Multipliers for the Weyl Transform

by

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Abstract

The purpose of this paper is to discuss the multipliers for the Weyl transform, which is related to the Schrödinger representation of the Heisenberg group, and its connection with the special Hermite operator.

1 Introduction

The Hermite functions $\tilde{h}_k$ on the real line are defined by

$$\tilde{h}_k(x) = H_k(x)e^{-\frac{1}{2}x^2}, k = 0, 1, 2, \ldots.$$

where $H_k(x)$ denotes the Hermite polynomial. These are eigen functions of the Hermite operator (harmonic oscillator) $-\Delta + x^2$ with the eigen values $2k + 1$. The normalised Hermite functions $h_k(x)$ are defined by

$$h_k(x) = (2^k k! \sqrt{\pi})^{-\frac{1}{2}} \tilde{h}_k(x)$$

which form a complete orthonormal family in $L^2(\mathbb{R}, dx)$. These Hermite functions are also eigen functions of the Fourier transform:

$$\hat{h_k}(\xi) = (-i)^k h_k(\xi).$$

Let μ be a multiindex and $x \in \mathbb{R}^n$. Then the n-dimensional Hermite functions $\Phi_\mu(x)$ are defined by

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taking the product of the one dimensional Hermite functions $h_{\mu_j}(x_j)$:

$$\Phi_\mu(x) = \prod_{j=1}^{n} h_{\mu_j}(x_j).$$

Then they form a complete orthonormal system for $L^2(\mathbb{R}^n, dx)$ and $H = -\Delta + |x|^2$ is the Hermite operator on $\mathbb{R}^n$.

The Laguerre polynomials may be regarded as generalizations of the Hermite polynomials in the sense that Laguerre polynomials of order $-\frac{1}{2}$ give the Hermite polynomials of even degree and those of order $\frac{1}{2}$ give the Hermite polynomials of odd degree.

The special Hermite functions, which occupy a central place in the study of Hermite and Laguerre expansions, are defined by

$$\Phi_{\mu\nu}(x) = (2\pi)^{-\frac{n}{2}} \int_{\mathbb{R}^n} e^{ix\xi} \phi_\mu(\xi + \frac{1}{2}y)\phi_\nu(\xi - \frac{1}{2}y) d\xi.$$

These are called special Hermite functions by Strichartz for the following reason: These are actually Hermite functions on $\mathbb{C}^n$ viewed as $\mathbb{R}^{2n}$ because

$$(-\Delta + \frac{1}{4}|z|^2)\Phi_{\mu\nu} = (|\mu| + |\nu| + n)\Phi_{\mu\nu}$$

but they do not give all Hermite functions on $\mathbb{C}^n$. These functions appear as the entry functions of the Schrödinger representation of the Heisenberg group (see Strichartz [9]). They form a complete orthonormal system in $L^2(\mathbb{C}^n)$. Let

$$L = -\Delta_z + \frac{1}{4}|z|^2 - iN$$

where

$$N = \sum_{j=1}^{n} \left(x_j \frac{\partial}{\partial y_j} - y_j \frac{\partial}{\partial x_j} \right).$$

Then $\phi_{\mu\nu}$ are eigen functions of L, with eigenvalue $(2|\nu| + n)$ and hence L is called the special Hermite operator.

Analysis on the Heisenberg group and expansions in terms of Hermite and Laguerre functions are interrelated. Infact, the first summability theorem for multiple Hermite expansions was deduced from the corresponding result on the Heisenberg group. A multiplier theorem for Laguerre expansions was likewise proved using a multiplier theorem on the Heisenberg group. The first multiplier theorem for Hermite expansions followed from considerations of the Weyl transform which is related to the Schrödinger representation of the Heisenberg group.

The Weyl transform W takes functions on $\mathbb{C}^n$ into bounded operators on $L^2(\mathbb{R}^n)$. As W enjoys many properties of the Fourier transform and is closely related to expansions in terms of Laguerre, Hermite and special Hermite expansions, it will be interesting to study multipliers for the Weyl transform analogous to Fourier multipliers.

We organise this paper as follows. In section 2, we give the basic concept of Fourier multipliers for L^p spaces. In section 3, we define Weyl transform and twisted convolution using the schrödinger representation of the Heisenberg group. In section 4, we discuss the Weyl multipliers for $L^p(\mathbb{C}^n)$. In section 5, we state two types of Weyl multiplier problems (without proofs) using special Hermite Operator.

2 Multipliers for the Fourier transform

Let m be a bounded measurable functions on $\mathbb{R}^n$. Define a linear transformation T_m on $L^2 \cap L^p(\mathbb{R}^n)$ by $\widehat{T_m f} = m\hat{f}$ for $f \in L^2 \cap L^p(\mathbb{R}^n)$. We say that m is a multiplier for $L^p \quad (1 \leq p < \infty)$ if whenever $f \in L^2 \cap L^p$, then $T_m f$ is also in L^p and T_m is bounded i.e.

$$\|T_m f\|_p \leq C\|f\|_p \qquad \forall\ f \in L^2 \cap L^p. \tag{1}$$

The smallest constant C for which (1) holds is called the norm of the multiplier. Clearly, the above equation shows that T_m has a unique bounded extension to L^p, which again satisfies the same inequality. We denote this extension also by T_m. Observe that the operator T_m commutes with translations. And for translation invariant operators, we have the following results.

Proposition 2.1 *Let T be a bounded linear transformation mapping $L^1(\mathbb{R}^n)$ to itself. Then a necessary and sufficient condition that T commutes with translations is that there exists a finite Borel measure μ on $\mathbb{R}^n$ such that $Tf = \mu * f$ for every $f \in L^1(\mathbb{R}^n)$ and $\|T\| = \|\mu\|$.*

Proposition 2.2 *Let T be a bounded linear transformation mapping $L^2(\mathbb{R}^n)$ to itself. Then a necessary and sufficient condition that T commutes with translations is that there exists a bounded measurable function ϕ such that $\widehat{Tf} = \phi\hat{f}$ for every $f \in L^2(\mathbb{R}^n)$ and $\|T\| = \|\phi\|$.*

Let $m(L^p)$ denote the class of L^p multipliers with the multiplier norm.Then $m(L^1)$ is the class of Fourier transforms of finite Borel measures on $\mathbb{R}^n$ $(M(\mathbb{R}^n))$ and the norm of $m(L^1)$ is identical with that of $M(\mathbb{R}^n)$. $m(L^2)$ is the class of all bounded measurable functions and multiplier norm is identical with the $L^\infty(\mathbb{R}^n)$ - norm.

Suppose that T is a bounded linear transformation on $L^p(\mathbb{R}^n), 1 \leq p < \infty$, which commutes with translations. Then, it is easy to see that

$$T(f * g) = Tf * g = f * Tg \quad \forall\ f \in C_c(\mathbb{R}^n).$$

Hence

$$\int_{\mathbb{R}^n} Tf(x)\tilde{g}(x)dx = \int_{\mathbb{R}^n} f(x)(\tilde{Tg})(x)$$

where $\tilde{g}(x) = g(-x)$.
Then, by usual duality argument, one can show that T is bounded on $L^{p'}, \frac{1}{p} + \frac{1}{p'} = 1$. It then follows from Marcinkewicz interpolation theorem that T is bounded on L^2 . Hence there exists a bounded measurable function m such that $\widehat{Tf} = m\hat{f} = \widehat{T_m f}$. By uniqueness of Fourier transform, we get $T = T_m$. Thus, on $L^p(\mathbb{R}^n)$, the concept of multipliers coincide with the study of bounded linear translation invariant operators.

Initially, no simple characterisation was known for $m(L^p)$ except for $p = 1$ and $p = 2$. However, there is an important sufficient conditions for L^p multipliers, $1 < p < \infty$:

Theorem 2.1 *Suppose that $m(x)$ is of class C^k in the complement of the origin of $\mathbb{R}^n$, where k is an integer greater than $\frac{n}{2}$. Suppose $m(x)$ satisfies*

$$|\frac{\partial^\alpha}{\partial x^\alpha} m(x)| \leq B|x|^{-\alpha} \text{ whenever } |\alpha| \leq k$$

where $\alpha = (\alpha_1, \dots \alpha_n)$ with $|\alpha| = \alpha_1 + \alpha_2 + \dots + \alpha_n$. Then $m \in m(L^p)$ for $1 < p < \infty$.

The proof is based on Littlewood-Paley theory. For a detailed study on Fourier multipliers on $L^p(\mathbb{R}^n)$, we refer to Hormander [4] or Stein [8].

Later, a necessary condition, namely, if m is a multiplier for $L^p(\mathbb{R}^n)$, then there exists a pseudomeasure σ such that $T_m f = \sigma * f$, was also obtained. Infact, this result was proved for a general locally compact abelian group G in place of $\mathbb{R}^n$. This is based on the works of Hormander [4] and Gaudry [3]. Infact, Gaudry in [3] obtained a representation theorem through quasi measures for an (L^p, L^q) multiplier. This idea motivates to obtain such type of representation theorems for various types of multipliers. To cite a few, Unni in [12] obtained the result for the multipliers between any two Segal algebras, Chan in [1] for the Banach valued multipliers $m(L^1(G, A))$, Radha in [6] for $m(L^1(G, A), L^p(G, A)), 1 < p < \infty$.

3 Weyl transform and twisted convolution

Consider the Heisenberg group H^n, which is a nilpotent Lie group, whose underlying manifold is $\mathbb{C}^n \times \mathbb{R}$ where the group operation is defined by

$$(z, t)(w, s) = (z + w, t + s + \frac{1}{2}\text{Im} z\overline{w})$$

and the Haar measure is the Lebesgue measure $dzdt$ on $\mathbb{C}^n \times \mathbb{R}$. The corresponding Lie algebra is generated by the $(2n+1)$ left invariant vector fields

$$X_j = \frac{\partial}{\partial x_j} - \frac{1}{2} y_j \frac{\partial}{\partial t}$$

$$Y_j = \frac{\partial}{\partial y_j} + \frac{1}{2} x_j \frac{\partial}{\partial t}$$

$$T = i \frac{\partial}{\partial T}$$

$j = 1, 2, ...n.$

By stone-Von Neumann theorem, every infinite dimensional irreducible unitary representation on the Heisenberg group is unitairly equivalent to the representation $\pi_\lambda, \lambda \in \mathbb{R} - \{0\}$, where π_λ is defined by

$$\pi_\lambda(z,t)\phi(\xi) = e^{i\lambda t} e^{i\lambda(x\xi + \frac{1}{2}xy)} \phi(\xi + y) \quad (z = x + iy)$$

for each $\phi \in L^2(\mathbb{R}^n)$. This is called Schrödinger representation of H^n with parameter λ. (Infact, there are only two types of representations on H^n, infinite dimensional and one dimensional. The one dimensional representations are not important to us in our discussions).

To define the Weyl transform and twisted convolution, we consider the representation π_1 of H^n. Notice that π_1 is not a faithful representation. The kernel of π_1 is the subgroup

$$K = \{(0,t)/ \ t \in \mathbb{Z}\}$$

For some purposes, it is better to throw away this kernel, so we define the reduced Heisenberg group H^n_{red} to be $H^n_{red} = H^n/K$. We still denote the elements of H^n_{red} by (z,t) with the understanding that $0 \leq t < 2\pi$. Then π_1 is a representation of H^n_{red}, which is now faithful. Since the central variable t always acts in a simple minded way, as multiplication by the scalar e^{it}, it is convenient to disregard it entirely. Therefore we define $W(z) = \pi_1(z,0)$.

The unitary representation π_1 of H^n_{red} determines a representation of the convolution algebra $L^1(H^n_{red})$, still denoted by π_1 in the usual way: if $f \in L^1(H^n_{red})$, then

$$\begin{aligned} \pi_1(f) &= \int_{H^n_{red}} f(z,t)\pi_1(z,t)dzdt \\ &= \int_{\mathbb{C}^n} \int_o^{2\pi} f(z,t)e^{it}W(z)dzdt \\ &= \sqrt{2\pi}\int_{\mathbb{C}^n} f_1(z)W(z)dz, \end{aligned}$$

where $f_1(z) = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} f(z,t)e^{it}dt$.
For details regarding representation theory of Heisenberg group, we refer to Folland [2].

Given a function f in $L^1(\mathbb{C}^n)$, the function $f^\#(z,t) = (2\pi)^{-1}e^{-it}f(z)$ belongs to $L^1(H^n_{red})$ and

$$\pi_1(f^\#) = \int_{\mathbb{C}^n} f(z)W(z)dz.$$

This transform which takes f into the operator $W(f) = \pi_1(f^\#)$ is called the Weyl transform.

The group convolution on H^n_{red} can be transferred to $\mathbb{C}^n$ as a non standard convolution. Given two function f, g in $L^1(\mathbb{C}^n)$, consider

$$\begin{aligned} f^\# * g^\#(z,t) &= \int_{H^n_{red}} f^\#(z-w, t-s-\frac{1}{2}\mathrm{Im} z\overline{w})g^\#(w,s)dwds \\ &= (2\pi)^{-1}\left(\int_{\mathbb{C}^n} f(z-w)g(w)e^{\frac{i}{2}\mathrm{Im} z\overline{w}}dw\right)e^{-it}. \end{aligned}$$

If we now define $f \times g$ by

$$f \times g(z) = \int_{\mathbb{C}^n} f(z-w)g(w)e^{\frac{i}{2}\mathrm{Im} z\overline{w}}dw,$$

then we have the relation $f^{\#} * g^{\#} = (f * g)^{\#}$. This convolution is called the twisted convolution and $L^1(\mathbb{C}^n)$ becomes a non commutative algebra under this.

The Weyl transform $W(f)$ enjoys many properties of the Fourier transform. For example, we have an analogue of the Fourier inversion formula

$$f(z) = (2\pi)^{-n}\mathrm{tr}(W(z)^* W(f))$$

and a Plancheral formula

$$\|f\|_2^2 = (2\pi)^{-n}\|W(f)\|_{HS}^2$$

for $f \in L^2(\mathbb{C}^n)$, where $W(z)^*$ denotes the adjoint of $W(z)$ and $W(f \times g) = W(f)W(g)$. Like ordinary convolution, twisted convolution extends from $L^1(\mathbb{C}^n)$ to other $L^p(\mathbb{C}^n)$ and satisfies the Young's inequality

$$\|f \times g\|_r \leq \|f\|_p \|g\|_q, \frac{1}{r} = \frac{1}{p} + \frac{1}{q} - 1.$$

We state here that twisted convolution has better behaviour in some contexts. For example, for $f, g \in L^2(\mathbb{C}^n), f \times g$ will also be in $L^2(\mathbb{C}^n)$ and

$$\|f \times g\|_2 \leq (2\pi)^{\frac{n}{2}}\|f\|_2\|g\|_2,$$

and the equations $f \times g = 0$ and $f \times f = f$ do have non trivial solution, given by special Hermite functions. For details of the above, we refer to Thangavelu [11].

4 Weyl multipliers for $L^p(\mathbb{C}^n)$

A bounded operator $M \in B(L^2(\mathbb{R}^n))$ is called a (left) Weyl multiplier for $L^p(\mathbb{C}^n)$ if the operator T_M defined on $f \in L^1 \cap L^p(\mathbb{C}^n)$ by $W(T_M f) = MW(f)$ extends to a bounded operator on $L^p(\mathbb{C}^n)$. We denote the Weyl multiplier class by m_W. As in the case of Fourier multipliers, $m_W(L^1(\mathbb{C}^n))$ is identified with $M(\mathbb{C}^n)$, the Banach algebra of finite Borel measures on $\mathbb{C}^n$. And $m_W(L^2(\mathbb{C}^n))$ is identified with $B(L^2(\mathbb{R}^n))$ of all bounded operators on $L^2(\mathbb{R}^n)$.

A Hormander type multiplier theorem, which gives sufficient conditions for L^p - multipliers for the Weyl transform, was proved by Mauceri in [5]. Let $A_j = \frac{-\partial}{\partial x_j} + x_j, A_j^* = \frac{\partial}{\partial x_j} + x_j$ be the "creation" and "annihilation" operators considered as unbounded operators on $L^2(\mathbb{R}^n)$ defined on their natural domains.

Let

$$\delta_j S = [A_j^*, S], \quad \overline{\delta}_j S = [S, A_j]$$

for every operator $S \in B(L^2(\mathbb{R}^n))$ whose range is contained in the domain of $A_j, A_j^*, j = 1, 2, \dots n$. Given an operator $S \in B(L^2(\mathbb{R}^n))$, we say that S is of class C^m if $\delta^\alpha \delta^\beta S$ is in $B(L^2(\mathbb{R}^n))$ for all $\alpha, \beta \in N^n$ such that $|\alpha| + |\beta| \leq m$. Let φ_α be the n-dimensional normalised Hermite functions. Let P_N denote the orthogonal projection on the subspace of $L^2(\mathbb{R}^n)$ generated by $\{\varphi_\alpha / |\alpha| = N\}$. For every $k \in N$, let ψ_k be the dyadic projection defined by $\psi_0 = P_0$ and $\psi_k = \sum_N P_N$ for $2^{k-1} \leq N < 2^k, k > 0$. Then we have

Theorem 4.1 [5] *Suppose that $M \in m_W(L^2(\mathbb{C}^n))$ is of class C^{n+1}. Assume also that*

$$\mathrm{Sup}_{k \in \mathbb{N}} 2^{k(|\alpha|+|\beta|-n)} \| \ (\delta^\alpha \overline{\delta}^\beta M) \psi_k \|_{HS}^2 \leq C \tag{1}$$

whenever $|\alpha| + |\beta| \leq n+1$. Then $M \in m_W(L^p(\mathbb{C}^n)), 1 < p < 2$.
If assumption (1) is replaced by

$$\mathrm{Sup}_{k \in \mathbb{N}} 2^{k(|\alpha|+|\beta|-n)} \| \psi_k (\delta^\alpha \overline{\delta}^\beta M) \|_{HS}^2 \leq C$$

then $M \in m_W(L^p(\mathbb{C}^n)), 2 \leq p < \infty$.

However, for necessary condition for L^p - multipliers, (for $p \neq 1, p \neq 2$), only the following was known.

Proposition 4.1 *Let $M \in m_W(L^p(\mathbb{C}^n)), 1 \leq p < \infty$. Then there exists a tempered distribution $\rho \in S'(\mathbb{C}^n)$ such that $T_M f = \rho \times f$ for every $f \in S(\mathbb{C}^n)$.*

Recently, Radha and Thangavelu in [7] obtained a concrete characterisation for $m \in m_W(L^p(\mathbb{C}^n)), 1 < p < \infty$. Let $A(\mathbb{C}^n)$ denote the space of functions f on $\mathbb{C}^n$ whose Weyl transforms $W(f)$ are in $B_1(L^2(\mathbb{R}^n))$. Then $A(\mathbb{C}^n)$ is a Banach algebra under $\|\cdot\|_A$ given by

$$\|f\|_A = \|W(f)\|_1 \quad \forall f \in A(\mathbb{C}^n)$$

and a multiplication as twisted convolution. Let $P(\mathbb{C}^n)$ denote the dual space of $A(\mathbb{C}^n)$. Then we have

Theorem 4.2 [7] *Let $M \in m_W(L^p(\mathbb{C}^n))$, $1 < p < \infty$. Then there exists a pseudomeasure σ such that $T_M f = \sigma \times f$ for every $f \in L^1 \cap L^p(\mathbb{C}^n)$.*

5 Connection with Special Hermite Operator

Let $\phi_{\mu\nu}$ be the special Hermite functions and L be the special Hermite operator. Given a function f in $L^p(\mathbb{C}^n), 1 \leq p \leq \propto$, we can formally expand f in terms of the special Hermite functions as

$$f(z) = \sum_{\mu} \sum_{\nu} < f, \overline{\Phi}_{\mu\nu} > \Phi_{\mu\nu}(z) \tag{1}$$

As $\Phi_{\mu\nu}$ are Schwarz class functions, the coefficients

$$< f, \Phi_{\mu\nu} > = \int_{\mathbb{C}^n} f(z)\overline{\Phi}_{\mu\nu}(z)dz$$

are well defined. As $\Phi_{\mu\nu}$ satisfies

$$\Phi_{\mu\nu} \times \Phi_{\alpha\beta} = 0 \quad \text{if } \nu \neq \alpha$$

$$\Phi_{\mu\nu} \times \Phi_{\nu\beta} = (2\pi)^{\frac{n}{2}} \Phi_{\mu\beta} \,,$$

we get

$$\begin{aligned} f \times \Phi_{\alpha\alpha} &= \sum_{\mu} \sum_{\nu} < f, \Phi_{\mu\nu} > \Phi_{\mu\nu} \times \Phi_{\alpha\alpha} \\ &= (2\pi)^{\frac{n}{2}} \sum_{\mu} < f, \Phi_{\mu\alpha} > \Phi_{\mu\alpha}, \end{aligned}$$

the series (1) can be put in the form

$$f(z) = (2\pi)^{-\frac{n}{2}} \sum_{\nu} f \times \phi_{\nu\nu}(z) \tag{2}$$

the special Hermite expansion of f. For various results concerning special Hermite expansions, we refer to [11].

Now, we can state a multiplier problem for special Hermite expansions, proof of which can be found in [11]. Let

$$T_m f(z) = (2\pi)^{-\frac{n}{2}} \sum_{\nu} m(\nu) f \times \phi_{\nu\nu}(z)$$

where m is a bounded function defined on the set of all multiindices. This operator is clearly bounded on $L^2(\mathbb{C}^n)$ but not on $L^p(\mathbb{C}^n), p \neq 2$. So we need to impose further conditions on m to ensure that T_m is bounded on $L^p(\mathbb{C}^n), 1 < p < \infty$, which is achieved by finite difference operators. Let

$$\Delta_j m(\mu) = m(\mu + \epsilon_j) - m(\mu)$$

and define Δ_j^k inductively. If β is a multiindex, we define

$$\Delta^\beta m(\mu) = \Delta_1^{\beta_1}, \ldots \Delta_n^{\beta n} m(\mu).$$

Then we have

Theorem 5.1 [11] *Assume that $k = n+1$ if n is odd and $k = n+2$ if n is even. Let m be a function defined on $\mathbb{N}^n$ which satisfies*

$$|\Delta^\beta m(\mu)| \leq C_\beta(1+|\mu|)^{-|\beta|}$$

for all β with $|\beta| \leq k$. Then T_m is bounded on $L^p(\mathbb{C}^n), 1 < p < \infty$.
The proof is based on Littlewood-Paley-Stein theory of g functions defined by the special Hermite semigroup e^{-tL} generated by the operator L.

To state another multiplier problem, let

$$\varphi_k(z) = L_k^{n-1}(\frac{1}{2}|z|^2)e^{-\frac{1}{4}|z|^2},$$

where L_k^{n-1} denotes the Laguerre polynomial of type $(n-1)$. As

$$\sum_{|\mu|=k} \Phi_{\mu\mu}(z) = (2\pi)^{-n/2}\varphi_k(z),$$

the special Hermite expansion can be written as

$$f(z) = (2\pi)^{-n}\sum_{k=0}^{\infty} f \times \varphi_k(z).$$

The Laguerre Sobolev space $W_L^{s,2}(\mathbb{C}^n)$ was introduced by Thangavelu in [10] in connection with spherical means on the Heisenberg group H^n.

For $m \in Z^+$ and $1 < p < \infty$, we define $W_L^{m,p}(\mathbb{C}^n)$ to be image of $L^p(\mathbb{C}^n)$ under L^{-m} where L^m is defined by

$$L^m f = (2\pi)^{-n}\sum_{k=0}^{\infty}(2k+n)^m f \times \varphi_k.$$

In otherwords, $f \in W_L^{m,p}(\mathbb{C}^n)$ if and only if

$$\|f\|_{m,p}^p = \|(2\pi)^{-n}\sum_{k=0}^{\infty}(2k+n)^m f \times \varphi_k\|_p^p$$

is finite. Then the problem is to characterise the multipliers for $W_L^{m,p}(\mathbb{C}^n)$. It turns out that

Theorem 5.2 [7] *The space of Weyl multipliers for $W_L^{m,p}(\mathbb{C}^n)$ coincides with the space of Weyl multiplier for $L^p(\mathbb{C}^n)$ for $1 < p < \infty$.*

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