The elliptic genus of K3 and CFT

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Katrin Wendland
Albert-Ludwigs-Universität Freiburg

[Taormina/W11] *The overarching finite symmetry group of Kummer surfaces in the Mathieu group M_{24}*, JHEP **1308**:152 (2013); arXiv:1107.3834 [hep-th]

[Taormina/W12] *A twist in the M_{24} moonshine story*; arXiv:1303.3221 [hep-th]

[Taormina/W13] *Symmetry-surfing the moduli space of Kummer K3s*; arXiv:1303.2931 [hep-th]

[W14] *Snapshots of conformal field theory*; arXiv:1404.3108 [hep-th]; to appear in "Mathematical Aspects of Quantum Field Theories", Mathematical Physics Studies, Springer

Motivation: The Atiyah-Singer Index Theorem

Atiyah-Singer Index Theorem and McKean-Singer Formula

For M: a compact oriented $2D$-dimensional spin manifold,
$W \longrightarrow M$: a vector bundle with associated Dirac operator $\not{D}$,

$$\int_M \widehat{A}(M)\mathrm{ch}(W) = \mathrm{ind}(\not{D}) = \mathrm{sTr}\left(e^{-t\not{D}^2}\right),$$

where:
$\widehat{A}(M) = \det^{1/2}\left(\frac{R/2}{\sinh(R/2)}\right)$ is the A-roof-genus, $R \in \mathcal{A}^2(M, \mathfrak{so}(TM))$ the Riemannian curvature wrt some metric, and $\mathrm{ch}(W)$ is the Chern character $\mathrm{ch}(W) = \mathrm{sTr}(\exp(-F^W))$, F^W the curvature of W.

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M: a Calabi-Yau D-fold,
$T := T^{1,0}M$ the holomorphic tangent bundle of M, $E \longrightarrow M$ a holomorphic bundle,
$\Longrightarrow$ holomorphic Euler characteristic: $\chi(E) = \int_M \mathrm{Td}(M)\,\mathrm{ch}(E)$
with $\mathrm{Td}(M) = \det\left(\frac{R^+}{1-e^{-R^+}}\right)$ the Todd class, R^+ the holomorphic curvature of T

The elliptic genus $\mathcal{E}_M$ of M [Hirzebruch88,Witten88]

$\mathcal{E}_M(\tau,z)$: weight 0 weak Jacobi form in $\tau, z \in \mathbb{C}$ $(\mathrm{Im}(\tau)>0, q=e^{2\pi i\tau})$,

$$\begin{aligned}
\mathcal{E}_M(\tau, z=0) &= \chi(M)\\
\mathcal{E}_M(\tau, z=\tfrac{1}{2}) &= (-1)^{D/2}\sigma(M) + \mathcal{O}(q),\\
q^{D/4}\mathcal{E}_M(\tau, z=\tfrac{\tau+1}{2}) &= (-1)^{D/2}\chi(\mathcal{O}_M) + \mathcal{O}(q)
\end{aligned}$$

$\mathcal{E}_M(\tau,z)$ is a regularization of an equiv. index on $\mathcal{L}M = C^0(\mathbb{S}^1, M)$.

[Landweber-Stong88,Ochanine88;Zagier88,Taubes89]

with $T = T^{1,0}M$ the holomorphic tangent bundle $(y = e^{2\pi iz})$:

$$\int_M \mathrm{Td}(M)\,\mathrm{ch}(\mathbb{E}_{q,-y}) = \mathcal{E}_M(\tau,z)$$

$$\mathbb{E}_{q,-y} := y^{-D/2}\bigotimes_{n=1}^{\infty}\left[\Lambda_{-yq^{n-1}}T^*\otimes\Lambda_{-y^{-1}q^n}T\otimes S_{q^n}T^*\otimes S_{q^n}T\right],$$

$$\Lambda_x E = \bigoplus_{p=0}^{\infty} x^p\Lambda^p E, \qquad S_x E = \bigoplus_{p=0}^{\infty} x^p S^p E,$$

$$\mathrm{ch}(\Lambda_x E) = \sum_{p=0}^{\infty} x^p\,\mathrm{ch}(\Lambda^p E), \qquad \mathrm{ch}(S_x E) = \sum_{p=0}^{\infty} x^p\,\mathrm{ch}(S^p E)$$

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[Zagier88,Taubes89,Eguchi/Ooguri/Taormina/Yang89]

with $T = T^{1,0}M$ the holomorphic tangent bundle ($y = e^{2\pi i z}$):

$$\int_M \mathrm{Td}(M)\,\mathrm{ch}(\mathbb{E}_{q,-y}) = \mathcal{E}_M(\tau,z) = \mathrm{sTr}_{\mathcal{H}_R}\left(y^{J_0} q^{L_0-D/8}\overline{q}^{\overline{L}_0-D/8}\right),$$

$$\mathbb{E}_{q,-y} := y^{-D/2} \bigotimes_{n=1}^{\infty} \left[\Lambda_{-yq^{n-1}}T^* \otimes \Lambda_{-y^{-1}q^n}T \otimes S_{q^n}T^* \otimes S_{q^n}T\right],$$

$\mathcal{H}_R$: Ramond sector of any superconformal field theory associated to M,

$J_0, L_0, \overline{L}_0$: zero modes of the $U(1)$-current and Virasoro fields in the SCA

The elliptic genus of K3

1. From indices to $U(1)$-equivariant loop space indices

[Hirzebruch78] with $c(T) = \prod_{j=1}^{D}(1+x_j)$ (splitting principle):

$$\begin{aligned}
\chi_y(M) &:= \sum_{p,q}(-1)^q y^p h^{p,q} \\
&= \sum_p y^p \sum_q (-1)^q \dim H^q(M, \Lambda^p T^*) \\
&= \sum_p y^p \chi(\Lambda^p T^*) \quad = \int_M \mathrm{Td}(M) \sum_p y^p \,\mathrm{ch}(\Lambda^p T^*) \\
&= \int_M \mathrm{Td}(M)\,\mathrm{ch}(\Lambda_y T^*) \quad = \int_M \prod_{j=1}^{D} x_j \frac{1+ye^{-x_j}}{1-e^{-x_j}}
\end{aligned}$$

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Let $\mathcal{L}M=C^0(\mathbb{S}^1,M)$,

q: a topological generator of $\mathbb{S}^1$; $\mathcal{L}M^{\mathbb{S}^1}=M\hookrightarrow\mathcal{L}M$ (constant loops),

so for $p\in M$: $T_p(\mathcal{L}M)=\mathcal{L}(T_pM)=T_pM\oplus\mathcal{N}$, $\mathcal{N}=\bigoplus_{n\in\mathbb{Z}\setminus\{0\}} q^nT_pM$,

where $q^nT_pM\cong T_pM$: the eigenspace of q_* with eigenvalue q^n, $n\in\mathbb{Z}$,

$$\chi_y(q,\mathcal{L}M) := \int_M \prod_{j=1}^{D}\left\{ x_j\frac{1+ye^{-x_j}}{1-e^{-x_j}} \prod_{n=1}^{\infty}\left[\frac{1+q^nye^{-x_j}}{1-q^ne^{-x_j}}\cdot\frac{1+q^ny^{-1}e^{x_j}}{1-q^ne^{x_j}}\right](-y)^{\zeta(0)}\right\}$$

$$= \int_M \mathrm{Td}(M)\,\mathrm{ch}(\mathbb{E}_{q,y})$$

$$\mathbb{E}_{q,y}=(-y)^{-D/2}\Lambda_yT^*\otimes\bigotimes_{n=1}^{\infty}\left[\Lambda_{yq^n}T^*\otimes\Lambda_{y^{-1}q^n}T\otimes S_{q^n}T^*\otimes S_{q^n}T\right]$$

Definition of the elliptic genus

Theorem **[Hirzebruch88,Witten88,Krichever90, Borisov/Libgober00]**

The elliptic genus

$$\mathcal{E}_M(\tau, z) := \int_M \mathrm{Td}(M)\,\mathrm{ch}(\mathbb{E}_{q,-y}) \quad \left(q=e^{2\pi i\tau},\, y=e^{2\pi iz}\right)$$

of a Calabi-Yau D-fold M is a weak Jacobi form of weight 0 and index $\frac{D}{2}$.

Chiral de Rham complex

Definition [Malikov/Schechtman/Vaintrob99]

Ω^{ch}_M: sheaf of vertex algebras over M

sections over $U \subset M$ with holomorphic coordinates $z_1, \ldots, z_D$:
vertex algebra generated by fields ϕ^j, p^j, ψ_j, ρ_j, $j \in \{1, \ldots, D\}$,
where $\phi^j \leftrightarrow z_j$, $p_j \leftrightarrow \frac{\partial}{\partial z_j}$, $\psi^j \leftrightarrow dz_j$, $\rho_j \leftrightarrow \frac{\partial}{\partial(dz_j)}$

ϕ^j, p^j, ψ_j, $\rho_j \in \mathrm{End}(\mathcal{H})[[x, x^{-1}]]$ with

$$\forall\, i,j,m,n\colon \quad \phi^j(x) = \sum_n \phi^j_n x^{-n}, \quad p^j(x) = \sum_n p_{j,n} x^{-n-1},$$
$$\psi^j(x) = \sum_n \psi^j_n x^{-n}, \quad \rho_j(x) = \sum_n \rho_{j,n} x^{-n-1},$$

where $[\phi^i_n, p_{j,m}] = \delta^i_j \delta_{n,-m}$, $\{\psi^i_n, \rho_{j,m}\} = \delta^i_j \delta_{n,-m}$ $\forall\, m,n \in \mathbb{Z}$;

$\mathcal{H}$: the Fock space built on $|0\rangle$ from ϕ^j_n, $p_{j,m}$, ψ^j_n, $\rho_{j,m}$, $n, m \in \mathbb{Z}$,

with $\phi^j_n|0\rangle = \psi^j_n|0\rangle = 0 \;\forall\, n > 0$ and $p_{j,m}|0\rangle = \rho_{j,m}|0\rangle = 0 \;\forall\, m \geq 0$

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Theorem **[Malikov/Schechtman/Vaintrob99;Borisov/Libgober00]**
There are globally well-defined fields on M,

$$L^{top} = -:p_j\partial\phi^j: - :\rho_j\partial\psi^j:, \; J = :\rho_j\psi^j:, Q = -:\psi^j p_j:, \; G = :\rho_j\partial\phi^j:,$$

which yield a (topological) $N=2$ superconformal algebra.
The elliptic genus $\mathcal{E}_M(\tau, z)$ is the bigraded Euler characteristic of Ω_M^{ch},
and $H^*(M, \Omega_M^{ch})$ is a topological $N=2$ superconformal vertex algebra.

...and topologically half twisted sigma model

Theorem [Kapustin05]

There is a fine resolution $\Omega_M^{ch,Dol}$ of Ω_M^{ch} obtained by introducing variables $\phi^{\bar{\jmath}}$, $\psi^{\bar{\jmath}}$,

where $\psi^{\bar{\jmath}}$ determines the grading and $d_{Dol} := \psi^{\bar{\jmath}} \frac{\partial}{\partial \phi^{\bar{\jmath}}}$,

such that

$$\mathcal{E}_M(\tau, z) = \mathrm{sTr}_{H^*(\Omega_M^{ch,Dol})} \left(y^{J_0 - D/2} q^{L_0^{top}} \right).$$

The d_{Dol}-cohomology $H^*(\Omega_M^{ch,Dol})$ is the large volume limit of the BRST-cohomology $\mathcal{H}_{NS}^{BRST}$ of Witten's half-twisted σ-model on M.

Conclusion:

$$\mathcal{E}_M(\tau, z) \overset{\text{spectral flow}}{=} \mathrm{sTr}_{\mathcal{H}_R^{BRST}} \left(y^{J_0} q^{L_0 - D/8} \right)$$

$$= \mathrm{sTr}_{\mathcal{H}_R} \left(y^{J_0} q^{L_0 - D/8} \overline{q}^{\overline{L}_0 - D/8} \right) = \mathcal{E}_{CFT}(\tau, z).$$

3. The elliptic genus of K3

For every K3 surface M,

$$\mathcal{E}_{K3}(\tau, z) = 8\left(\frac{\vartheta_2(\tau,z)}{\vartheta_2(\tau,0)}\right)^2 + 8\left(\frac{\vartheta_3(\tau,z)}{\vartheta_3(\tau,0)}\right)^2 + 8\left(\frac{\vartheta_4(\tau,z)}{\vartheta_4(\tau,0)}\right)^2.$$

For every $N = (2,2)$ SCFT at central charges $c = \overline{c} = 6$ with space-time SUSY and integral $U(1)$ charges:
The theory has $N = (4,4)$ SUSY, and its CFT elliptic genus either vanishes, or it agrees with $\mathcal{E}_{K3}(\tau, z)$.

Definition (K3 THEORY):
An $N = (2,2)$ SCFT at $c = \overline{c} = 6$ with space-time SUSY, integral $U(1)$ charges and CFT elliptic genus $\mathcal{E}_{K3}(\tau, z)$.

Decomposition into irreducible $N = 4$ characters

3 types of $N = 4$ irreps $\mathcal{H}_\bullet$ with $\chi_\bullet(\tau, z) = \mathrm{sTr}_{\mathcal{H}_\bullet}\left(y^{J_0} q^{L_0 - 1/4}\right)$:

- vacuum $\mathcal{H}_0$ with $\chi_0(\tau, 0) = -2$
- massless matter $\mathcal{H}_{1/2}$ with $\chi_{1/2}(\tau, 0) = 1$
- massive matter $\mathcal{H}_h$ $(h \in \mathbb{R}_{>0})$, $\chi_h(\tau, z) = q^h \widetilde{\chi}(\tau, z)$, $\chi_h(\tau, 0) = 0$

Ansatz:

$$\begin{aligned}\mathcal{H}_R \;=\;& \mathcal{H}_0 \otimes \overline{\mathcal{H}}_0 \,\oplus\, 20\, \mathcal{H}_{1/2} \otimes \overline{\mathcal{H}}_{1/2} \\ &\oplus \left(\bigoplus_{0<n\in\mathbb{N}} \left[f_n \mathcal{H}_n \otimes \overline{\mathcal{H}}_0 \oplus \overline{f}_n \mathcal{H}_0 \otimes \overline{\mathcal{H}}_n\right]\right) \\ &\oplus \left(\bigoplus_{0<m\in\mathbb{N}} \left[g_m \mathcal{H}_m \otimes \overline{\mathcal{H}}_{1/2} \oplus \overline{g}_m \mathcal{H}_{1/2} \otimes \overline{\mathcal{H}}_m\right]\right) \\ &\quad \oplus \bigoplus_{0<h,\overline{h}\in\mathbb{R}} k_{h,\overline{h}} \mathcal{H}_h \otimes \overline{\mathcal{H}}_{\overline{h}}\end{aligned}$$

where all f_n, $\overline{f}_n$, g_m, $\overline{g}_m$, $k_{h,\overline{h}}$ are non-negative integers.

$$\Longrightarrow \quad \begin{aligned}\mathcal{E}_{K3}(\tau, z) &= -2\chi_0(\tau, z) + 20\chi_{1/2}(\tau, z) + 2e(\tau)\widetilde{\chi}(\tau, z), \\ 2e(\tau) &= \sum_{n=1}^{\infty} (g_n - 2f_n) q^n\end{aligned}$$

Conjecture [Ooguri89,W00]

$g_n - 2f_n \geq 0$ for all $n \in \mathbb{N}$ proved in **[Eguchi/Hikami09]**.

4. Some conjectures

Conjecture **[Eguchi/Ooguri/Tachikawa10]**
For all n, $g_n - 2f_n$ gives the dimension of a non-trivial representation of the Mathieu group M_{24}.
Proved in **[Gannon12]**, using results of **Cheng, Duncan, Gaberdiel, Hohenegger, Persson, Ronellenfitsch, Volpato**.

4. Some conjectures

Theorem [Eguchi/Ooguri/Tachikawa10,Gannon12]

There exists a representation $\mathcal{R}_n$ of M_{24} for every $n \in \mathbb{N}$, such that

$$\lim_{vol\to\infty}(\mathcal{H}_R^{BRST}) \cong (-2)\mathcal{H}_0 \oplus 20\,\mathcal{H}_{1/2} \oplus \bigoplus_{n=1}^{\infty}(\mathcal{R}_n \oplus \overline{\mathcal{R}}_n) \otimes \mathcal{H}_n$$

as a representation of M_{24} and of the N=4 superconformal algebra.

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WHY?

Theorem [Mukai88]

If G is a symmetry group of a K3 surface M,

> that is, G fixes the two-forms that define the hyperkähler structure of M,

then G is isomorphic to a subgroup of the Mathieu group M_{24}, and $|G| \leq 960 \ll 244.823.040 = |M_{24}|$.

Some open conjectures

Observation **[Taormina/W10-13]**
The map $\mathcal{H}_R \twoheadrightarrow \lim_{vol\to\infty}(\mathcal{H}_R^{BRST})$ depends on the choice of a geometric interpretation; SO: restrict to geometric symmetry groups.

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as a representation of the geometric symmetry group $G \subset M_{24}$; the rhs collects the symmetries from distinct points of the moduli space.

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Evidence [Taormina/W13]
$\mathcal{R}_1$ as common representation space of all geometric symmetry groups of Kummer K3s yields an action of the maximal subgroup $\mathbb{Z}_2^4 \rtimes A_8 \subset M_{24}$ induced from an irrep of M_{24} on $\mathcal{R}_1$.

A simpler open conjecture

Recall:

$$\begin{aligned}\mathcal{E}_{\mathrm{K3}}(\tau, z) &= \int_{\mathrm{K3}} \mathrm{Td}(\mathrm{K3})\mathrm{ch}(\mathbb{E}_{q,-y}) \\ &= -2\chi_0(\tau, z) + 20\chi_{1/2}(\tau, z) + \sum_{n=1}^{\infty} (g_n - 2f_n)\chi_n(\tau, z)\end{aligned}$$

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Conjecture [W13]

There are polynomials p_n for every $n \in \mathbb{N}$, such that

$$\mathbb{E}_{q,-y} = -\mathcal{O}_{\mathrm{K3}}\chi_0(\tau,z) - T\chi_{1/2}(\tau,z) + \sum_{n=1}^{\infty} p_n(T)\chi_n(\tau,z),$$

where $2\dim(\mathcal{R}_n) = g_n - 2f_n = \int_{\mathrm{K3}} \mathrm{Td}(\mathrm{K3}) p_n(T)$ for all $n \in \mathbb{N}$.
Moreover, $p_n(T) \twoheadrightarrow \mathcal{R}_n \oplus \overline{\mathcal{R}}_n$ carries a natural action of every geometric symmetry group $G \subset M_{24}$ of K3.

The End

**Thank you
for your attention!**