Arthur and Merlin as Oracles

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Abstract

We study some problems solvable in polynomial time, given oracle access to the promise versions of the Arthur-Merlin classes. We show that

- $\mathrm{S}_2^p \subseteq \mathrm{P}^{\mathrm{prAM}}$
- $\mathrm{BPP}_{\|}^{\mathrm{NP}} \subseteq \mathrm{P}_{\|}^{\mathrm{prAM}}$

Then, as a direct consequence of the Miltersen-Vinodchandran result [FOCS'99], we get that $\mathrm{S}_2^p = \mathrm{P}^{\mathrm{NP}}$ and $\mathrm{BPP}_{\|}^{\mathrm{NP}} = \mathrm{P}_{\|}^{\mathrm{NP}}$, under the hardness hypothesis used for derandomizing AM. This gives an alternative (and perhaps, a more direct) proof of the same result obtained by Shaltiel and Umans [CCC'05].

We also derive some corollaries related to the Karp–Lipton theorem and learning circuits for SAT. Finally, we shall briefly discuss a $\mathrm{P}^{\mathrm{prAM}}$ algorithm for finding near-optimal strategies of succinctly presented zero-sum games; Fortnow et. al [CCC'05] described a $\mathrm{ZPP}^{\mathrm{NP}}$ algorithm for the same problem.