On tree 3-spanners in 2-sep directed path graphs

Abstract

A *tree t-spanner* T of a connected graph G is a spanning tree of G such that the distance between any two vertices in T is at most t times their distance in G. A graph that has a tree t-spanner is called a *tree t-spanner admissible graph*. While the problem of recognizing whether a graph is tree t-spanner admissible is NP-complete for any fixed $t \geq 4$ and is polynomially solvable for $t \leq 2$, the case $t = 3$ is still open despite lot of efforts from various researchers over last one decade.

Given an integer $t \geq 1$, the problems associated with tree t-spanner are:

(1) **Characterization:** Characterize tree t-spanner admissible graphs.

(2) **Recognition:** Recognize whether a given graph is tree t-spanner admissible.

(3) **Construction:** Construct a tree t-spanner of a given tree t-spanner admissible graph.

Though the above three problems concerning tree 3-spanner are open for general graphs, it has been shown by various researchers that these problems can be solved in polynomial time for various graph classes with special structures. However, most of the graph classes for which the tree 3-spanner problems have been solved admit tree 3-spanners, i.e., every graph in the graph class admits a tree 3-spanner. So, problem (1) and problem (2) turn out to be trivial within these graph classes.

In this talk we will discuss the above three problems for $t = 3$ in a special class of graphs called 2-*sep directed path graphs*. First we see that 2-sep directed path graphs need not admit tree 3-spanner in general. So, there is a need for characterization of tree 3-spanner admissible 2-sep directed path graphs. First we will present a characterization of tree 3-spanner admissible 2-sep directed path graphs. Then, a linear time algorithm to recognize such graphs will be discussed. Also, we will discuss a linear time algorithm to construct a tree 3-spanner of a tree 3-spanner admissible 2-sep directed path graph.