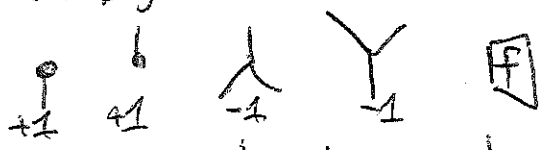
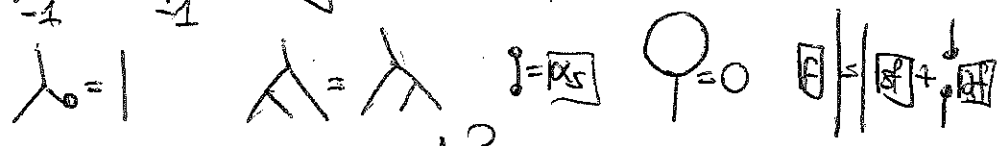
Ben Elias Chennai Lectures Part IV Catchup 1/2014 (1)

Recap: For each $s \in S$, we've encoded certain morphisms $B_s^{\otimes n} \to B_s^{\otimes m}$ diagrammatically



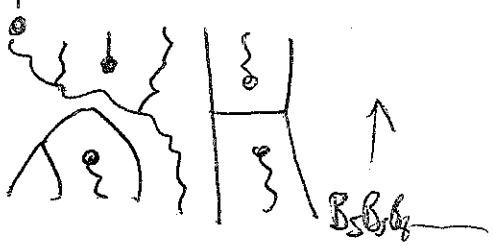
w/ relations: isotopy,



Now, when S is not just one elt, what else does one get?

For each pair $s,t \in S$ one can already construct lots of maps

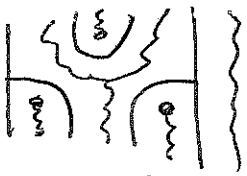
What can we already say?



Recall the $\mathcal{TL}$ category

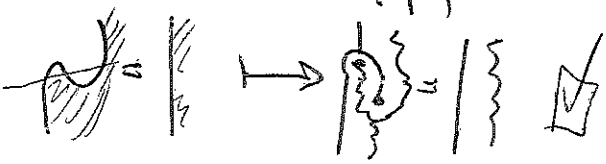
encoding sl_2 reps

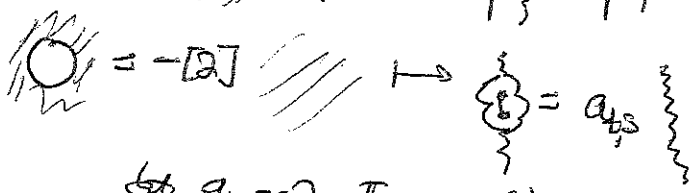
Color each region w/ alternating colors, then deformation retract



get a map of degree 0!

Is this an algebra map?





So

$$\mathrm{Hom}_{\mathcal{TL}}(n,m)/_{q=\zeta_{2m}} \longrightarrow \mathrm{Hom}^{\circ}_{BS}\Big(\underbrace{B_s B_t B_s \ldots}_{n+1}, \underbrace{B_s B_t \ldots}_{m+1}\Big)$$

let $a_{t,s} = -2\cos\frac{\pi}{m_{st}} = -(q+q^{-1})$ for $q = \zeta_{2m}$ (or $q=1$ when $m=\infty$)

LHS has idempotents — Jones Wenzl projectors — defined at $q=\zeta_{2m}$ for JW_k, $k \le m-1$

This implied that $V \otimes V_k \cong V_{k+1} \oplus V_{k-1}$ (for $1 \le k \le m-2$.)

Transferring to RHS, let $B_{\underbrace{stst\ldots}_{k+1}}$ be image of JW_k

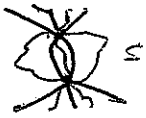
then $B_t B_{\underbrace{stst\ldots}_{k}} = B_{\underbrace{stst\ldots}_{k+1}} \oplus B_{\underbrace{stst\ldots}_{k-1}}$ for $2 \le k \le m-1$

this categorifies $\underline{H}_t \underline{H}_{\underbrace{stst\ldots}_{k}} = \underline{H}\ldots + \underline{H}\ldots$ so we were correct to do this.

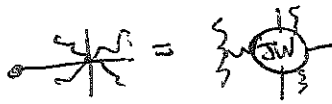
When $n<\infty$.

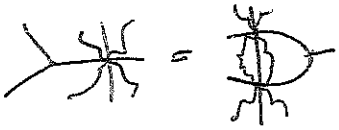
But now we have a description of B_{stab_n} and $B_{\text{stab}_{n+1}}$ and they should be isomorphic. Another way: $B_{\text{stab}_n} \otimes B_{n,n} \otimes B_{\text{stab}_n}$

so $\exists$ $\rightleftharpoons$ with $\curvearrowright$ = pg to $B_{n,n}$.

Draw these as and . Then  =

Can use this to ~~prove~~ prove some other relation:

 a different way, degree 42 ... or vice versa.



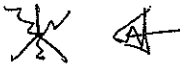
Note: Writing down as a braid rep is really hard! No closed formula

Sends C_{Bat_n} to C_{Bat}, and sends any element to something in R-bar ~~reduced~~ and generated by C_{Bat} (since $\cong$ to $R \otimes_{R^{S_n}} R(n)$)

But hard to write down explicitly.

Final relations: W_n beyond ctt of $A_3, B_3, H_3, A_1 \times I_2(m), A_1 \times A_1 \times A_1$ finite rank 3 Cox gps

Γ = reduced expression graph. Each vertex $\rightsquigarrow BS(w)$

Each edge $\rightsquigarrow$  not an isomorphism! cohomology lectures

A path goes to a composition of 2m valent vertex. A new rule

Two paths needn't be equal! But equal "up to lower terms" (agree on C_{Bat} clearly, lower terms all kill C_{Bat}) to be explained soon

Ex: =

One new rel'n for the cycle appearing in each graph.

Too lower terms appear in this!!

Ex: $A_1 \times A_1 \times A_1$