Additive Combinatorics, abstract

Additive combinatorics is a fascinating new branch of number theory. There is a very thin border line between classical additive number theory and additive combinatorics, while the former mainly deals with direct problems, sums of squares, for example, the later deals with indirect problems, additive properties of general sets of integers, or understanding the structure of a set from information of the sum set. Many of the basic results can be expressed a little more generally, for an arbitrary additive group, or ring, or field. However, we will confine ourselves to the most important special cases, $\mathbb{Z}$, the ring of integers, $\mathbb{R}$, the field of real numbers, $\mathbb{Z}/N\mathbb{Z}$, the ring of residue classes mod N, especially $\mathbb{F}_p$ the finite field of p elements. Another remarkable difference is that the proofs always have some combinatorial flavor, and much more, they use tools from many diverse fields of mathematics, including harmonic analysis, convex geometry, incidence geometry, graph theory, probability theory, algebra, and ergodic theory. This wealth of perspectives makes the subject rich and fascinating.

As a young subject, there are only a few comprehensive books to study from, we can suggest the following two: Terence Tao, Van Vu, Additive Combinatorics, Cambridge University Press, 2006, and Melvyn B. Nathanson, Additive Number Theory, Inverse Problems and the Geometry of Sumsets, Springer Verlag, 1996.

We plan to cover the following subjects with detailed proofs.

– Elementary estimates for sum sets, Sidon sets.

– Sum–product estimates.

– Arithmetic progressions I, van der Waerden's theorem, the circle method, Roth's theorem.

– Plünnecke's inequality and the structure theorem of Freiman.

– Sums along a graph, statistical results.

We also plan to sketch the main ideas of the proofs of the deepest results of the subject.

– Arithmetic progressions II, Szemerédi's theorem, Green–Tao's theorem about dense sets of primes.