The Kroblem

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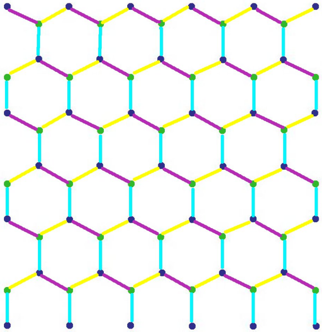
Topological order in $2-d$ Tensor Network States

- Given a $2+1$ Toplological Field Theory (TFT), construct a class of Tensor Network States that realises it in the long wavelength limit.
- Given tensor network states in $2-d$, for the ground state and low lying excitations, what is the field theory that describes their long wavelength correlations.

The Kitaev Model

Alexei Kitaev,“Anyons in an exactly solved model and beyond", cond-mat/0506438, Annals of Physics.



$$H = J_x \sum_{<ij>} \sigma_i^x \sigma_j^x + J_y \sum_{<ij>} \sigma_i^y \sigma_j^y + J_z \sum_{<ij>} \sigma_i^z \sigma_j^z$$

The Kitaev Honeycomb Model

- Simple exact solution. Same degree of difficulty as 1-d transverse field Ising model.
- Solution is a spin liquid with topological order, spinons and non-abelian anyons.

Open problem: Open problem:
What is the ground state wave function in the spin basis ?

$$|GS\rangle = \sum_{x_1,\ldots,x_N} \Psi(x_1, x_2, \ldots, x_N)|x_1, x_2, \ldots, x_N\rangle$$

Is there a nice tensor network representation (TNR)?

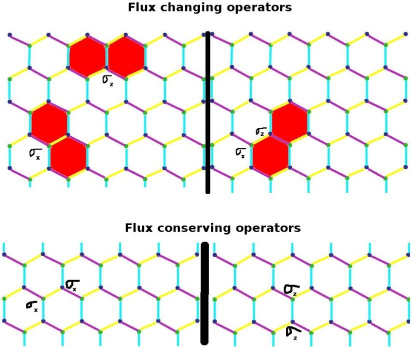
Evidence for TNR

- At $J_z \to \infty$, the model maps on to the Toric Code Model which has a TNR.
- At $J_z = 0$, the model maps on to decoupled transverse-field Ising chains which have TNR.

The flux basis

$$W_p|\{n_p\},\alpha\rangle = (1-2n_p)|\{n_p\},\alpha\rangle, \quad n_p = 0,1$$

$$\langle\{n'_p\},\alpha|\{n_p\},\beta\rangle = \left(\prod_p \delta_{n_p n'_p}\right) \langle\{n_p\},\alpha|\{n_p\},\beta\rangle$$



2-spin correlations

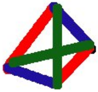
Only non-zero 2-spin correlators:

$$\langle\{n_p\}|\sigma_i^x\sigma_{i+\hat{x}}^x|\{n_p\}\rangle,\ \langle\{n_p\}|\sigma_i^y\sigma_{i+\hat{y}}^y|\{n_p\}\rangle,\ \langle\{n_p\}|\sigma_i^z\sigma_{i+\hat{z}}^z|\{n_p\}\rangle$$

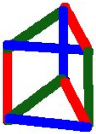
G. Baskaran, Saptarshi Mandal and R. Shankar, PRL **98**, 247201 (2007)

- This form is true for every state in the flux basis, for all values of S
- When the hamiltonian commutes with W_p for all p, then above form is true for all simultaneous eigenstates, independent of the energy.
- Can we write TNS which reproduces these correlations ?

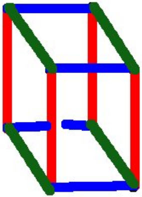
Easier problems: Klusters



Kitrahedron



Krism



Kube