Kinematical Conservation Laws (KCL): A mathematical model to describe evolution of curves and surfaces

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Abstract

d-D kinematical conservation laws (KCL) are equations of evolution of a moving surface Ω_t in d-dimensional $(x_1, x_2, \ldots, x_d)$-space $\mathbb{R}^d$. The KCL are derived in a specially defined ray coordinates $(\xi_1, \xi_2, \ldots, \xi_{d-1}, t)$, where $\xi_1, \xi_2, \ldots, \xi_{d-1}$ are surface coordinates on Ω_t and $t > 0$ is time. The 2-D KCL were derived by Morton, Prasad and Ravindran in 1992 and though 3-D KCL were derived by Giles, Prasad and Ravindran in 1995, the theory of this system remained incomplete. Later the analysis of the 3-D KCL system was completed by Arun and Prasad and published in 2008. Here we discuss various properties of 2-D and 3-D KCL systems. We first review the important properties of 2-D KCL and some of its applications. KCL are the most general equations in conservation form, governing the evolution of Ω_t with singularities which we call kinks and which are points on Ω_t when Ω_t is a curve in $\mathbb{R}^2$ and are curves on Ω_t when it is a surface in $\mathbb{R}^3$. Across a kink the normal $\mathbf{n}$ to Ω_t and amplitude w on Ω_t are discontinuous. From 3-D KCL we derive a system of six differential equations and show that the KCL system is equivalent to the ray equations for Ω_t. The six independent equations and an energy transport equation for small amplitude waves in a polytropic gas involving an amplitude w, which is related to the normal velocity m of Ω_t, form a completely determined system of seven equations but this system must satisfy three *geometric solenoidal constraints*. We have determined eigenvalues of the system by a novel method and find that the system has two distinct nonzero eigenvalues and five zero eigenvalues and the dimension of the eigenspace associated with the multiple eigenvalue zero is only four. For an appropriately defined m, the two nonzero eigenvalues are real when $m > 1$ and pure imaginary when $m < 1$. For the numerical simulation of this *weakly hyperbolic system* of conservation laws ($m > 1$) we employ a high resolution central scheme, which is second order accurate. A constrained transport type technique is used to enforce the geometric solenoidal constraints. Finally, we have presented results of some numerical solution.