal{A}) = N f(\alpha)$$

$$< \sigma > = \frac{\partial F}{\partial \mathcal{A}} = \frac{\partial f}{\partial \alpha}|_{\alpha=\alpha_0} = 0.$$

  This solves part (a)

- Physical explanation. As you forcibly expand the area, you create gaps These are quickly filled in by molecules from the solution Chemical potential difference is zero at equilibrium. So no energy cost to stretch the membrane. No surface tension.
- what about part (b)?

Fluid Membranes: Cosmological constant part b

- Part (b) can also be addressed σ has fluctuations about its mean value

$$< (\Delta\sigma)^2 > = < (\sigma - < \sigma >)^2 > = T\frac{\partial^2 F}{\partial \mathcal{A}^2} = \frac{T}{N}\frac{\partial^2 f}{\partial \alpha^2}|_{\alpha=\alpha_0}$$

naturally expect $Tf'' \sim 1$ and so

$$(\Delta\sigma) \sim \frac{1}{\sqrt{N}} \quad \frac{T}{l_{Mol}^2}$$

in complete analogy to Sorkin's proposal.

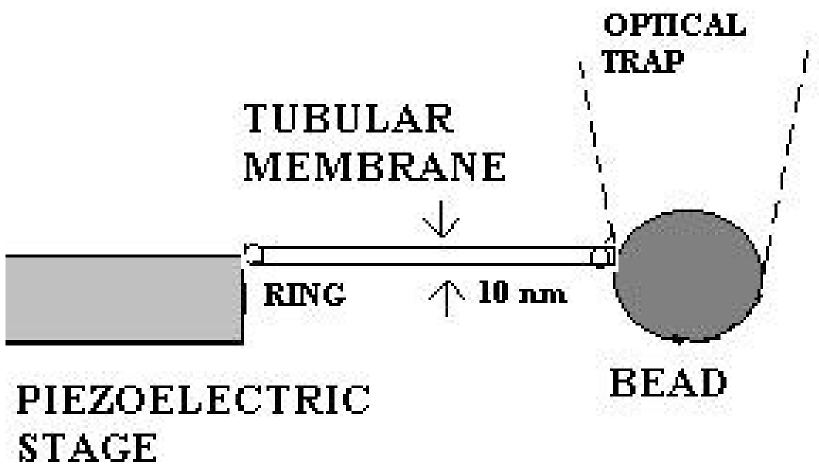
- Fluctuating σ is a standard Stat mech effect. Consider $S(x^i)$, where $x^i, i = 1, 2...$ are any set of quantities Einstein: fluctuation probability$\propto \exp \Delta S$ Taylor expansion about maximum (say $x = 0$)

$$S(x) = S(0) + 1/2\frac{\partial S}{\partial x^i \partial x^j}x^i x^j + ...$$

- $P(x) \propto \exp -x^i C_{ij} x^j$
- mean square fluctuations of intensive quantities go as $1/N$ Landau and Lifshitz.
- Brownian Motion observable effect. Fluctuations of Mesoscopic systems

Fluid Membranes: experiment

- This fluctuation can be measured by laboratory experiments
- Experiment: How can we measure this fluctuating surface tension?

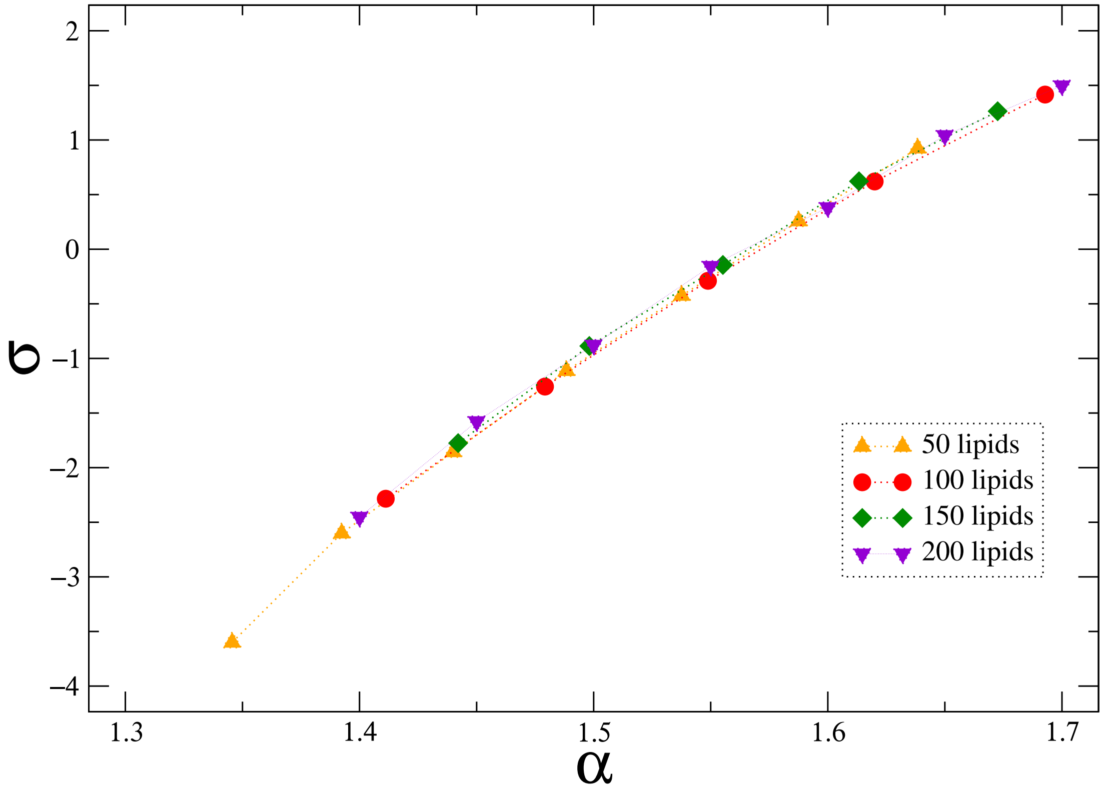


Two nanometer size rings one attached to a translation stage;
the other to a micron sized bead placed in an optical trap. Fix separation L by a feedback loop. Force on the bead is related to surface tension expect to see fluctuations in σ as an extra r.m.s. fluctuation of the position of the bead in the trap.

Impractical in lab, need nano sized membranes, different technique. Simulations!

Fluid Membranes: DPD Simulations

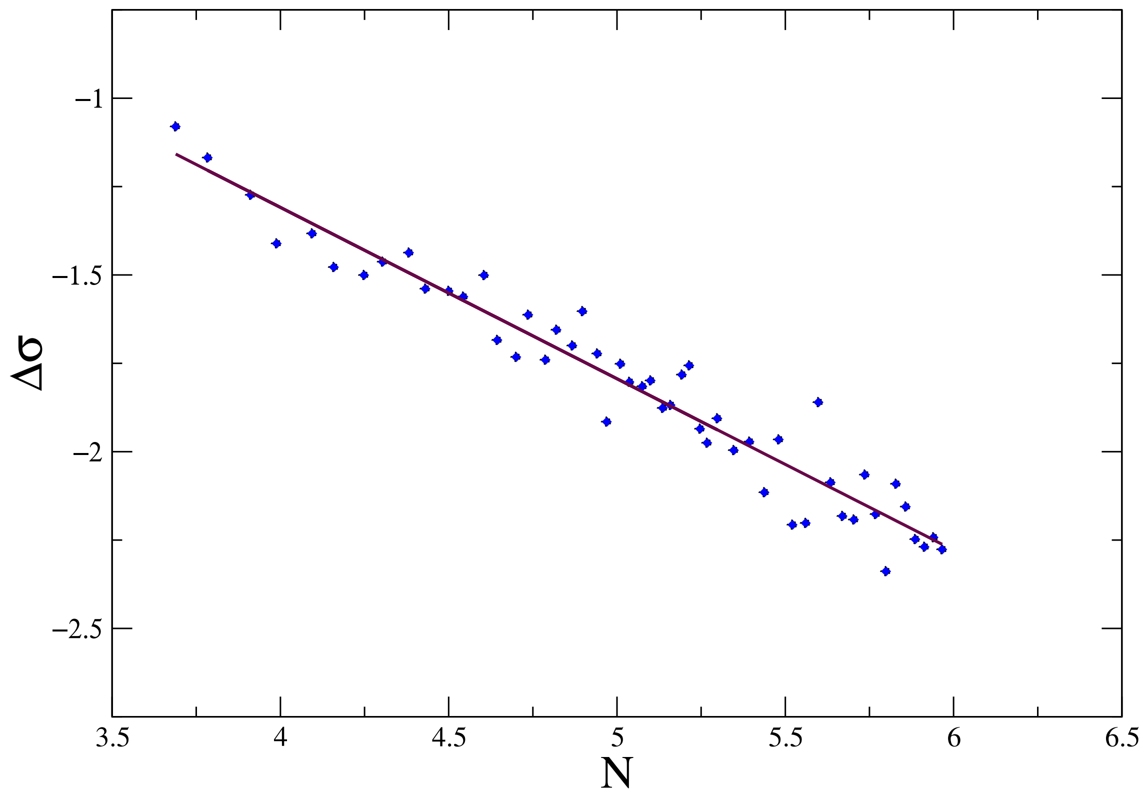
part a solved: Simple theory



surface tension depends only on area per lipid! in accord with simple theory

Fluid Membrane: DPD simulations

part b solved: Simple theory



(Rohit katti) using software developed by M. Venturoli (thanks!)

Log Log Plot best fit straight line slope .48 vs .5 (theory)

Conclusion:summary

- Described the cosmological constant problem and Sorkin's quantum gravity explanation for it
- Developed Analogy between surface tension and the Cosmological Constant
  Standard mapping between Quantum Field Theory and Statistical Mechanics.
- Noticed that dimensional arguments work well for the surface tension of most interfaces
- Noticed the exception: Fluid membranes which have practically vanishing tension.
  Analogue of the cosmological constant problem in soft condensed matter physics. Translated exotic physics into known physics, laboratory physics.
- Suggested an experiment for measuring a fluctuating surface tension. Other realisations also possible and may be advantageous.
- Connection between two disparate fields, transport wisdom both ways
  (a) discussed in fluid membranes but not in cosmology
  (b) discussed in cosmology (Sorkin) but not in fluid membranes

Conclusion: What have we learned?

- Sorkins suggestion of a fluctuating λ was made in the context of causal sets and unimodular gravity.
- How essential are these inputs? What is really needed? Can we develop a minimalist picture?
- What seems essential is
  dynamical λ (unimodular gravity)
  discrete spacetime (Causets)
- let us consider these in turn

Conclusion: dynamical λ and Unimodular Gravity

- dynamical λ. In GR λ is a fixed coupling constant, no fluctuations. Consider the soft matter context. For a membrane with tension σ, we would write

$$Z[\sigma] = \sum_{\mathcal{C}} \exp[-\frac{\mathcal{E}_2(\mathcal{C})}{k_B T}] \exp[-\frac{\sigma \mathcal{A}}{k_B T}]$$

This is in the constant surface tension ensemble. Gibbs We can equally well work in the constant area ensemble.

$$Z[\mathcal{A}] = \sum_{\mathcal{C}} \delta(\mathcal{A} - \mathcal{A}(\mathcal{C})) \exp[-\frac{\mathcal{E}_2(\mathcal{C})}{k_B T}]$$

Helmholtz. The two descriptions are just a Laplace transform away from each other! Thermodynamically, a Legendre transform. This discussion translates easily to the gravity context, where the Laplace transform is replaced by a Fourier transform. In a quantum version of gravity, there is no reason to treat λ as a coupling constant whose value is eternally fixed. In this age of the renormalisation group and running coupling constants, this is surely an outdated attitude. We should regard λ as a chemical potential for creating spacetime, subject to fluctuations! To summarise, unimodular gravity is not an essential input to Sorkin's idea. Rather, GR and unimodular gravity are closely related theories, just Legendre transforms of each other.

Conclusion: graininess of spacetime and Causets

- *discrete spacetime*: This aspect is supplied by Causets, but more generically present in all quantum gravity approaches.
- Yet, there is a further ingredient in Sorkin's argument which seems to need Causets: the Poisson nature of the number fluctuations $\Delta N \propto \sqrt{N}$.
- But consider again the analogue system. The distribution of molecules is far from Poisson. When N is large, the central limit theorem assures us of $\sqrt{N}$ fluctuations quite independent of Poisson. $\sqrt{N}$ fluctuations are $exact$ for Poisson statistics, but in cosmology, we needn't be anxious on this score: $N = 10^{244}$ is comfortably large.
- Poisson statistics are not essential for Sorkin's argument to work.

Conclusion: What we learn

- We conclude by that we will have quantum fluctuations in the cosmological constant in any approach to quantum gravity which has discreteness of spacetime and a dynamical λ.
- This is both good and bad news. Bad, because the experiment does not seem to discriminate between the competing theories. Any approach that gets black hole entropy right will have discreteness in some form and produce a fluctuating cosmological constant of the right magnitude to fit observations. Good, because now we may now have a general quantum gravity explanation for the cosmological constant problem.
- Sorkin's idea solves part b): The cosmological constant *is* zero, as close to zero as it can be given quantum gravity fluctuations.
- What about part a) Why is it nearly zero? The analogue system suggests an explanation along the following lines: The cosmological constant is a low energy residue resulting from an imperfect cancellation between high energy processes. There are many instances of this in physics: Protons and electrons are so strongly attracted to each other that they neutralise each other's charge and all we see in chemistry are the weak van der Waal's forces between atoms resulting from an imperfect cancellation of charge. Quarks are so strongly attracted to each other that they confine and the nuclear forces are a low energy residual force between nucleons.

Conclusion: Virtues and Charges

I can't express this idea better than Isaac Newton (Opticks):

"...There are therefore Agents in Nature able to make the Particles of Bodies stick together by very strong Attractions. And it is the business of experimental Philosophy to find them out.

Now the smallest particles of Matter may cohere by the strongest Attractions, and compose bigger Particles of weaker Virtue; and many of these may cohere and compose bigger Particles whose Virtue is still weaker, and so on for divers Successions, until the Progression end in the biggest particles on which the Operations in Chymistry, and the Colours of natural Bodies depend, and which by cohering compose bodies of a sensible magnitude."

He seems to be talking about running coupling constants. Perhaps the explanation for the cosmological constant is along these lines. There is a fixed point at $\lambda = 0$. We will get there only when the univese is infinitely old. I can't wait for that to happen!

Acknowledgements

Y. Hatwalne	:	Fluid Membranes
Ashoke Sen	:	Discreteness in String Models
J. Henson		
R.D. Sorkin	:	For comments

Thank you!