

Introduction to Lehmer's Problem

In this short course, we shall introduce Lehmer's problem, claiming a universal lower bound for the Mahler measure of an algebraic number. After an introductory lecture explaining the original problem in its historical context, and explaining the earlier results obtained towards this still open conjecture, we shall cover the methods and results obtained during the last decade.

During the course, we shall cover classical tools from diophantine geometry, such as Siegel's lemma (both the classical versions and the absolute ones over algebraic numbers), Schwarz lemmas (with an emphasis on the ultrametric case). We shall pay special attention to zero estimates.

We shall then give an overview of the situation over abelian extensions and explain the natural generalizations of the classical problem to the higher dimension. We shall then give complete proofs of at least one partial result towards these more general conjectures.

If time permits, the series of lectures will be concluded by an overview of the situation over abelian varieties.

Prerequisites: we shall make utmost efforts to make the lectures as self contained as possible. However, standard algebraic number theory, some notions of height theory will be assumed to be known. For part of the lectures, we shall also need the audience to know some commutative algebra and basic algebraic geometry. The last lectures will assume that the audience has had some exposition to abelian varieties.