

The Shlafly Double Six

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Algebra versus Geometry

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This lecture may provide a counter-example!

what is the Shlafly Double-Six

A double-six consists of a pair of ordered 6-tuples of lines (P_0, \dots, P_5) and (Q_0, \dots, Q_5) such that each P_i meets all the Q_j except Q_i .

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3. Then S contains the lines

$$P_1 := X = Z = 0 \quad ; \quad P_2 := Y = T = 0$$

$$P_3 := X = T \wedge Y = Z$$

$$Q_4 := X = Z \wedge Y = T$$

$$Q_5 := Y = Z = 0 \quad ; \quad Q_6 := X = T = 0$$

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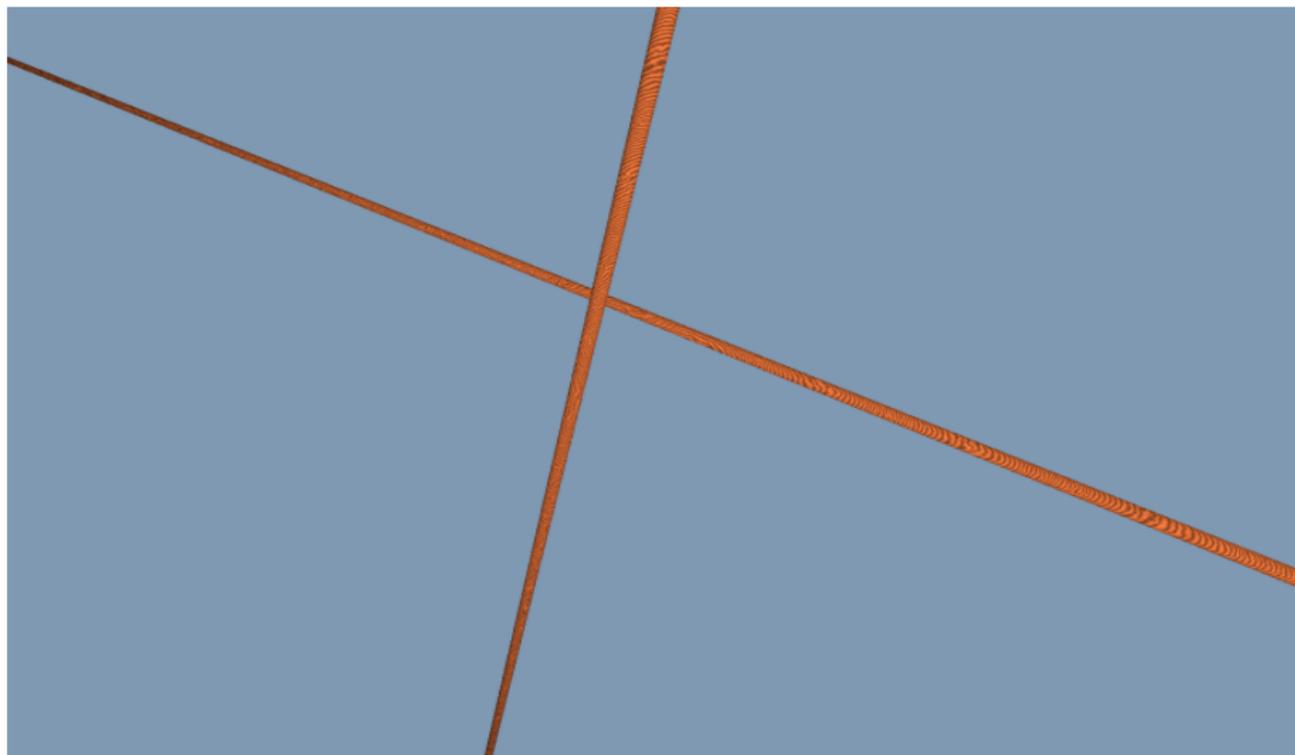
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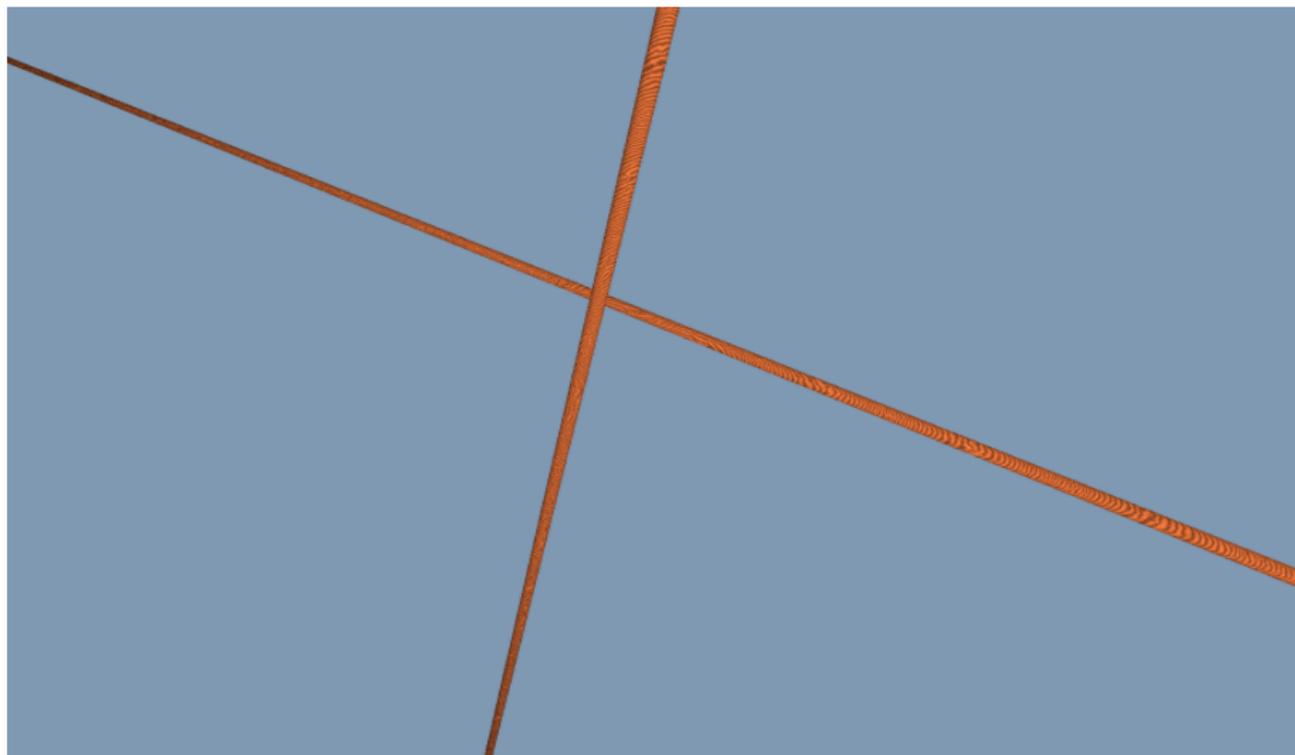
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Using this one can easily write down 19 more lines in S . The Shlafly Double-six is part of this collection of 27 lines!

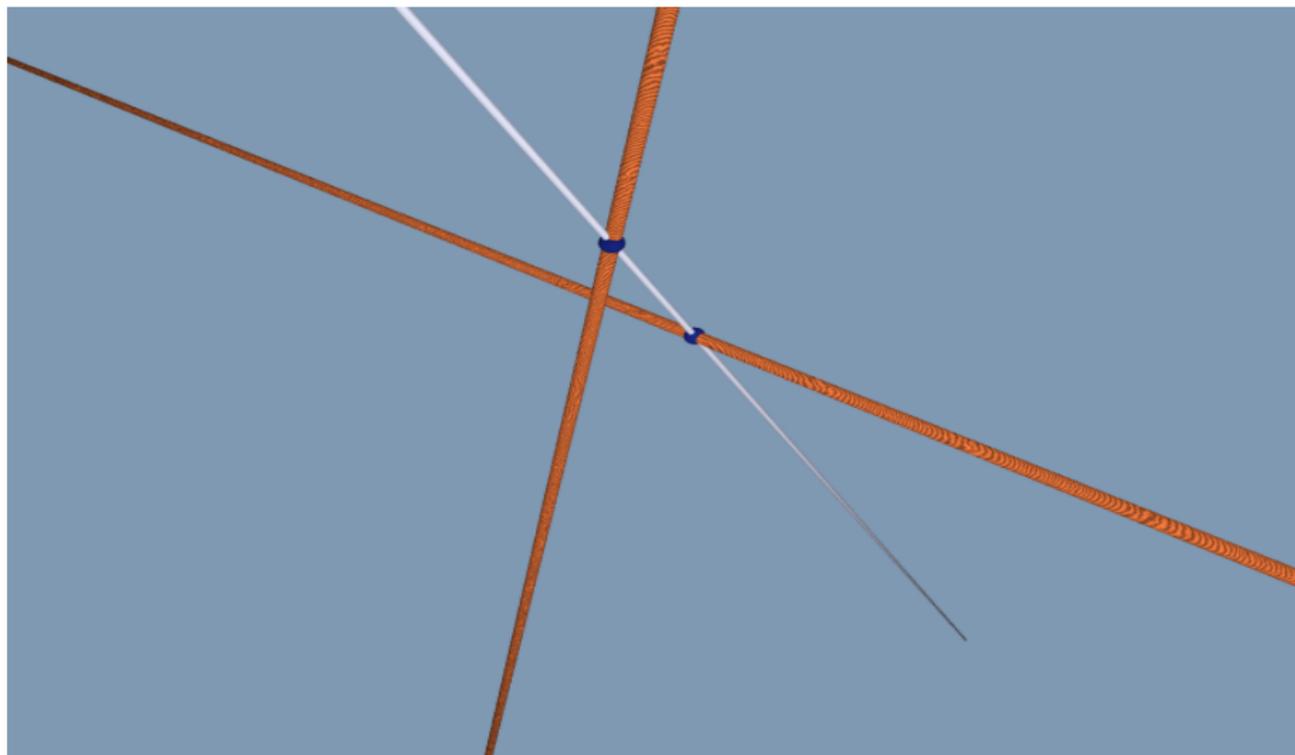
The geometric construction



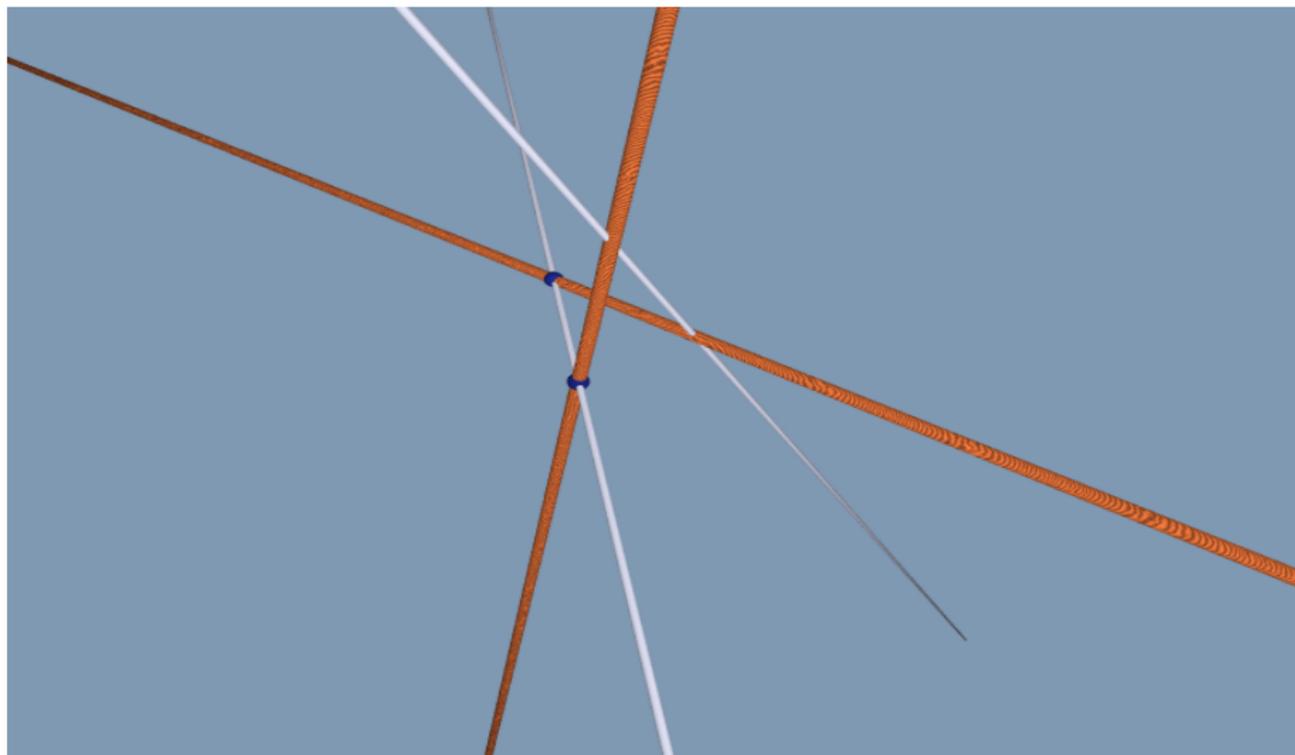
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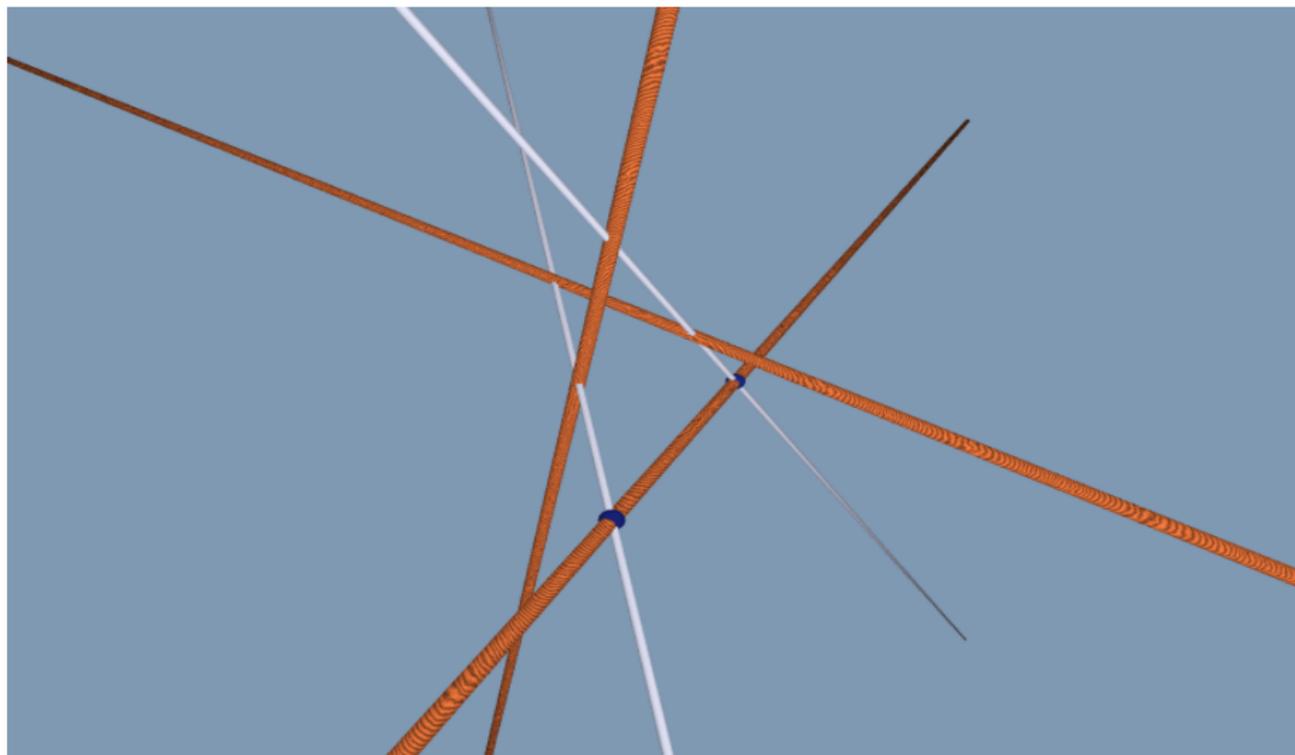
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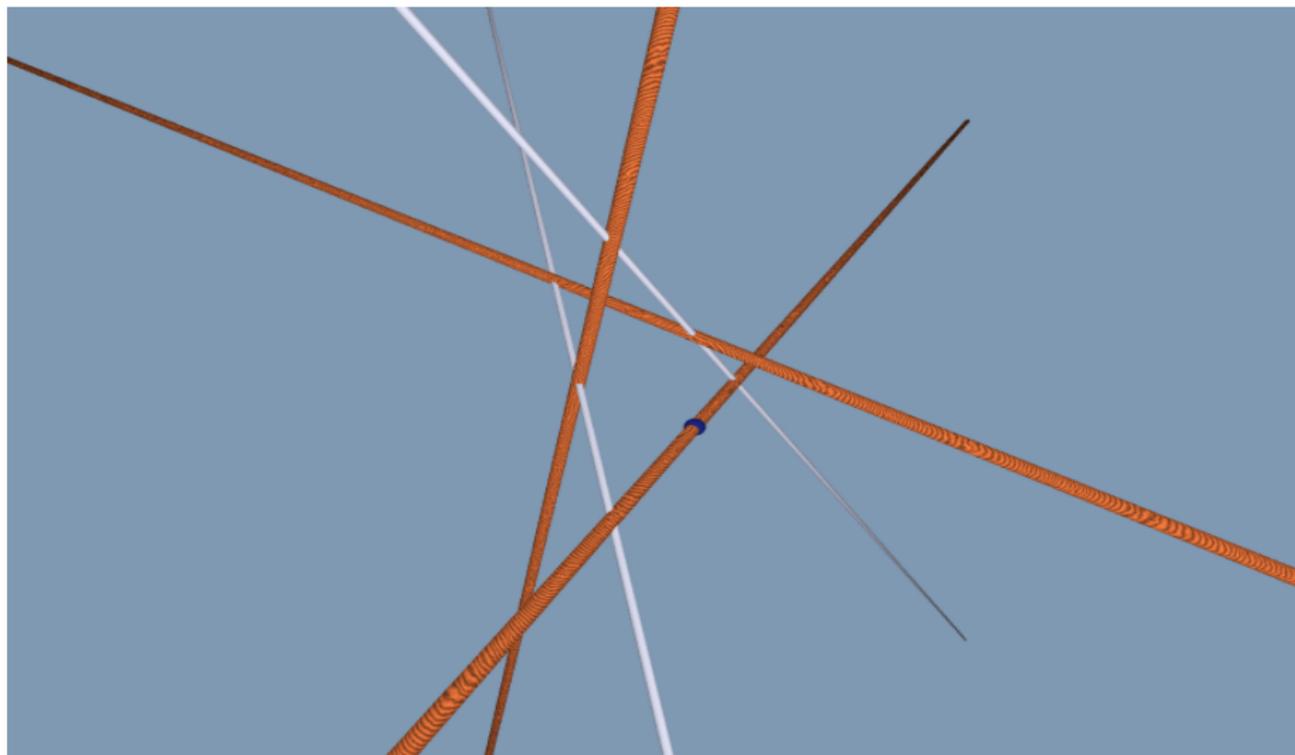
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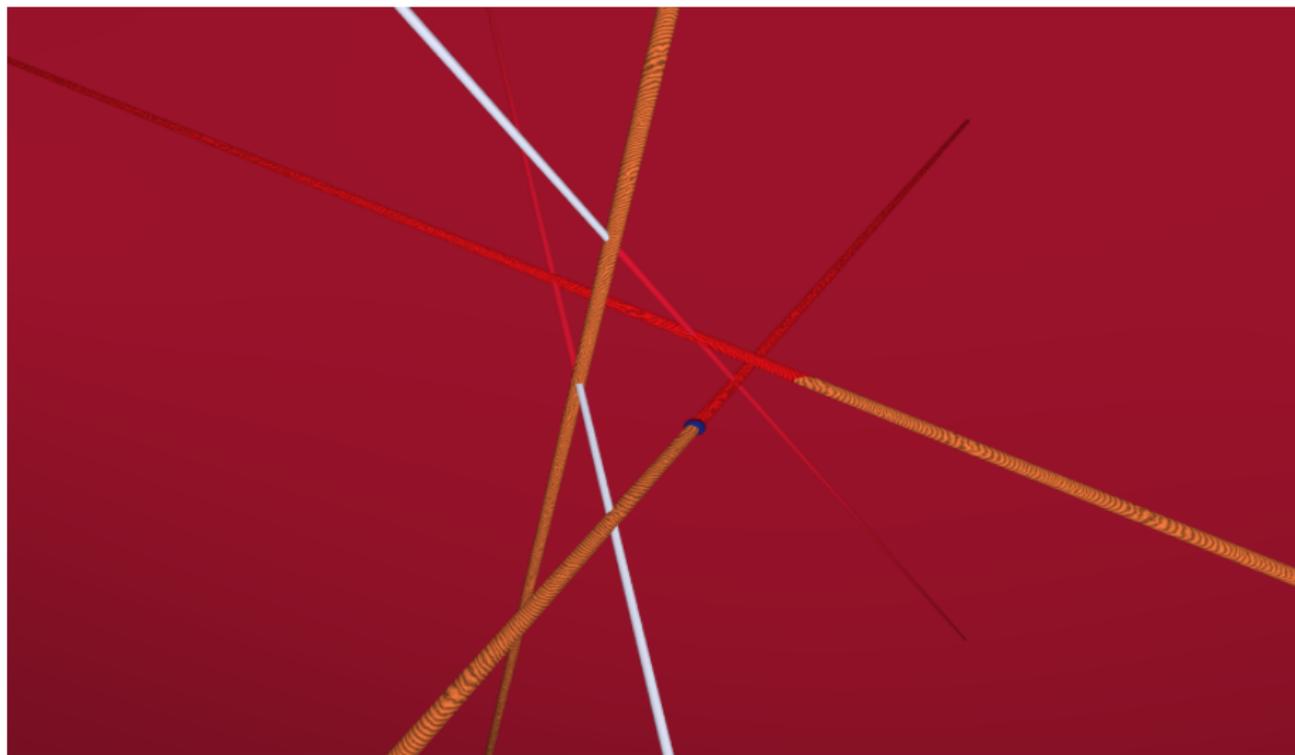
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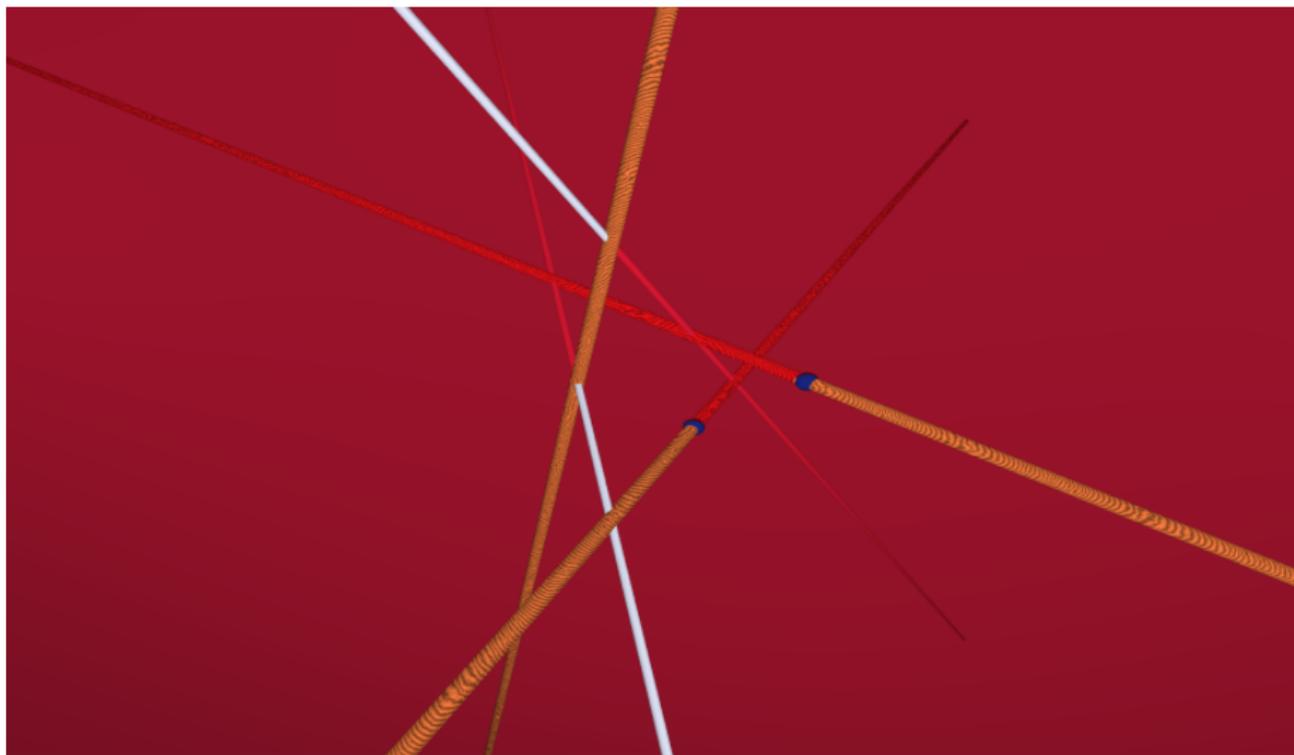
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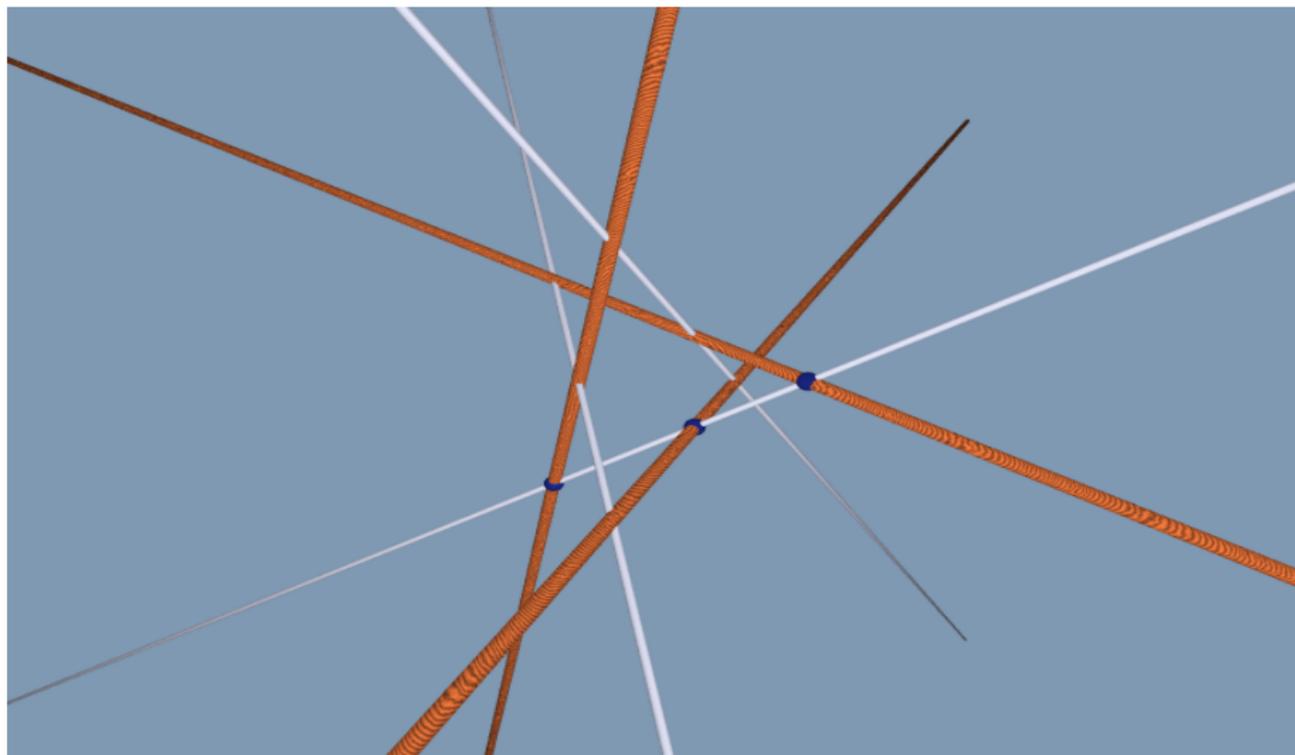
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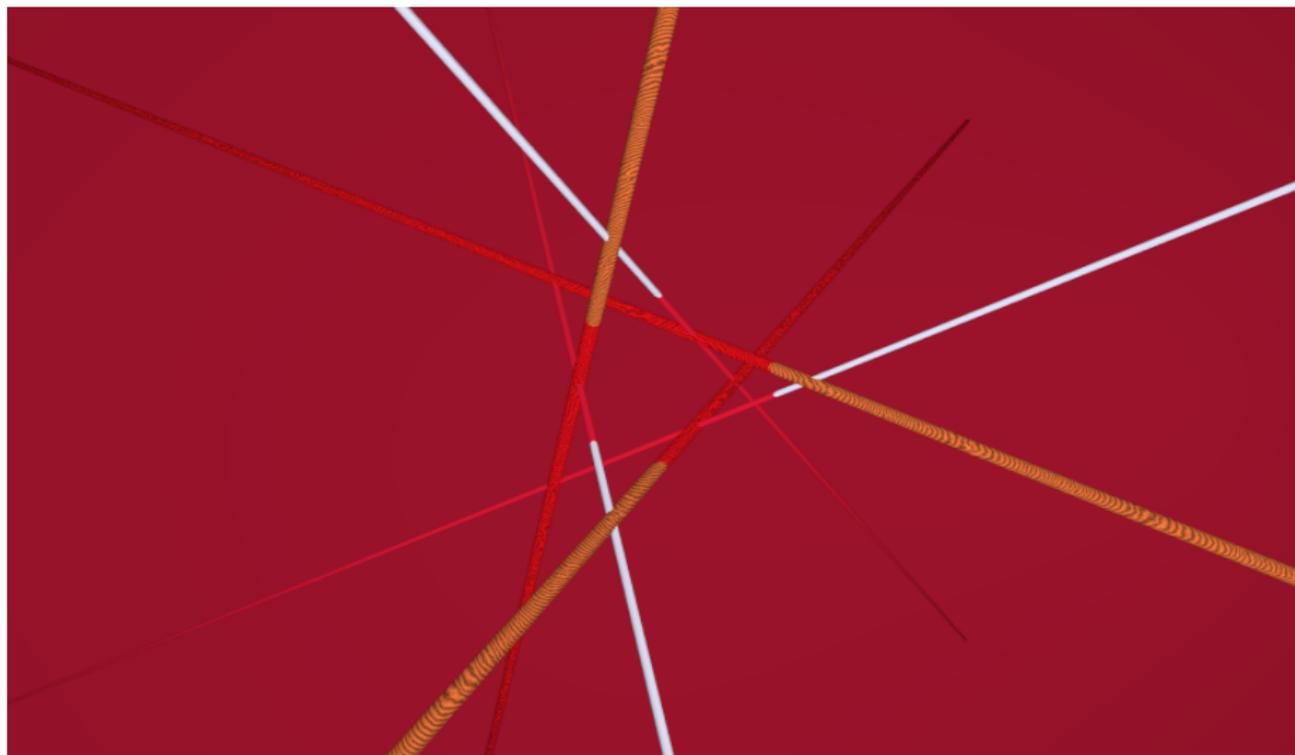
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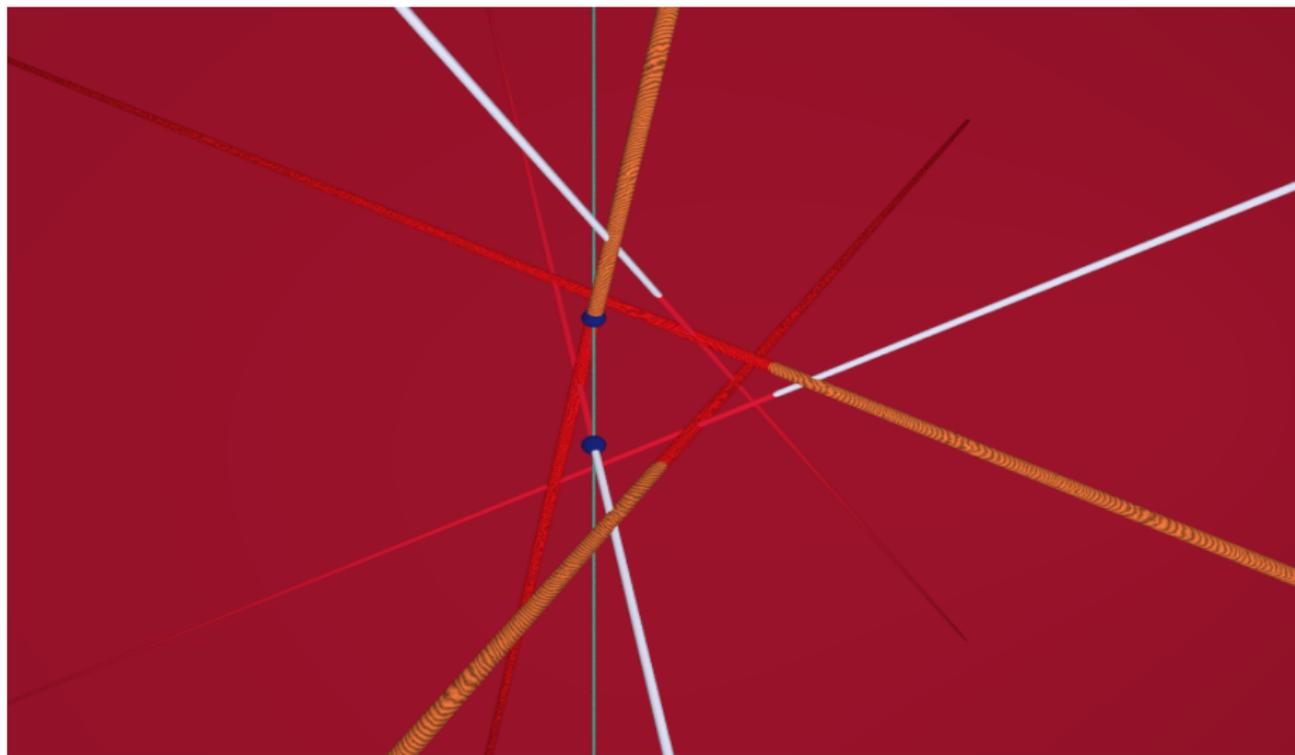
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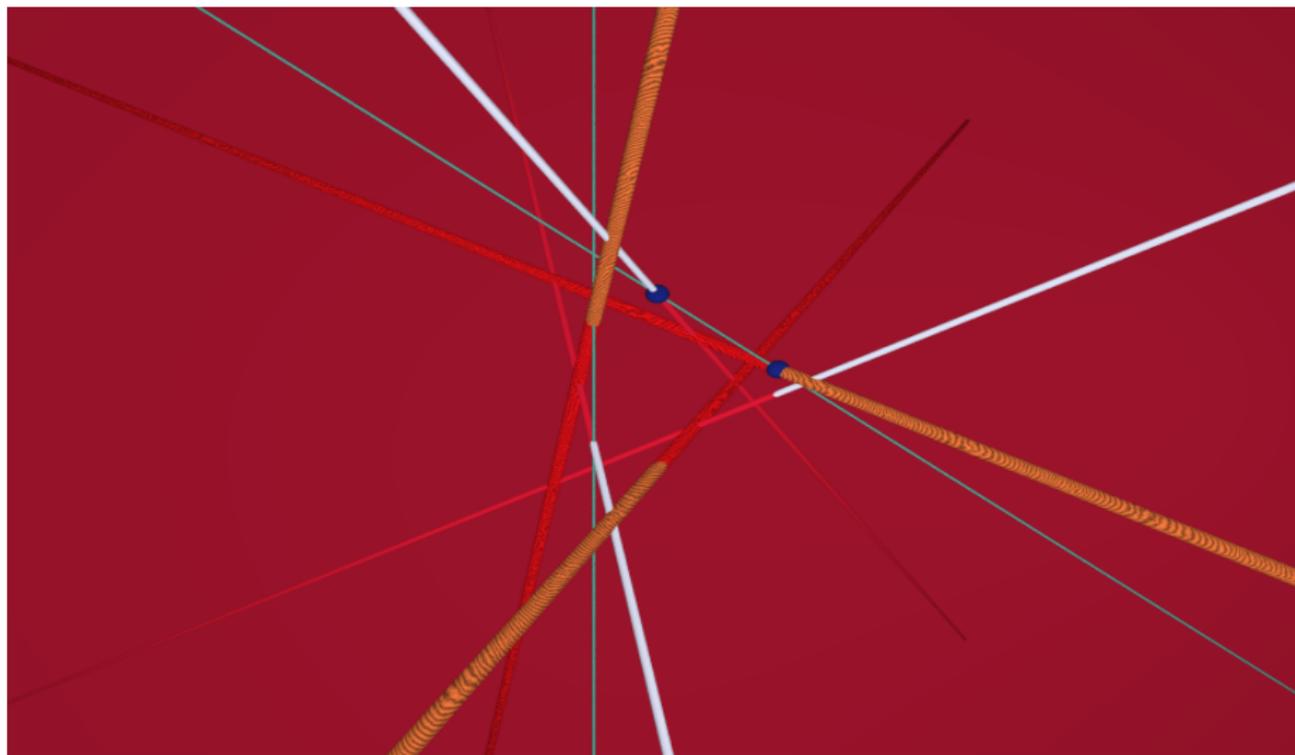
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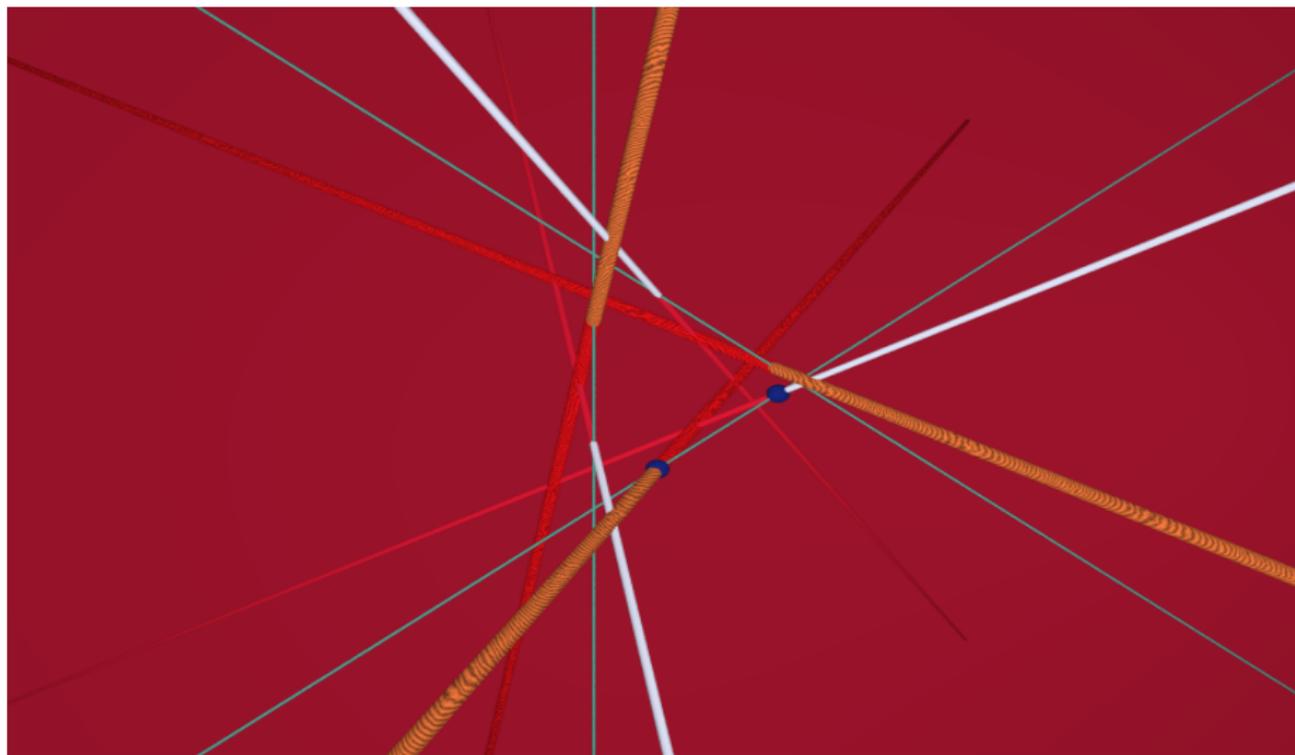
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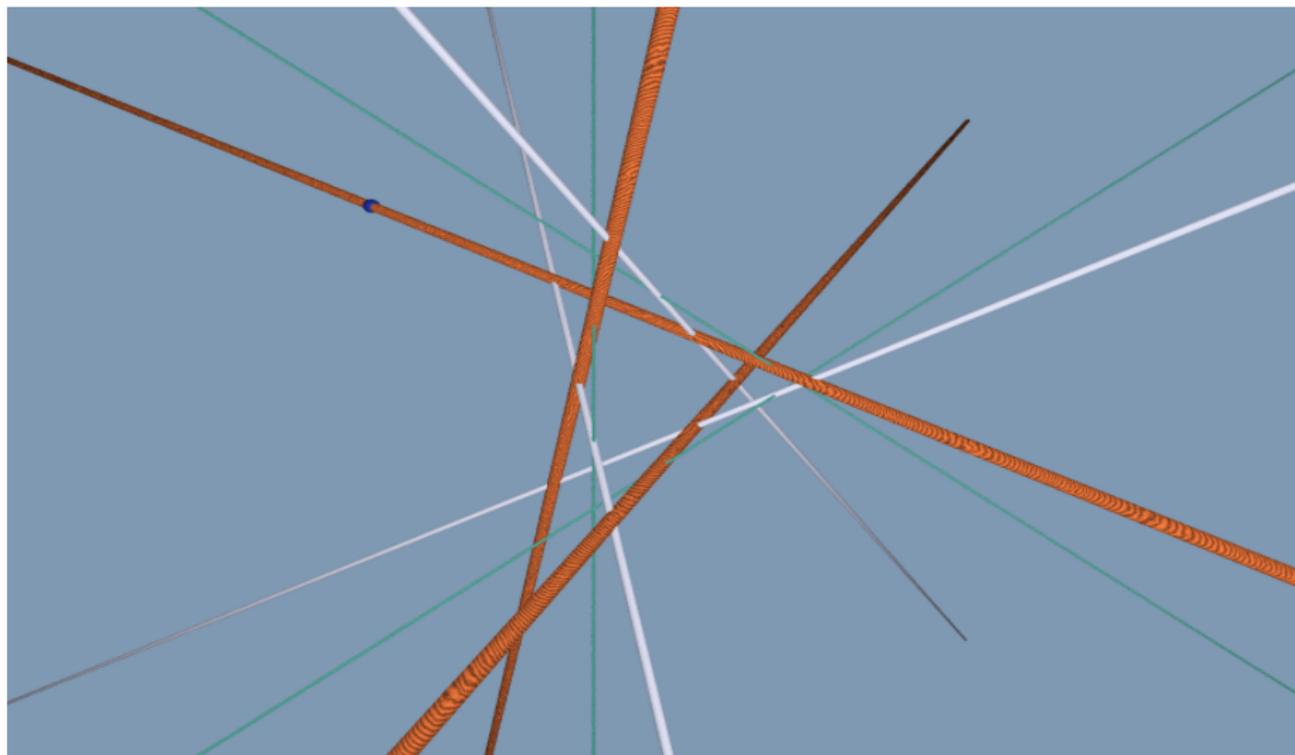
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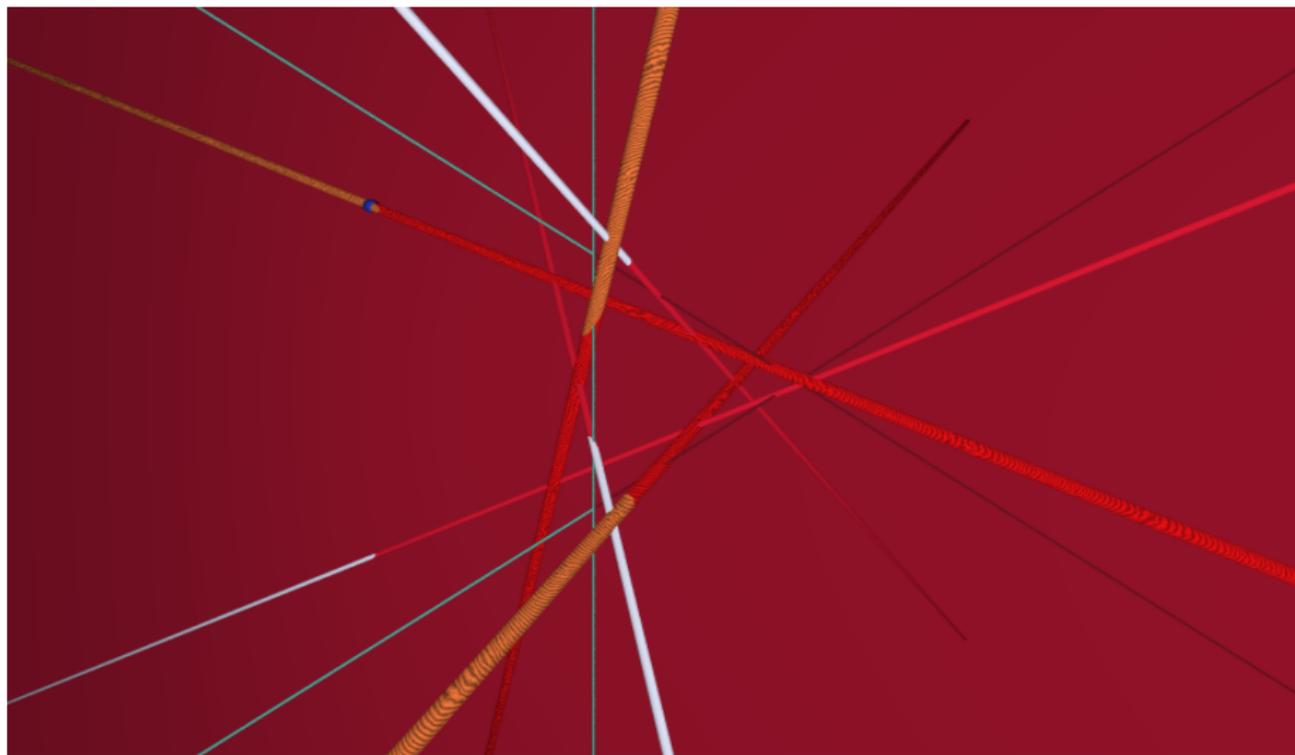
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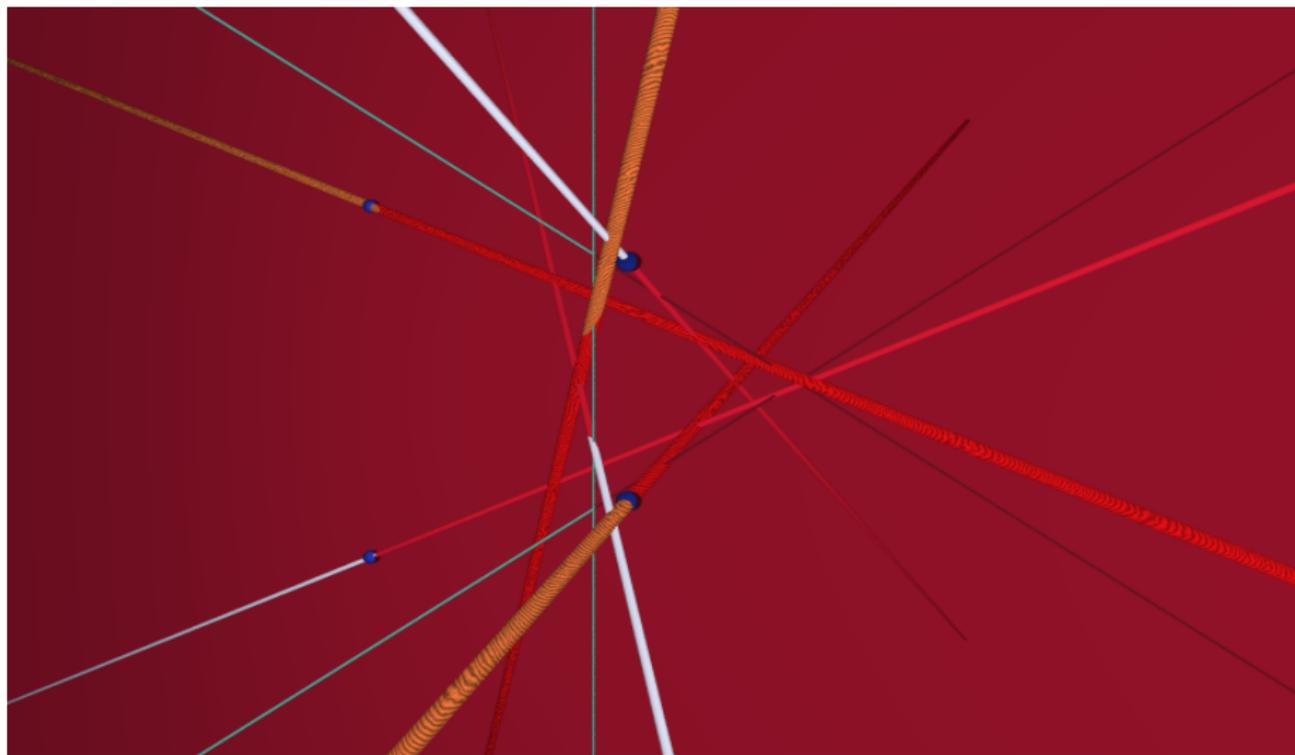
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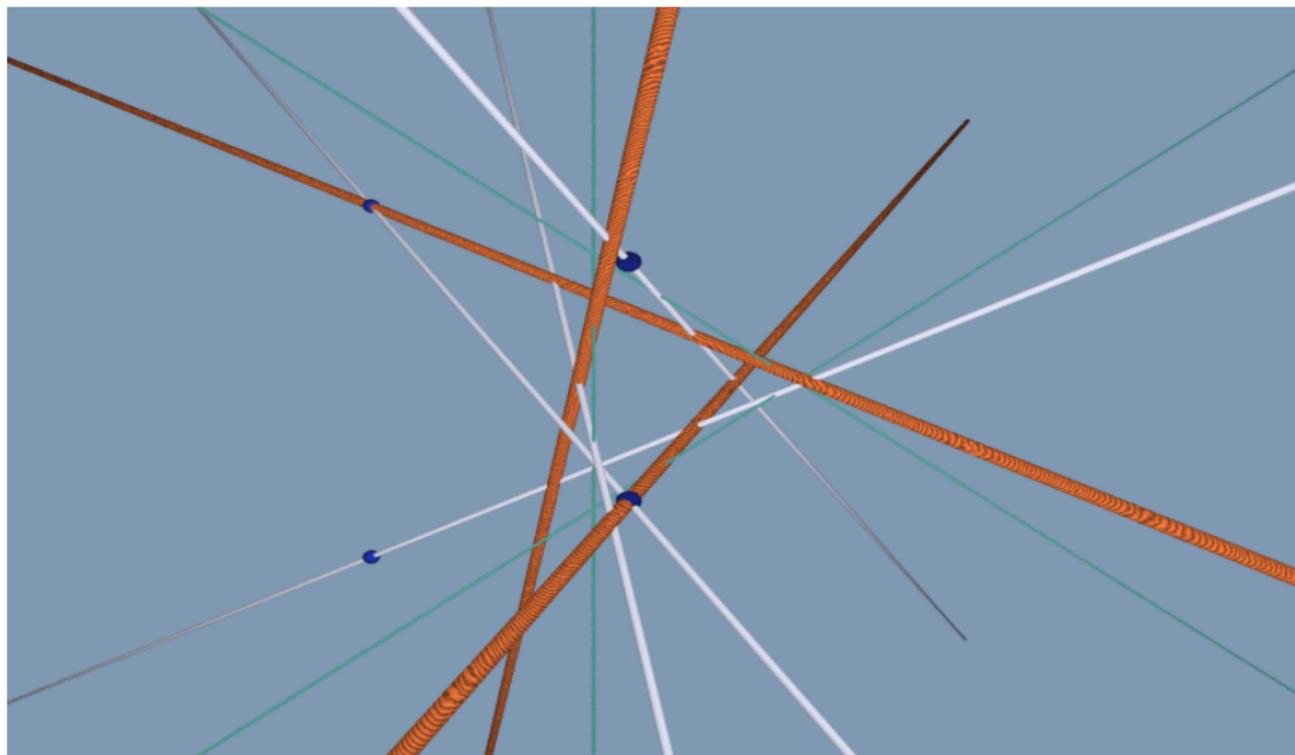
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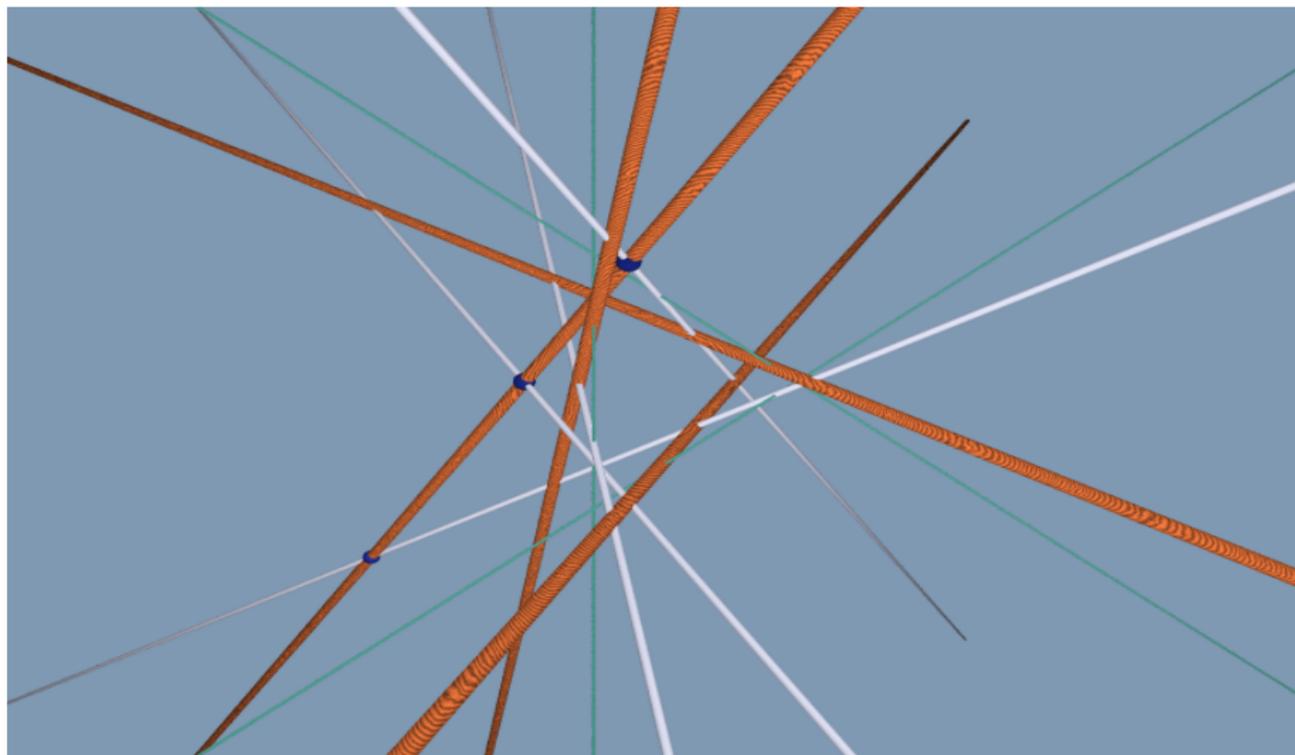
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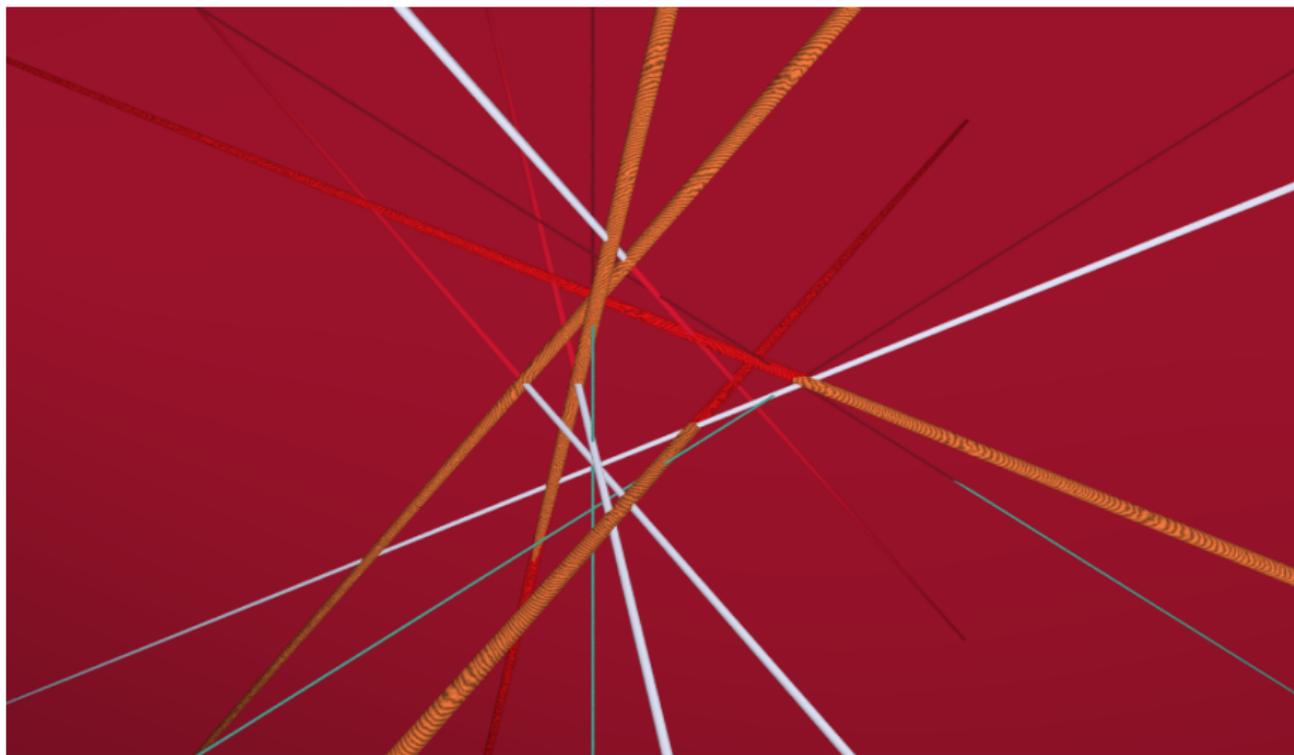
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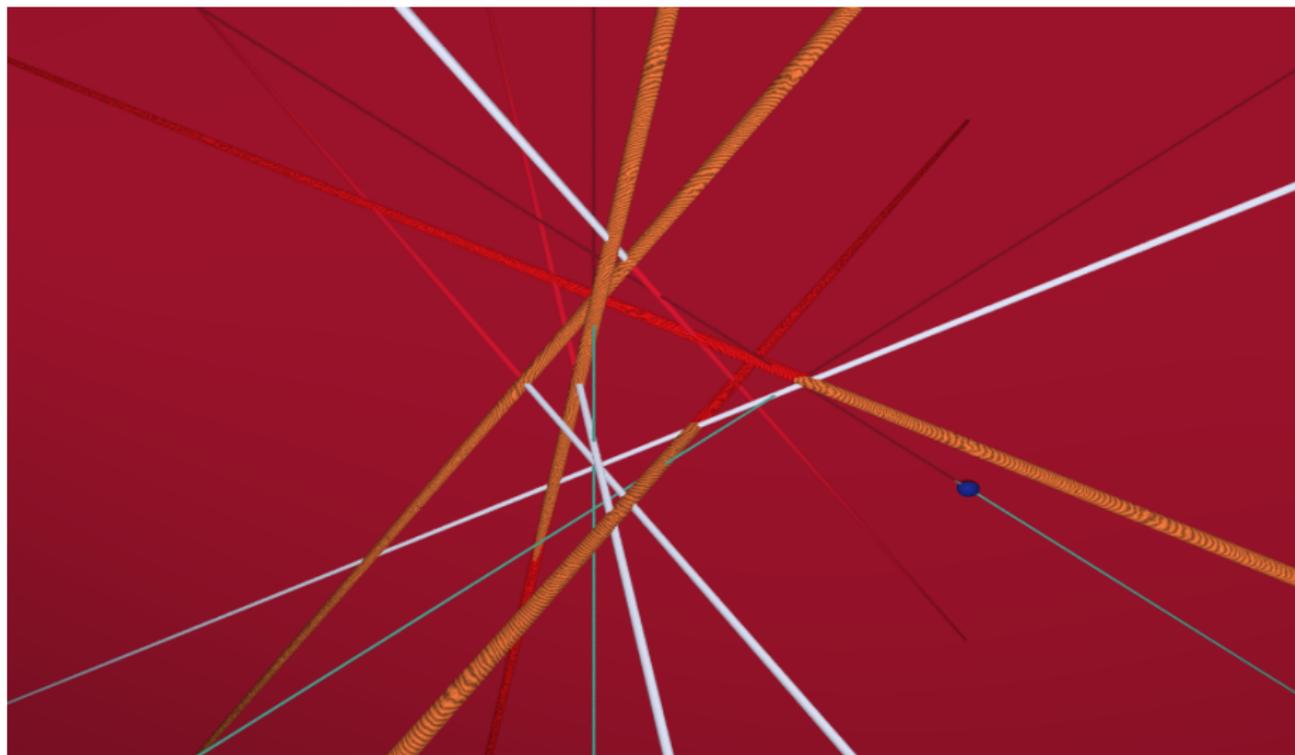
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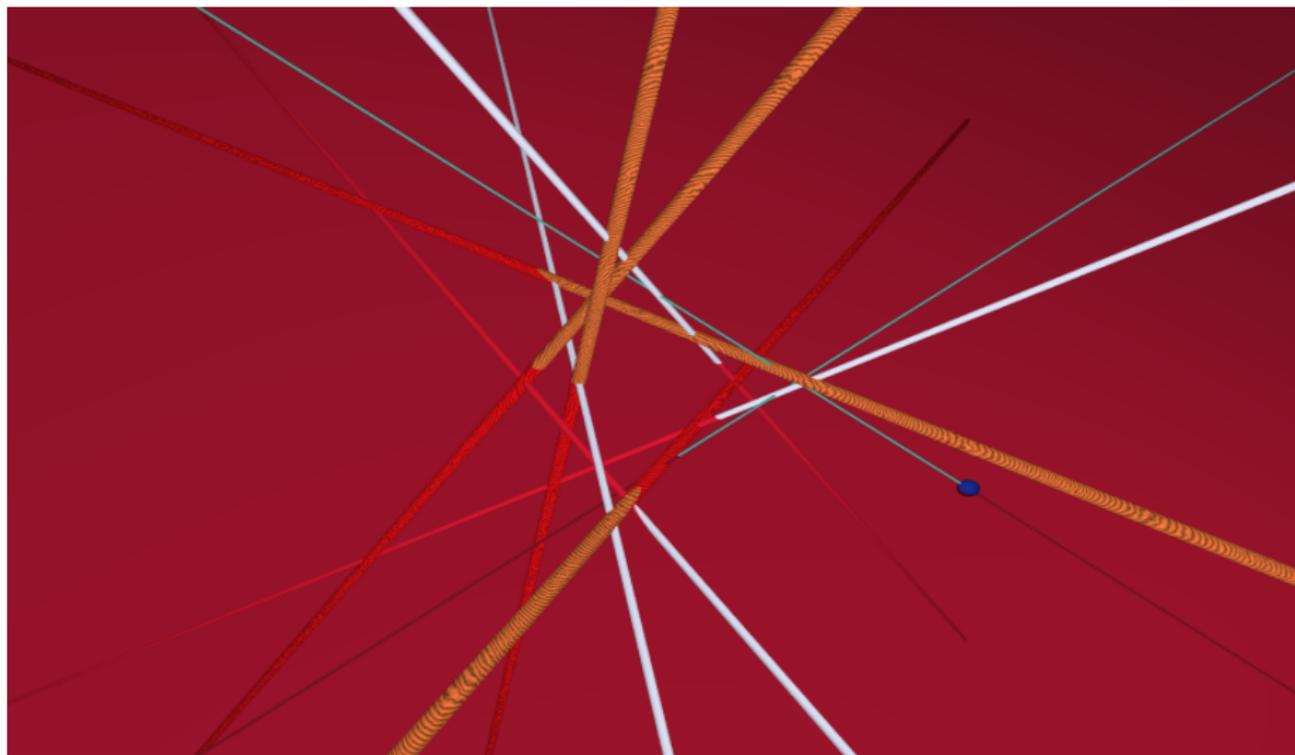
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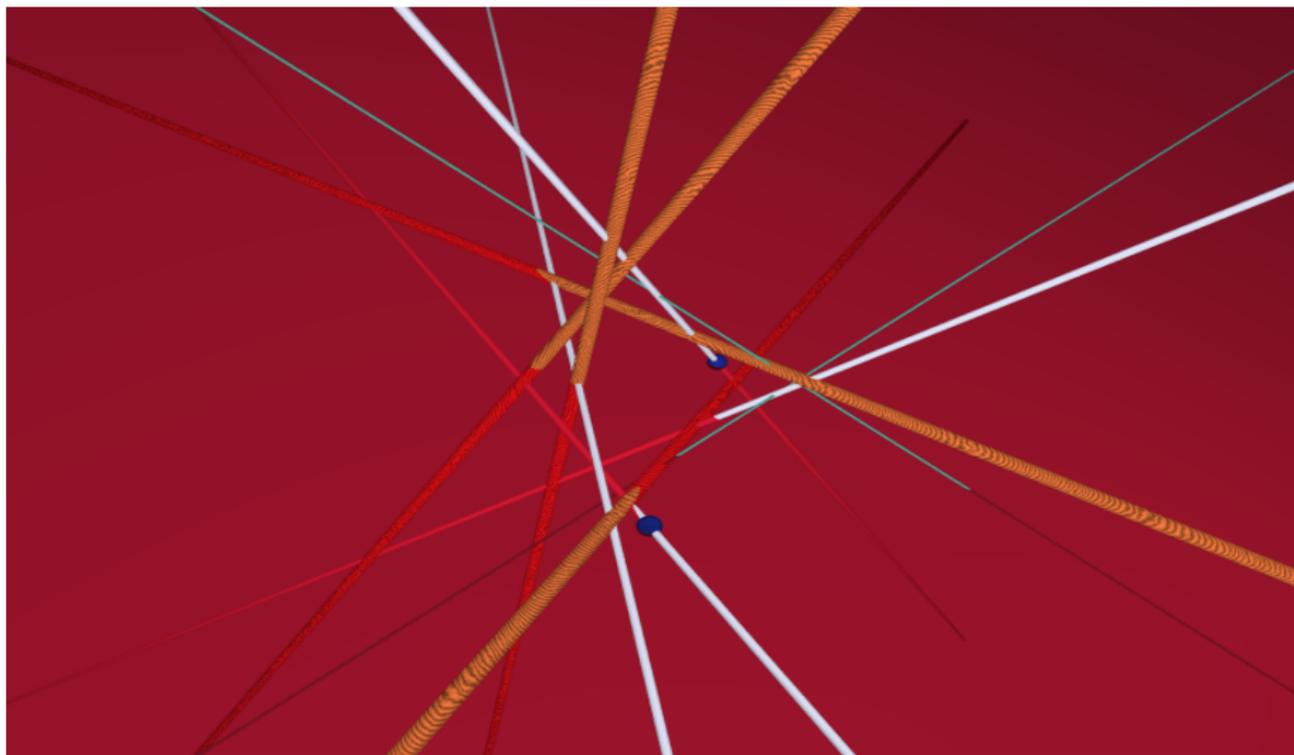
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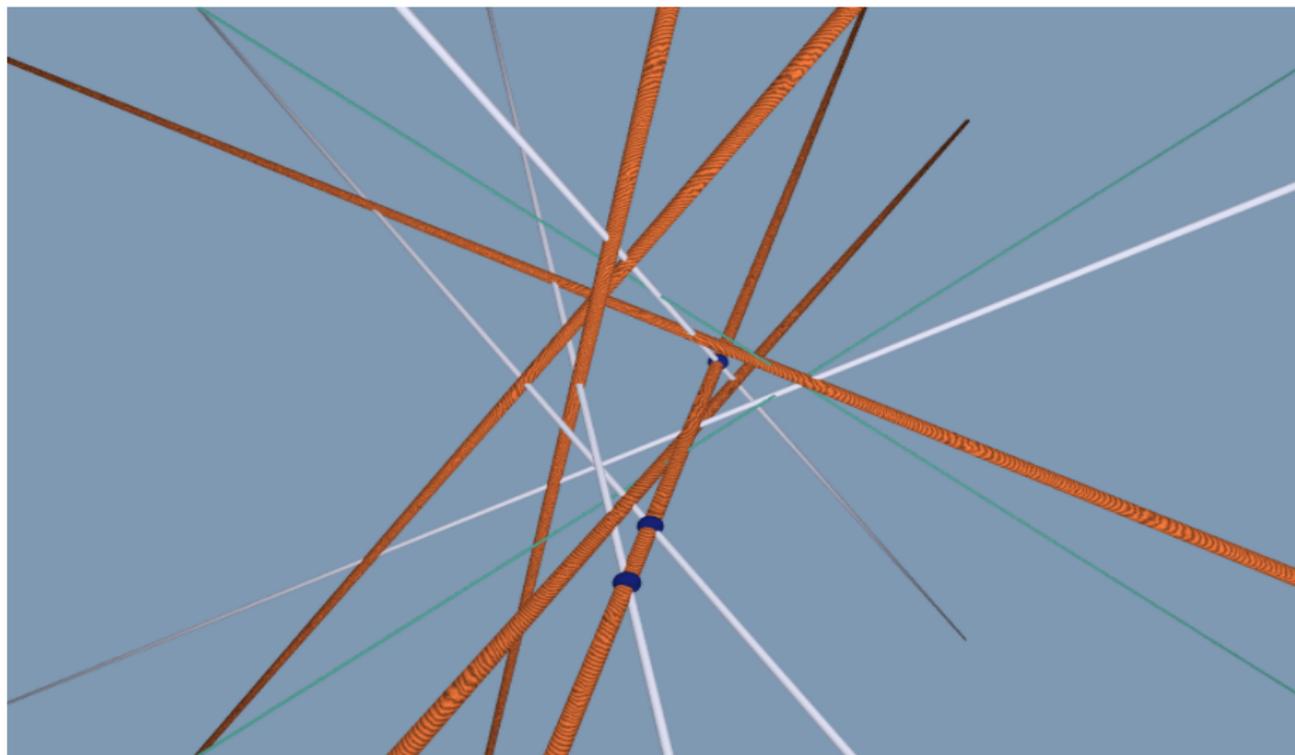
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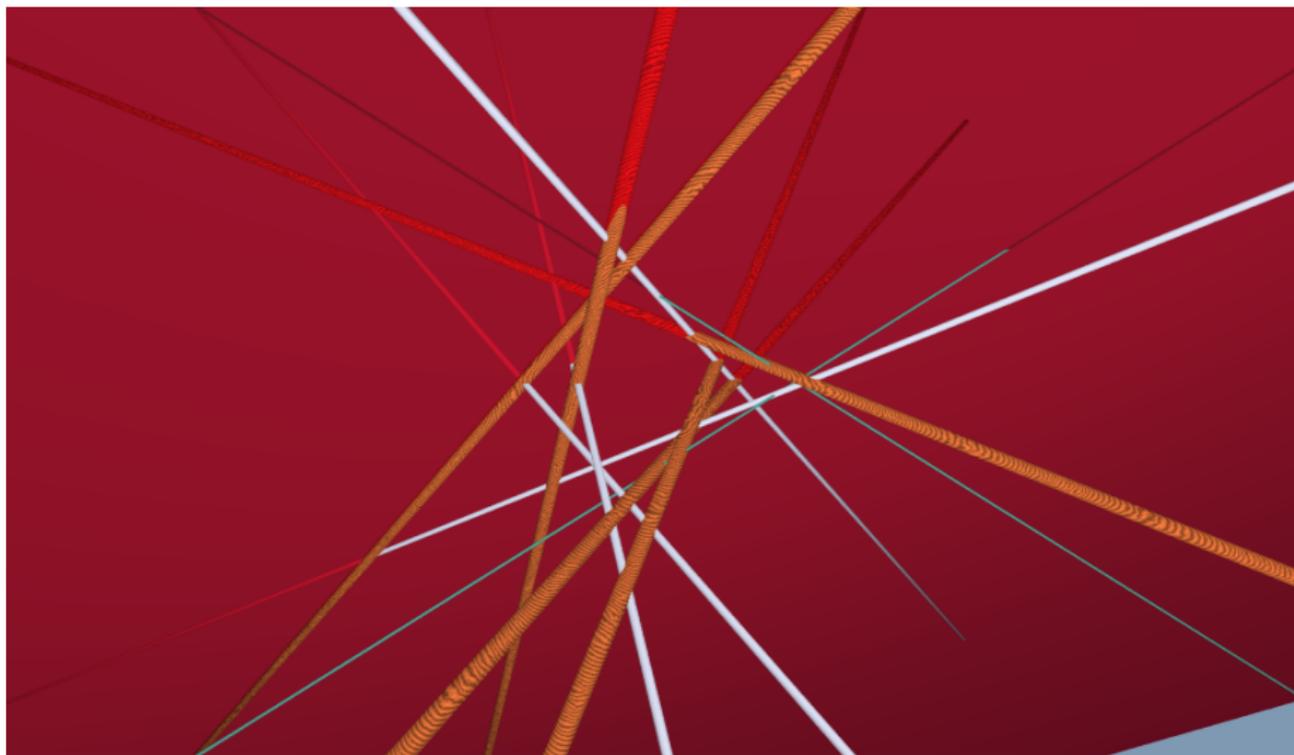
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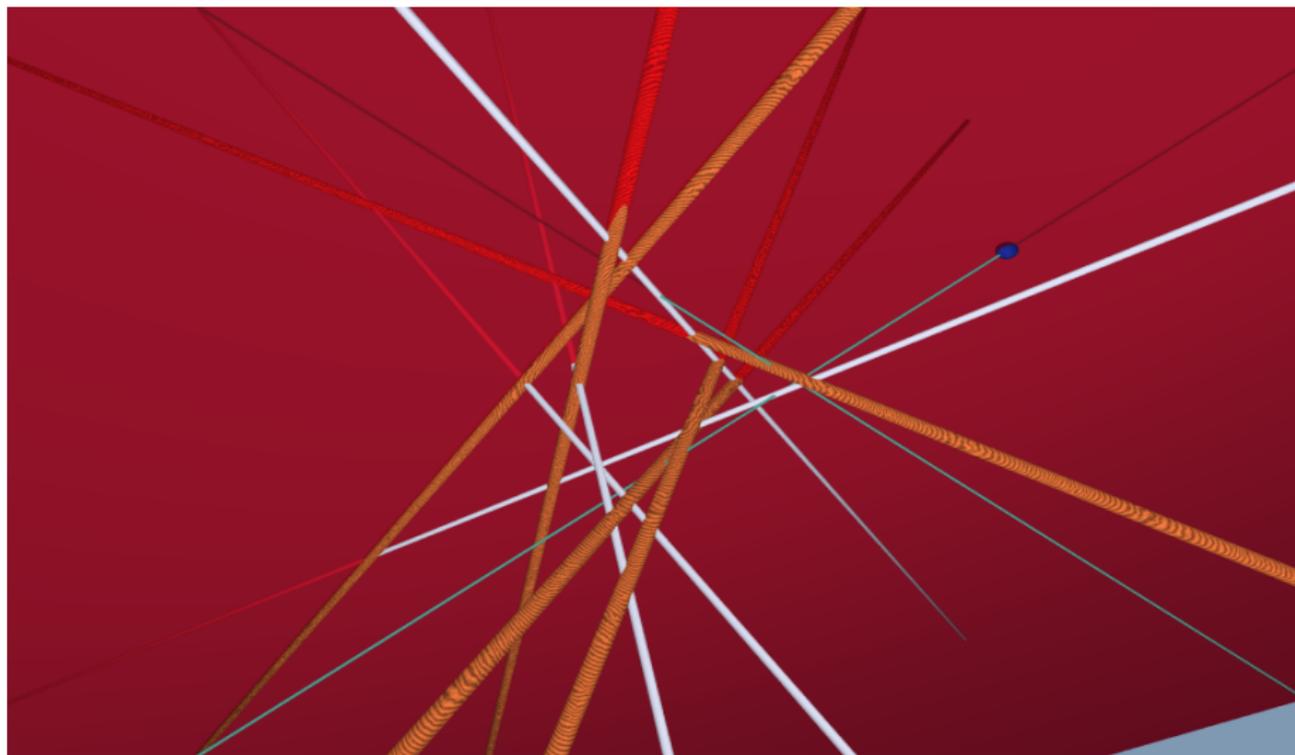
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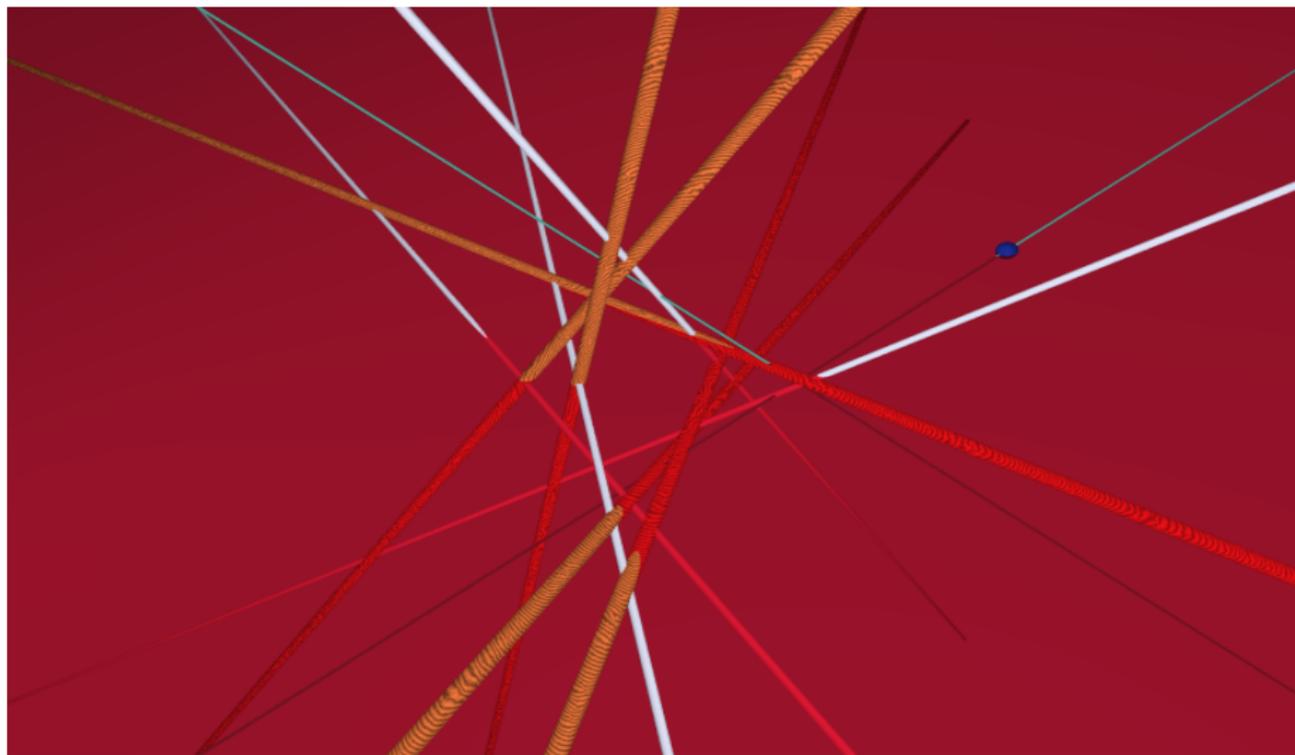
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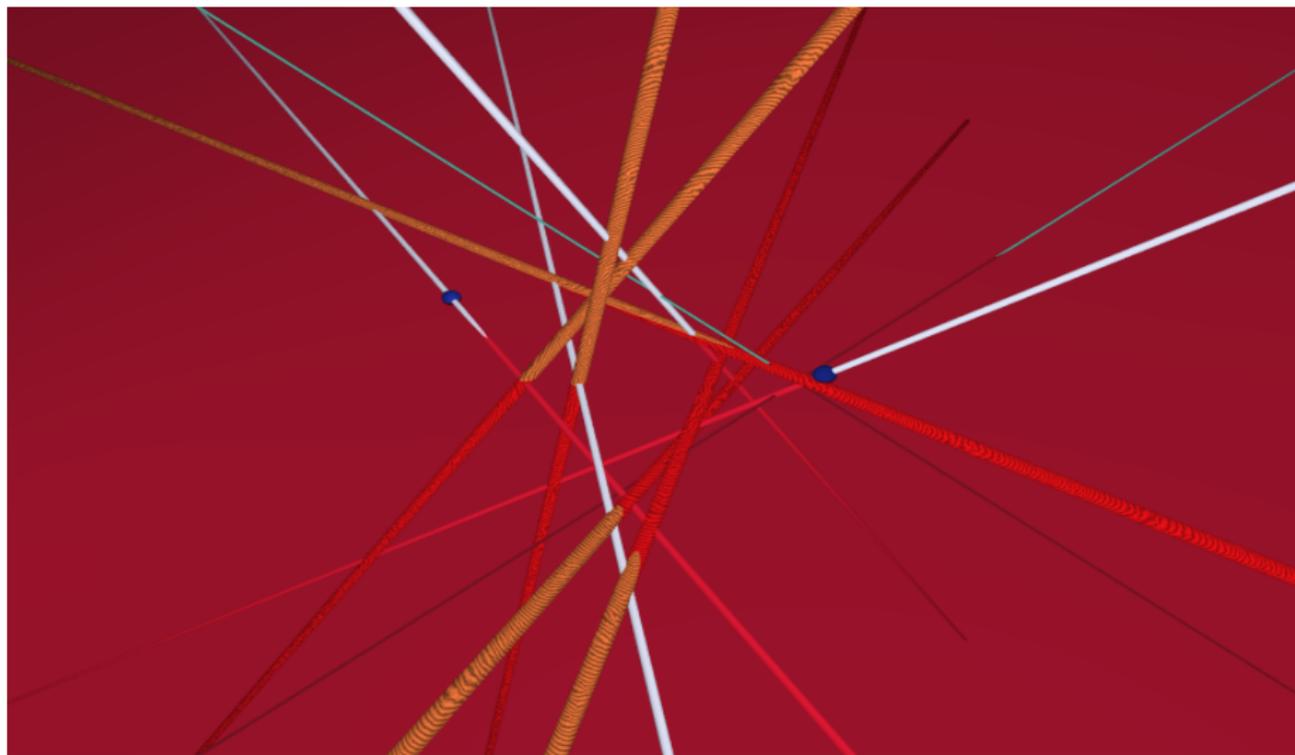
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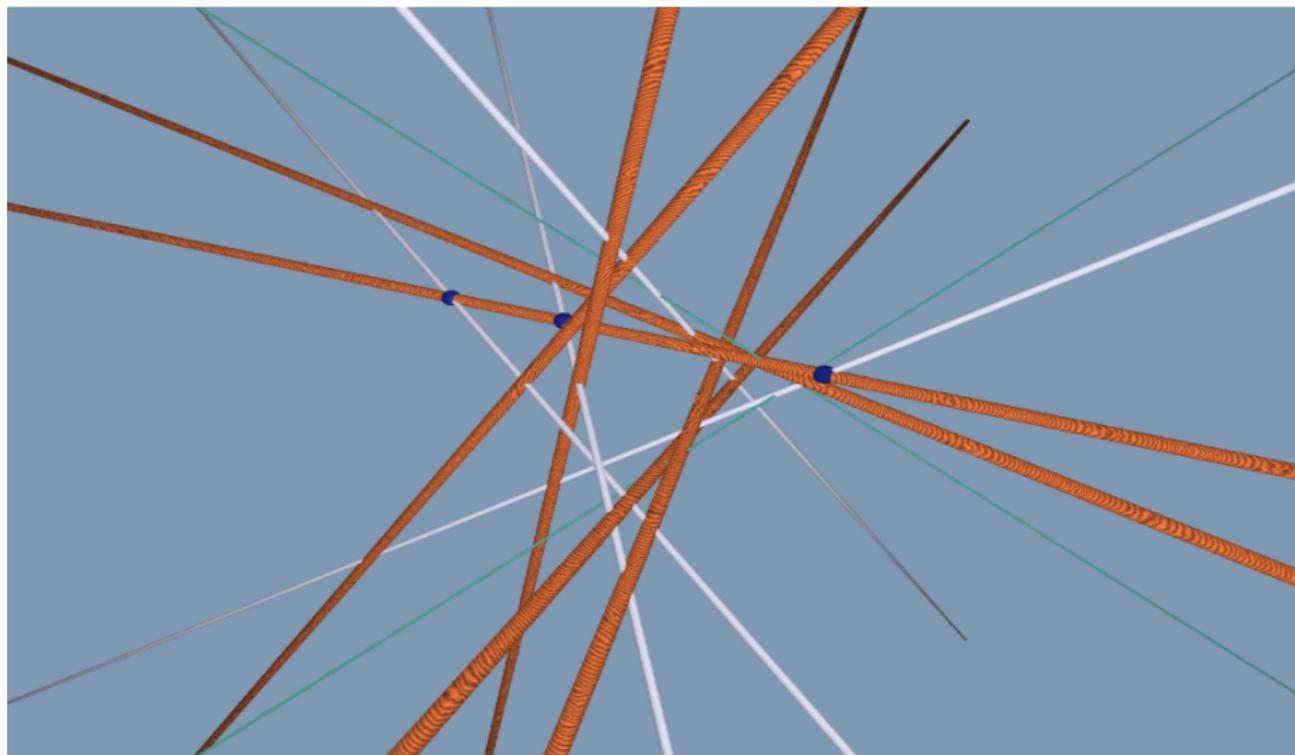
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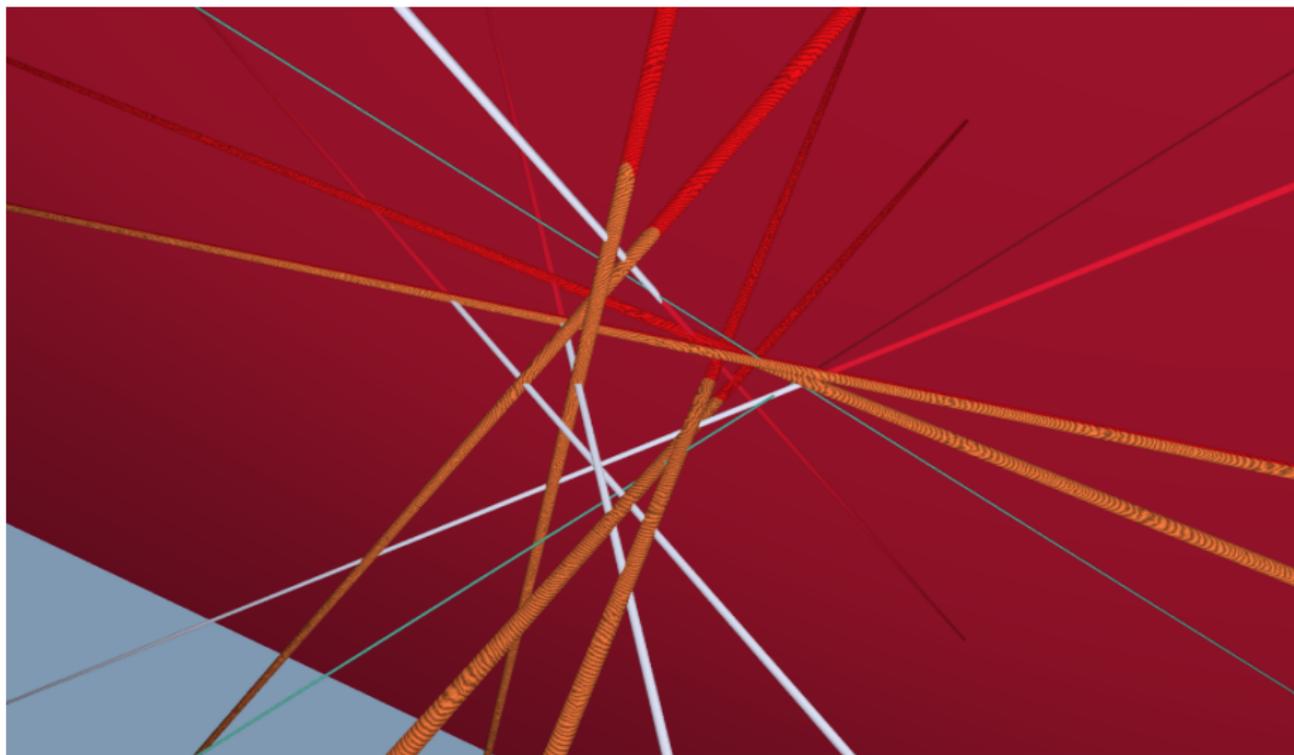
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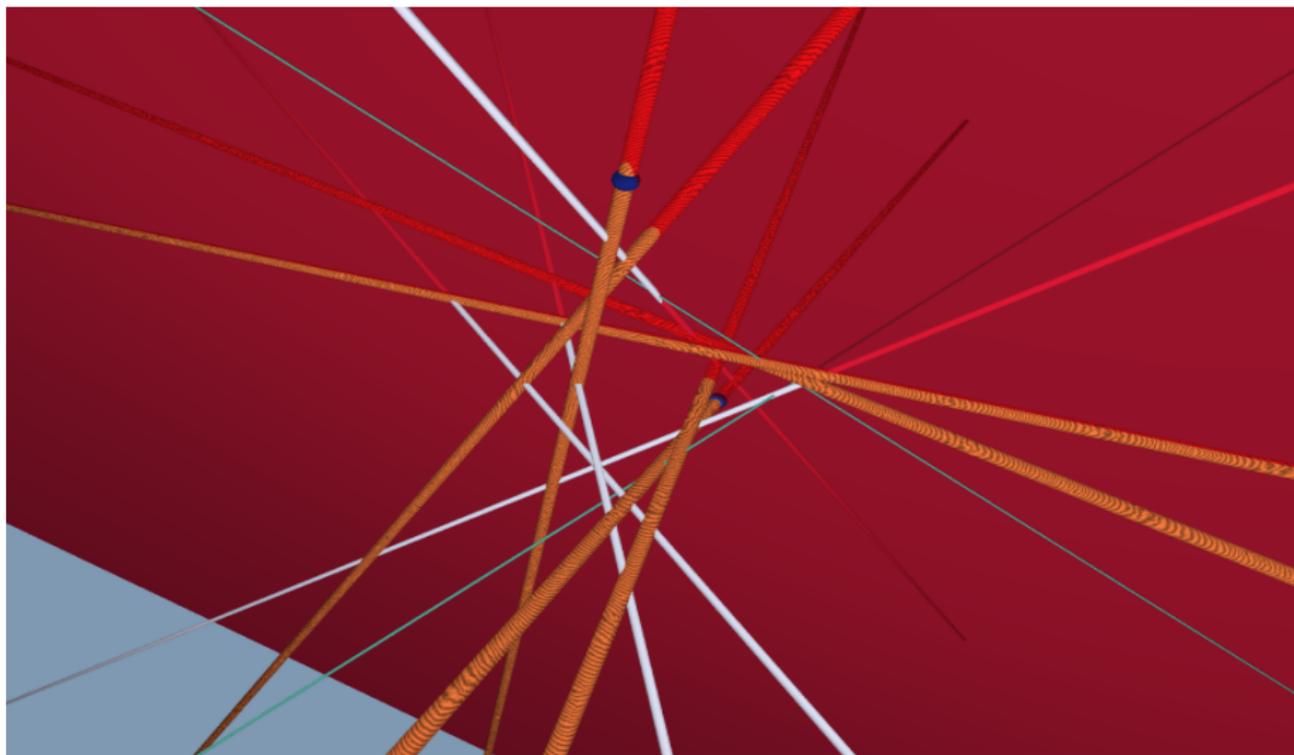
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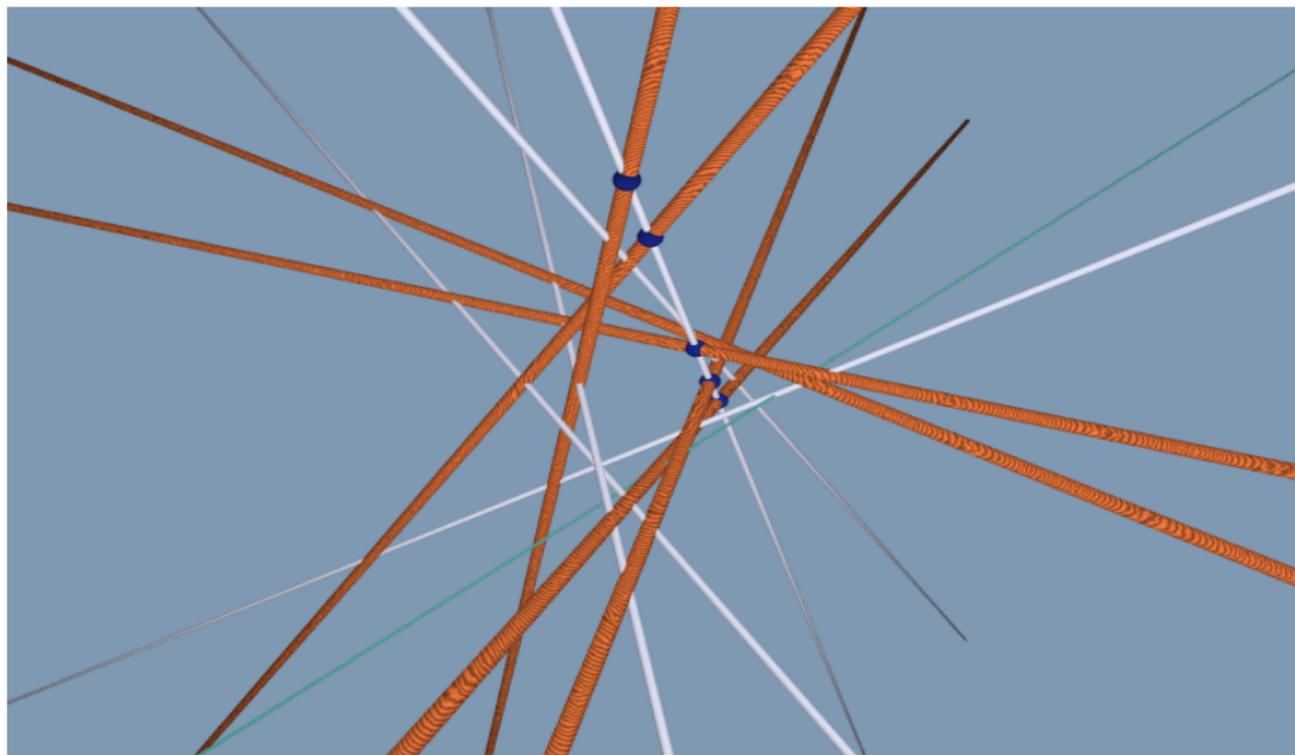
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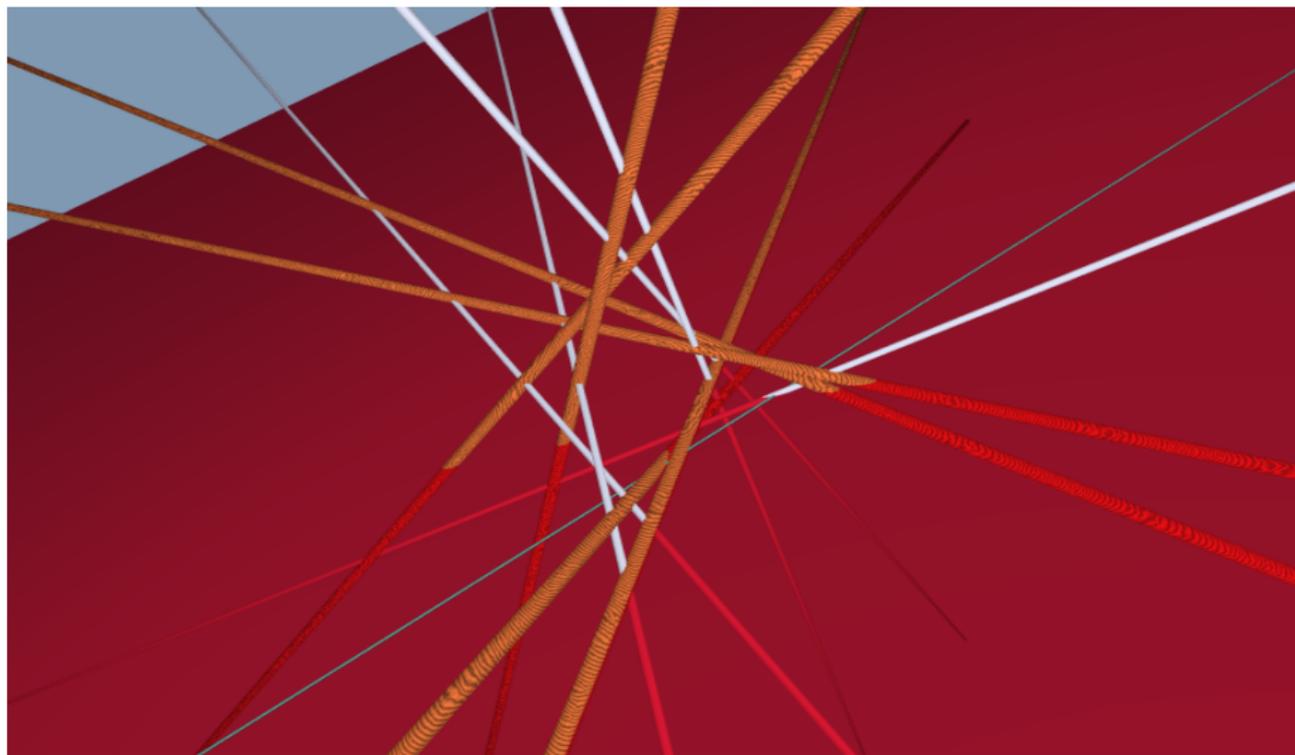
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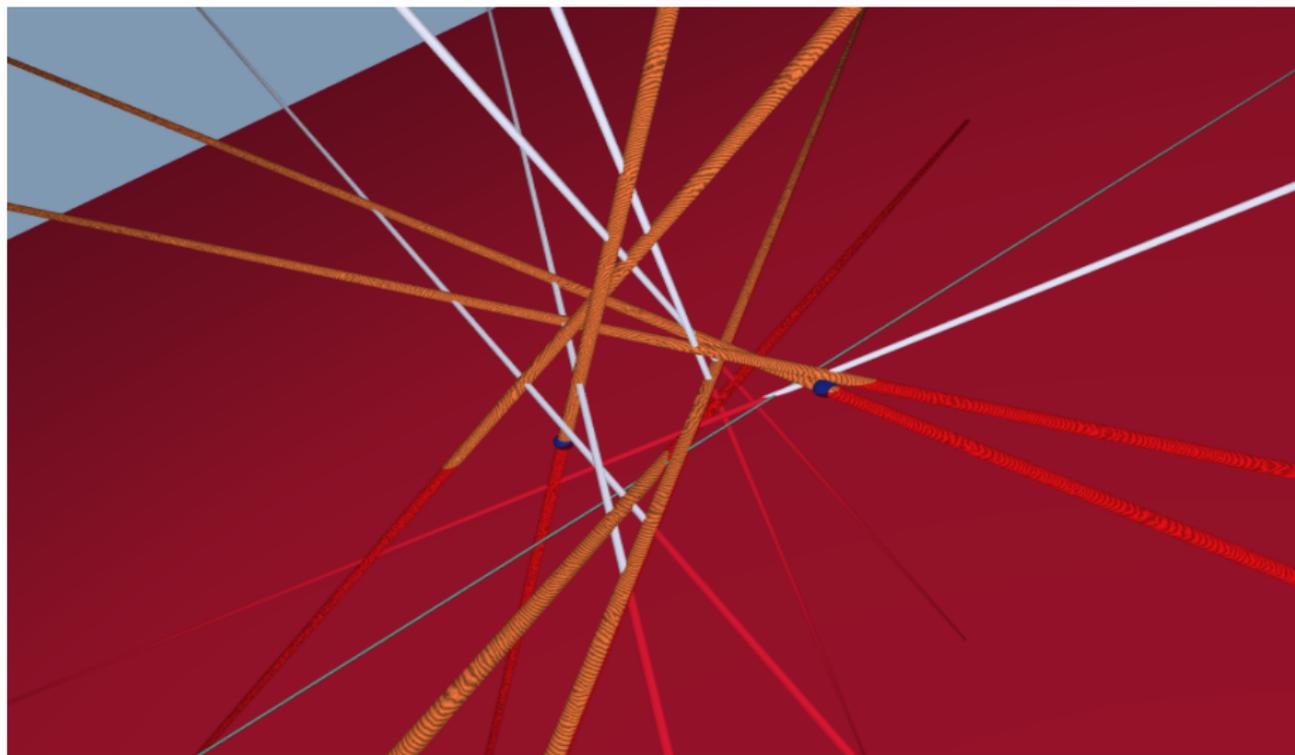
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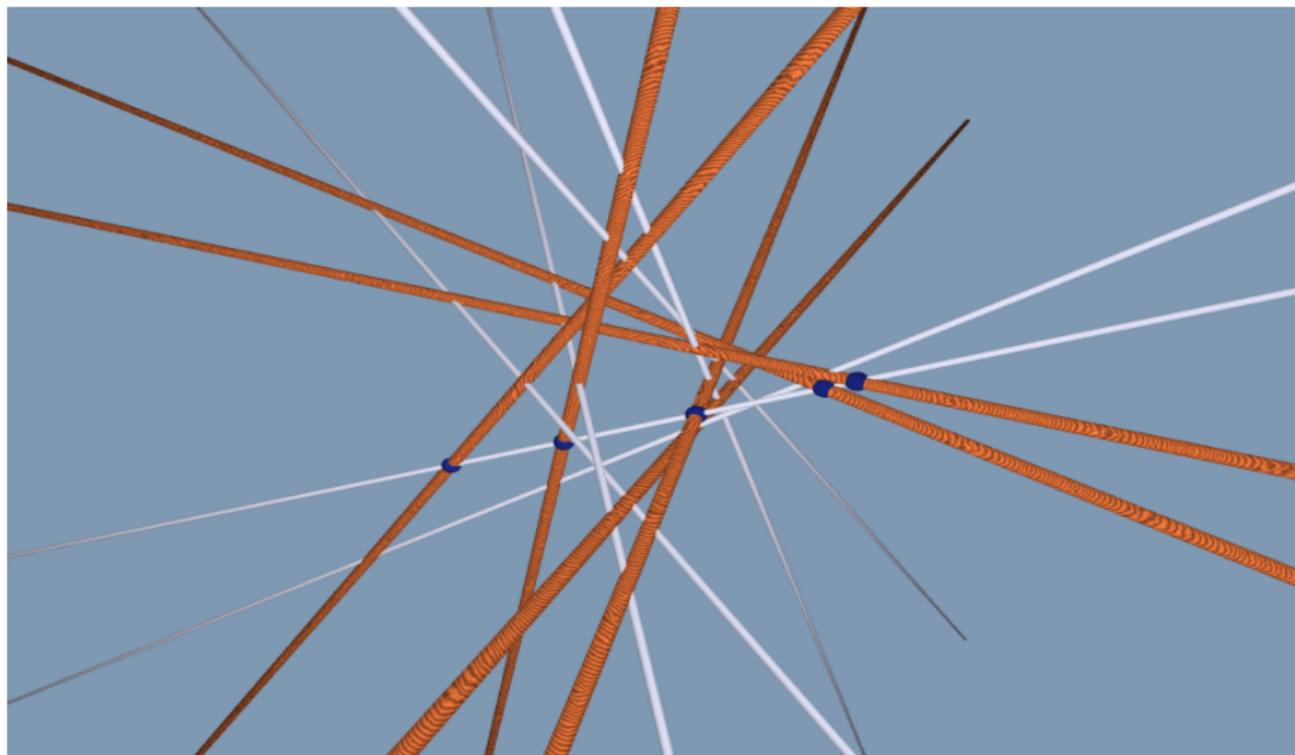
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The geometric construction



The geometric construction



The completed figure

