Combinatorics
Practice Problem Set
Answers

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1. How many straight lines can be formed by 8 points of which 3 are collinear?

   Answer ${}^8C_2 - {}^3C_2 + 1$ (general formula ${}^nC_2 - {}^rC_2 + 1$)

2. How many triangles can be formed by 8 points of which 3 are collinear?

   Answer ${}^8C_3 - {}^3C_3$ (genral formula${}^nC_3 - {}^rC_3$)

3. How many committees of 5 students can be selected from a class of 25?

   Answer Choose 5 out of 25 $= {}^{25}C_5 = \frac{25\times24\times23\times22\times21}{1\times2\times3\times4\times5} = 53130$

4. How many 10-letter patterns can be formed from the letters of the word "BASKETBALL"?

   Answer toatal letters=10,B=2,A=2,S=1,K=1,E=1,T=1,L=2 so total no of 10 letter words $= \frac{10!}{2!2!1!1!1!1!2!} = 604800$

5. A box contains 12 black and 8 green marbles. How many ways can 3 black and 2 green marbles be chosen?

   Answer ${}^{12}C_3 + {}^8C_2 = 248$

6. 8 students on a student council are assigned 8 seats around a U-shaped table.
   a) How many different ways can the students be assigned seats at the table?
   b) How many ways can a president and a vice-president be elected from the 8 students?

   Answer a) 8! ways b)${}^8C_1 \times {}^7C_1 = 56$

7. A Club consists of 20 members, of which 9 are male and 11 are female.Seven members will be selected to form an event-planning committee. How many committees of 4 females and 3 males can be formed?

Answer Total no of committees $= {}^{11}C_4 \times {}^{9}C_3$

8. How many 7-digit telephone numbers can be formed if the first digit cannot be 0 or 1?

   Answer total no of telephone numbers $= 8 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

9. Six people are seated at a round table to play a game of cards. a) Is the seating arrangement around the table a linear or circular permutation? b) How many possible seating arrangements are there?

   Answer a)circular permutation b)$(6-1)! = 120$ possible arrangements

10. How many different 5-digit street addresses can have the digits 4, 7, 3, 4, and 8?

    Answer Total no of 5-digit addresses $= \frac{5!}{2!} = 60$

11. Three hardcover books and 5 paperbacks are placed on a shelf. How many ways can the books be arranged if all the hardcover books must be together and all the paperbacks must be together?

    Answer $2! \times 3! \times 5!$

12. How many permutations are there of the word "SCHOOL"?

    Answer total letter = 6,S=1,C=1,H=1,O=2,L=1 So total no of Permutation$= \frac{6!}{2!} = 360$

13. How many ways can you choose 4 groups of 4 people from 16 people, assuming the groups are distinct?

    Answer ${}^{16}C_4 + {}^{12}C_4 + {}^{8}C_4 + {}^{4}C_4$

14. In a race with 30 runners where 8 trophies will be given to the top 8 runners (the trophies are distinct: first place, second place, etc), how many ways can this be done?

    Answer ${}^{30}C_8 \times 8$

15. How many ways can you do the above problem if a certain person, Ram, must be one of the top 3 winners?

    Answer $3 \times {}^{29}C_7 \times 7$

16. How many ways can you arrange 16 people into 4 rows of 4 desks each?

    Answer Total no of ways =16!

17. How many ways can you pair up 8 boys and 8 girls?

Answer total no of ways of pairing $= 8 \times 8 = 64$

18. In how many ways can a party of 4 men and 4 women be seated at a circular table so that no two women are adjacent?

    Answer The 4 men can be seated at the circular table such that there is a vacant seat between every pair of men in (4-1)! =3! Ways. Now 4 vacant seats can be occupied by 4 women in 4! Ways. Hence the required number of seating arrangements $= 3!X4! = 144$

19. Make all arrangement of letters of the word TAMIL so that
    a) T is always next to L
    b) T and L are always together

    Answer a) Let's keep LT together and consider it one letter. Now we have 4 letters which can be arranged in a row
    in 4P4 =24
    b) Now L and T can interchange their places in 2! Ways. So total arrangements $= 4!2! = 48$

20. Out of 2 Women and 5 Men, a committee of 3 is to be formed. In how many ways can it be formed if at least one woman is to be included?

    Answer A Committee can be formed in the following ways 1 woman and 2 men OR 2 women and 1 man ${}^2C_1 \times {}^5C_2 + {}^2C_2 \times {}^5C_1 = 20 + 5 = 25$

21. In how many ways can a cricket eleven be chosen out of a batch of 15 players if
    a)There is no restriction on the selection.
    b)A Particular Player is always chosen.
    c)A Particular Player is never chosen.

    Answer a)There is no restriction on the selection $= {}^{15}C_{11}$
    b) A Particular Player is always chosen $= {}^{14}C_{10}$
    c)A Particular Player is never chosen $= {}^{14}C_{11}$