Erratum to "Arithmetizing classes around NC^1 and L"

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Status

Membership in regular languages is known to be complete for NC^1, due to Barrington's result [1] about bounded width branching programs. For VPAs, which are provably more powerful than NFAs, Dymond showed membership to be in NC^1 in [2]. In [3], we claimed that an arithmetic analogue of this also holds: counting accepting paths in VPAs is no harder than counting the same in NFAs. Unfortunately, the proof of this claim is incorrect. Thus, as of now, we do not know where exactly #VPA lies, in the range

$$\#\text{NFA} \subseteq \#\text{NC}^1 \subseteq \text{FL} \subseteq \#\text{L} \subseteq \#\text{LogCFL}$$

It could well be complete for #LogCFL, though such a completeness would have to be under Turing reductions since an equal-to-0 test is easier for VPAs than for PDAs.

Why the proof in [3], claiming that #BP-VPA is in #BP-NFA, is wrong

Consider the following VPA: $M = (Q, \Delta, Q_{in}, \Gamma, \delta, Q_F)$ where

- $Q = \{q_0, q_1, q_2, q_3, q_4\}$ with $Q_{in} = \{q_0\}$, $Q_F = \{q_4\}$.
- $\Delta = (\Delta_c, \Delta_r, \Delta_{int})$ with $\Delta_c = \{a\}$, $\Delta_{int} = \{b\}$, $\Delta_r = \{c\}$,
- $\Gamma = \{X, Y, Z\}$
- δ contains the following transitions:
  $\delta(q_0, a) = \{(q_1, X), (q_1, Y)\}$
  $\delta(q_1, a) = \{(q_2, Z)\}$
  $\delta(q_2, b) = \{q_2\}$
  $\delta(q_2, c, Z) = \{q_3\}$
  $\delta(q_3, c, X) = \{q_4\}$
  $\delta(q_3, c, Y) = \{q_4\}$

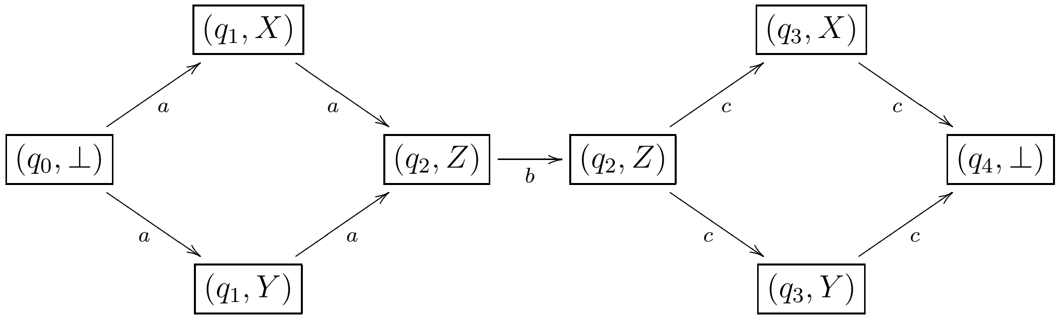
On input $aabcc$, this VPA has 2 paths:

$$(q_0, \perp) \to_a (q_1, X) \to_a (q_2, ZX) \to_b (q_2, ZX) \to_c (q_3, X) \to_c (q_4, \perp)$$

and

$$(q_0, \perp) \to_a (q_1, Y) \to_a (q_2, ZY) \to_b (q_2, ZY) \to_c (q_3, Y) \to_c (q_4, \perp)$$

The relevant portion of the BWBP constructed according to the prescription in [3] is shown below. As can be seen, it has 4 paths, of which 2 do not correspond to any computation path of the VPA.



References

[1] D. Barrington. Bounded-width polynomial-size branching programs recognize exactly those languages in NC^1. *JCSS*, 38(1):150–164, 1989.

[2] P. W. Dymond. Input-driven languages are in $\log n$ depth. *Information Processing Letters*, 26:247–250, 1988.

[3] N. Limaye, M. Mahajan, and B. V. R. Rao. Arithmetizing classes arround NC^1 and L. In *Proc. 24th STACS*, pages 477–488, 2007.