The goal of this course is to present a general overview of
Quantum Probability with emphasis on open challenges and applications .

1) Emergence of quantum theory from classical probability .

Goal : to illustrate the idea that QP is not a generalization,
but a deeper level of understanding of classical probability.

1a) Orthogonal polynomials and the quantum decomposition of
classical
random variables with all moments.

1b) Deduction of generalized commutation relations from commutativity
and one–to–one correspondence between quantum theories and equiva-
lence
classes of probability measures.

1c) The classical Bernoulli process and the probabilistic meaning of
the

q–deformation parameter: birth of the q–q–bit.

1d) The probabilistic origins of Boson Fock space and its interacting
generalizations .

1e) An open problem in classical probability solved with quantum
probabilistic thechniques.

2) Stochastic independences and central limit theorems (CLT).

Goal : to illustrate the infinitely many notions of stochastic independence
and the corresponding central limit theorems, as well as their functional
formulation , that gives rise to the quantum fields (white noises).

2a) The singleton condition in CLTs and the role of pair partitions .

2b) The strict singleton condition and the equivalence between CLTs and entangled ergodic theorems.

2c) Central limits of Bernoulli processes: Bosons, Fermions, various forms of
q–deformations.

2d) Singleton but not strict singleton independences: Monotone, Boolean
.

2e) Non singleton independences.

2d) Notions of Gaussianity.

2e) Functional CLTs and quantum fields.

2f) Axiomatic approaches to stochastic independences.

3) Statistical dependences and Markovianity.

Goal : to describe Markovianity as prototype form of statistical dependence .
To explain the principle differences between classical and quantum conditioning .

3a) Markov chains and Markov fields on graphs: constructive
approach .

3b) Conditioning. Markov chains and Markov states: structure theory .

3c) Non triviality of Fermi Markov states.

3d) Expected Markov processes and semi–groups.

4) Basic ideas of the stochastic limit of quantum theory (SLQT).

Goal : to illustrate how the basic quantum noises, White Noise Hamiltonian

Equations (WNHE) and Stochastic Differential Equations (SDE) arise from physics
through the stochastic limit of quantum theory.

4a) Slow and fast observables: emergence of WNHE from usual Hamiltonian Equations. Equivalence of first and normally ordered second order WNHE with SDE.

4b) Markov semi–groups of stochastic limit type. The interaction graph of a Markov semi–group. Classical sub–processes of quantum processes .

4c) The similarity principle and non–equilibrium quantum field theory .

4d) Short description of Level (II) (low density) and Level (III) (strongly non–linear effects) problems in SLQT.
Emergence of the non–crossing diagrams in physics.

5) Non linear quantization .

Goal : to describe the attempts made in the past 15 years to answer the question:
if all, classical and quantum, SDE are covered by 1–st and normally ordered 2–d powers of white noise, what about higher order WNHE?

5a) Quadratic quantization, $sl(2, \mathbb{R})$, renormalization ,
emergence of the no-go theorems, connections with infinite divisibility ,
the quadratic quantization functor.

5b) Quantization of higher powers of white noise, ew renormalization and the Virasoro–Zamolodchikov hierarchy.

5c) Renormalization and central extensions (2), cohomology of ∞–dimensional Lie algebras.

5d) Interplay between infinite divisibility and non–linear quantization :

the Galilei algebra, C^*–2d–quantization.

6) Public lecture, colloquium style (1 h).

(if there is sufficient interest to justify such a lecture)

Non–Kolmogorovian probability

Goal: to explain how the study of the foundational problems of quantum mechanics led to the conclusion that 'non–Kolmogorovian' probabilities exist in physics and how this idea was developed in analogy with non –Euclidean geometries , thus introducing 'statistical invariants' in analogy with geometrical invariants and a unifying set of axiom that allows to deduce (in a non–trivial way, as far as the q–model is concerned) the known models as well as new ones. Examples of 'non–Kolmogorovian' statistical data outside quantum physics will be discussed.