THE MANY AVATARS OF GALILEAN CONFORMAL SYMMETRY

Arjun Bagchi

IISER PUNE

Indian Strings Meet 2014

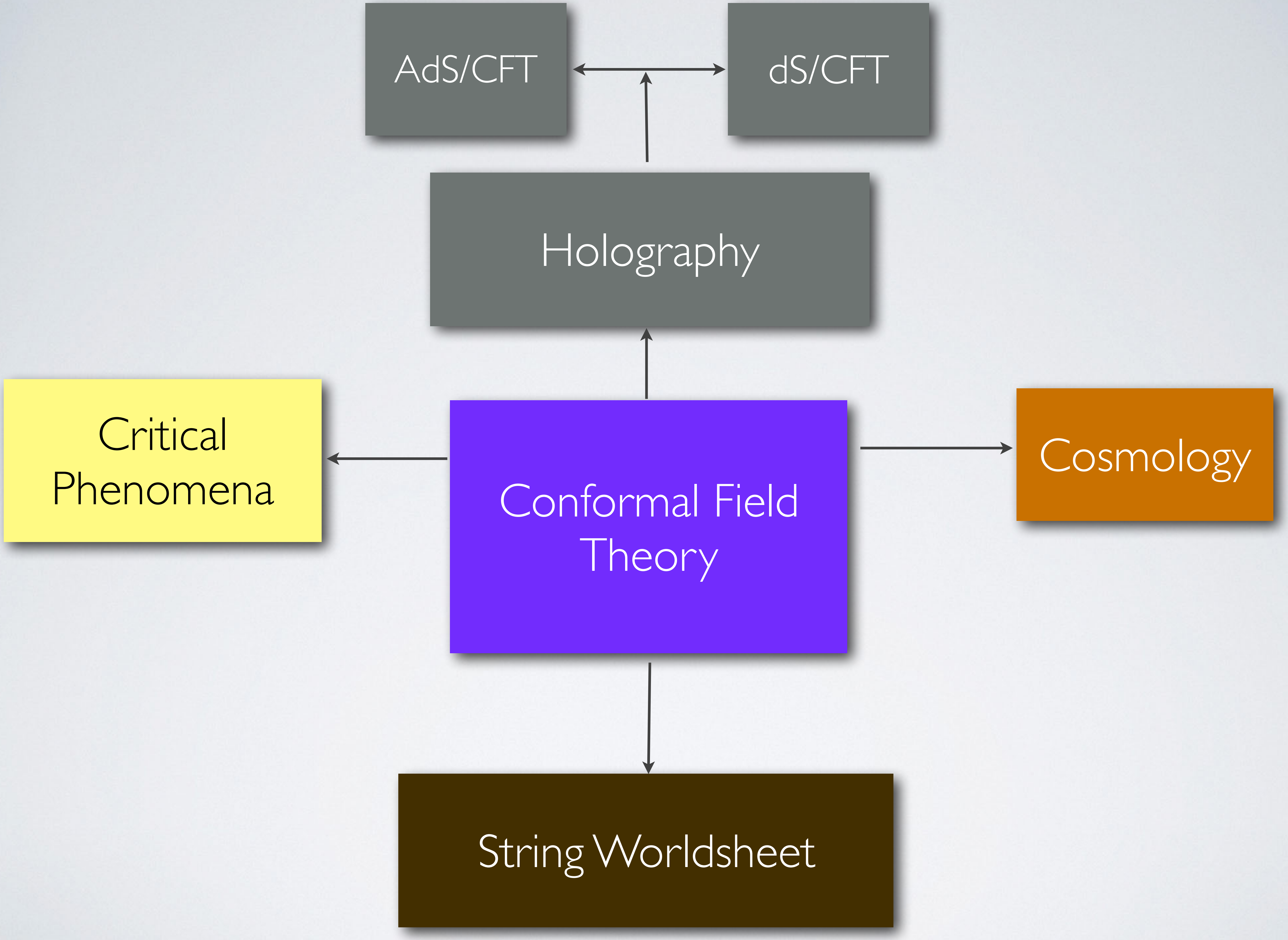
December 18, 2014.

CONFORMAL FIELD THEORY

✦ Conformal field theories are amongst the most powerful tools in modern theoretical physics.

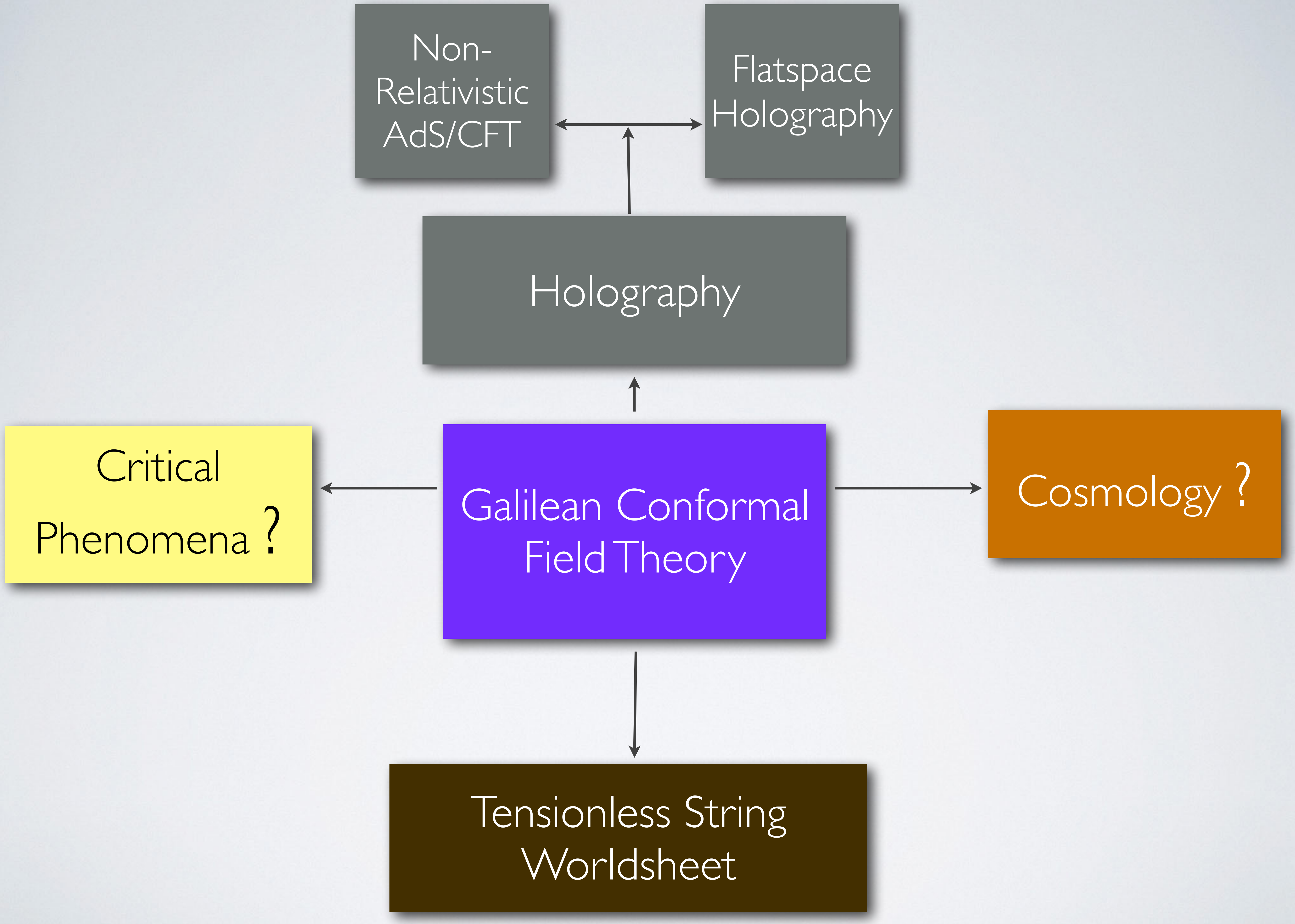
✦ This is especially true for 2D CFTs where the underlying symmetry is two copies of the Virasoro algebra which is infinite-dimensional.

✦ Uses diverse...


AdS/CFT
dS/CFT
Holography
Critical Phenomena
Conformal Field Theory
Cosmology
String Worldsheet

GALILEAN CONFORMAL FIELD THEORY

- We seem to have found another very useful symmetry.
- These are field theories with symmetries that are contractions of the relativistic conformal symmetry.
- Contractions first understood a non-relativistic limit hence the name.
- This has the promise of being even more powerful as unlike CFTs, the symmetries are infinite dimensional in all spacetime dimensions and not just D=2.



Non-Relativistic AdS/CFT
Flatspace Holography
Holography
Critical Phenomena ?
Galilean Conformal Field Theory
Cosmology ?
Tensionless String Worldsheet

WHAT IS A GCFT?

- Start with a CFT in D dimensions. Symmetry: SO(2,d)
- Use Inonu-Wigner contractions to contract the algebra.
- Number of generators stay the same. (d+2)(d+1)/2
- Re-write this algebra in a suggestive form.
- Give it an infinite dimensional lift.

Poincare: $\{J_{\mu\nu}, P_\mu\}$

Relativistic conformal : $\{D, K_\mu\}$

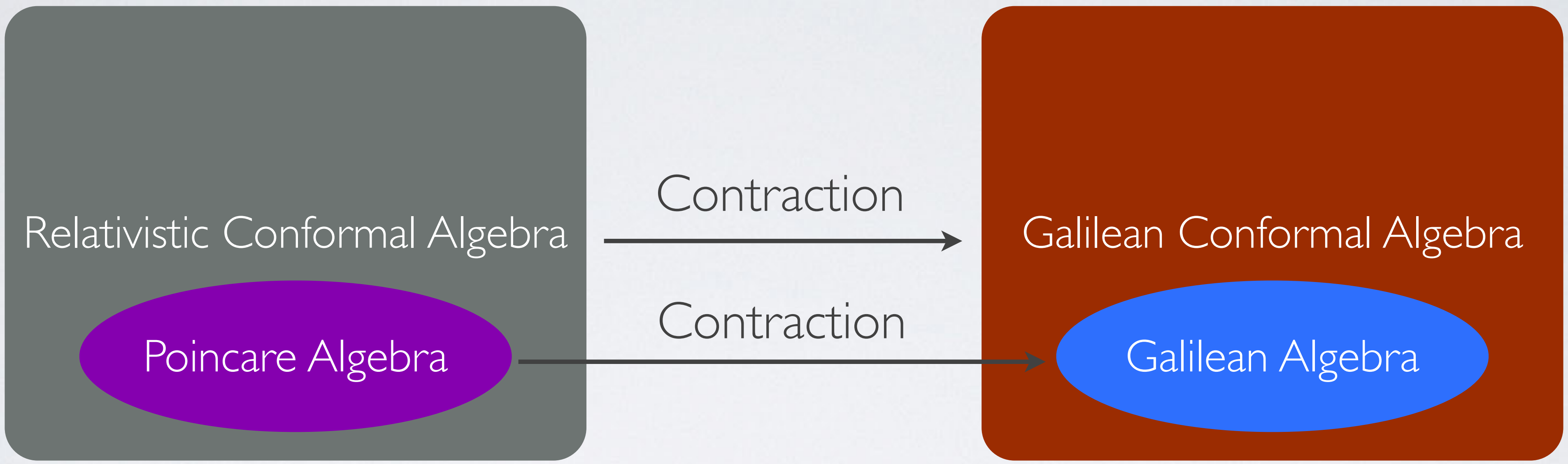
Non Relativistic Limit: $x_i \to \epsilon x_i,\, t \to t,\, \epsilon \to 0 (\Rightarrow v_i \sim \frac{x_i}{t} \to \epsilon \to 0)$

Example of a contraction

$$J_{0i} = x_i \partial_t + t\partial_i \to \epsilon x_i \partial_t + \frac{1}{\epsilon} t\partial_i$$

$$B_i = \lim_{\epsilon \to 0} \epsilon\, J_{0i} = t\partial_i$$

WHAT IS A GCFT?



GCA generators:

$$\{H, D, K_0\} \rightarrow \{L_{-1}, L_0, L_{+1}\} \Rightarrow L_n = t^{n+1}\partial_t + (n+1)t^n x_i \partial_i$$
$$\{P^i, B^i, K^i\} \rightarrow \{M^i_{-1}, M^i_0, M^i_1\} \Rightarrow M^i_n = t^{n+1}\partial_i$$

Symmetry Algebra:

$$[L_n, L_m] = (n-m)L_{n+m}$$
$$[L_m, M^i_n] = (n-m)M^i_{n+m}$$
$$[M^i_n, M^j_m] = 0.$$

Now if we say that these vector fields are valid for any integer n, we get an infinite dimensional symmetry algebra in which this finite algebra is embedded. This is the infinite lift of the GCA.

Non Relativistic AdS/CFT

★ Holography mainly considered in the relativistic setting.

★ Many systems are inherently non-relativistic,- condensed matter systems, hydrodynamics..

★ [*AB, Gopakumar 2009*]: First systematic NR limit of AdS/CFT.

★ Boundary theory: GCFT in (d+1)-dimensions. Infinite symmetry in all dimensions.

★ Bulk theory: Novel Newton-Cartan like $AdS_2 \times R^d$

★Many aspects explored since.

★[*AB, Mandal 2009*]: Correlation functions and representations.

- Can build representations in a way similar to CFTs.
- Label states with L_0, M_0
- Primary states annihilated by $L_n, M_n \; (n > 0)$, representations built on this by acting with raising operators.
- Two and three point functions entirely fixed by global Galilean Conformal invariance.

Current Investigations: In search of field theories

AB, R Basu, A Mehra 1408.0810

The "Narrow" Picture

- Electrodynamics: Maxwell's equations are invariant under conformal transformations in D=4.
- Galilean Electrodynamics: non-relativistic limit of ED [Le Ballac, Levy-Leblond 1973]
- Two distinct limits: Electric and Magnetic.
- Is there a symmetry enhancement for D=4? Yes!
- Is this an infinite enhancement? Yes!
- First example of a GCFT for D>2. Infinite symmetries for D>2 is remarkable.

The Broad Picture

- Do such enhancements exist for Yang-Mills theory? [AB, R Basu, A Kakkar, A Mehta (work in progress).]
- Initial investigations indicate that this is the case.
- Super-Yang-Mills theory?
- If these infinite symmetries exist, they could be pointing to integrable structures.
- New integrable sub-sectors in SYM ... new integrable sub-sectors in AdS/CFT?

FLAT HOLOGRAPHY

- Flat spacetimes can be obtained as a limit of AdS where radius of AdS goes to ∞
- Flat holography: limit of AdS/CFT?
- Interesting successes recently. Key ingredient is the notion of asymptotic symmetries.

$$\text{Asymptotic Symmetry Group} = \frac{\text{Allowed Diffeomorphisms}}{\text{Trivial Diffeomorphisms}}$$

- Ofter ASG = isometry of vacuum. e.g. ASG(AdS) for d=4 and higher is SO(d,2)
- But there are famous exceptions: ASG (AdS3) = Virasoro $*$ Virasoro.
- States of Quantum gravity in AdS3 would form representations of infinite algebra.
- Much of the magic of the AdS3/CFT2 correspondence is due to the underlying infinite dimensional symmetry.
- Similar infinite dimensional ASGs exist for flat spacetimes in 3D and 4D.
- We use them for constructing flat holography.

FLAT HOLOGRAPHY

The BMS/GCA Correspondence

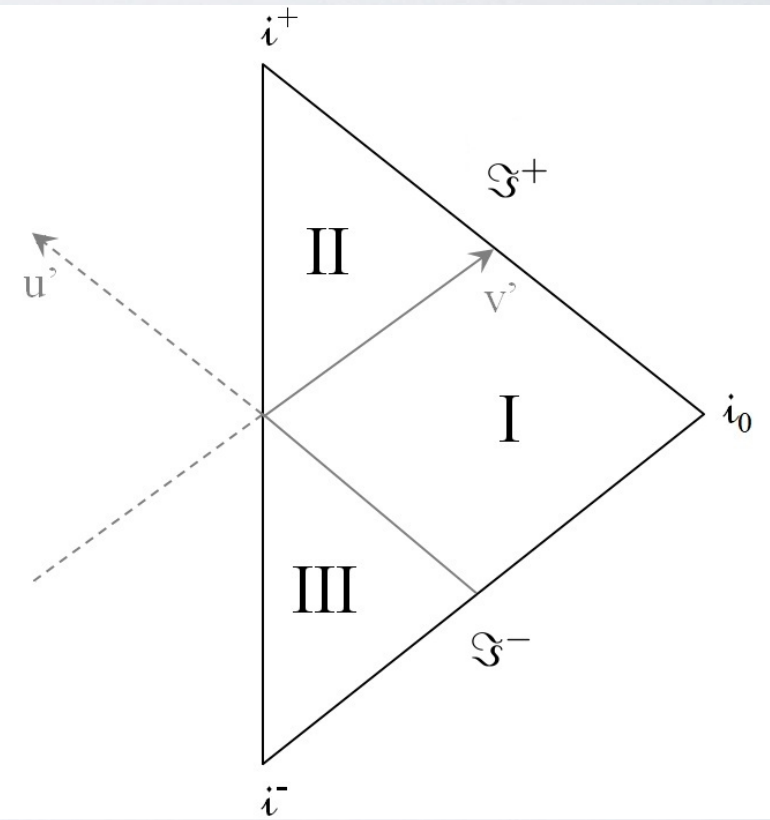
- We will concentrate on 3D flat space.
- Asymptotic symmetries at null infinity in 3D flatspace is given by the infinite dimensional BMS algebra.

BMS$_3$: Infinite-dimensional algebra!

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} \\ [L_m, M_n] &= (m-n)M_{m+n}, \\ [M_m, M_n] &= 0. \end{aligned}$$

L_m = Diff (S^1).
M_m = "Super" translations (f(θ)).

[Ashtekar, Bicak, Schmidt 1996]; [Barnich-Compere 2006].



Central Extension: Can compute Virasoro-like central extensions.

$$c_{LL} = c_{MM} = 0; \; c_{LM} = \frac{3}{G}\delta_{m+n,0}(m^3 - m).$$

$\boxed{\text{Observation (BMS/GCA): BMS}_3 \text{ is } \textit{isomorphic} \text{ to GCA}_2.}$ [AB 2010.]

FLAT HOLOGRAPHY

The BMS/GCA Correspondence

- The dual of 3D flat space is a field theory with the symmetries of a 2D GCA i.e. a 2D GCFT.
- Can obtain the symmetries of flat space from the symmetries of AdS.

Asymptotic symmetry group of AdS_3 = Vir $\otimes$ Vir.

Asymptotic symmetry algebra: $[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m} + \frac{c}{12}\delta_{n+m,0}(n^3 - n)$ and similarly for $\bar{\mathcal{L}}_n$. Here $c = \bar{c} = \frac{3\ell}{2G}$. [Brown, Henneaux 1986.]

Flat space arises as a limit of AdS when the AdS radius is taken to infinity. This is a contraction from the algebraic sense.

BMS algebra is generated by a simple contraction of the linear combinations of $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$.

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \frac{1}{\ell}(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}) \tag{1}$$

where ℓ is the AdS radius.

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + c_L\delta_{n+m,0}(n^3 - n). \\ [L_n, M_m] &= (n-m)M_{n+m} + c_M\delta_{n+m,0}(n^3 - n). \\ [M_n, M_m] &= 0. \end{aligned} \tag{2}$$

Naturally generates the central charges: $c_M = \frac{1}{\ell}(c + \bar{c}) = \frac{3}{G}$ and $c_L = c - \bar{c} = 0$ as $c = \bar{c} = \frac{3\ell}{2G}$.

The Curious Case of the Flat BTZ

- Can perform the same flat limit on the BTZ black hole.
- Obvious immediate problem: no black holes in 3d flat space. What is going on?
- Start with the non-extremal BTZ

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2\ell^2} dt^2 + \frac{r^2\ell^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 + r^2\left(d\phi + \frac{r_+ r_-}{\ell r^2} dt\right)^2 .$$

$$r_\pm = \sqrt{2G\ell(\ell M + J)} \pm \sqrt{2G\ell(\ell M - J)}$$

Flat limit $\ell \to \infty$: $r_+ \to \ell\sqrt{2GM} = \ell \hat{r}_+, \quad r_- \to r_0 = \sqrt{\frac{2G}{M}}\, J.$

r_+ is pushed out to infinity. One is left with the inside.

Radial coordinate r becomes time-like and the time coordinate t becomes spatial.

Cosmology!

Will call this a Flat Space Cosmology (FSC).

$$\text{Metric:} \quad ds^2_{\text{FSC}} = \hat{r}_+^2 dt^2 - \frac{r^2\, dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 d\phi^2 - 2\hat{r}_+ r_0 dt d\phi$$

Successes of (our version of) Flat Holography

AB, S. Detournay, J. Simon, R. Fareghbal. 1208.4372

Matching of FSC Entropy by a GCFT calculation.

Can associate a temperature to the FSC horizon. $T_{\text{FSC}} = \frac{\kappa}{2\pi} = \frac{\hat{r}_+^2}{2\pi\, r_0}$.

Using Bekenstein-Hawking area law to define the entropy of the FSC we get

$$\boxed{S_{\text{FSC}} = \frac{\text{Area of horizon}}{4G} = \frac{\pi r_0}{2G} = \frac{\pi J}{\sqrt{2GM}}}$$

$$\boxed{S_{\text{GCFT}} = \ln d(h_L, h_M) = \pi\left(c_L\sqrt{\frac{2h_M}{c_M}} + h_L\sqrt{\frac{2c_M}{h_M}}\right).}$$

$$h_L = h - \bar{h} = j + \frac{c_L}{2}, \quad h_M = \frac{1}{\ell}(h + \bar{h}) = m + \frac{c_M}{2}; \quad c_L = 0, \quad c_M = \frac{1}{4G}$$

$$\boxed{S_{\text{GCFT}} = S_{\text{FSC}}}$$

• Sub-leading term also is of an expected form.

AB, R Basu. 1312.5748

$$\boxed{S_{\text{FSC}} = \frac{2\pi r_0}{4G} - \frac{3}{2}\log\left(\frac{2\pi r_0}{4G}\right) - \frac{3}{2}\log\kappa + \text{ constant}}$$

Successes of (our version of) Flat Holography

AB, S. Detournay, D. Grumiller, J. Simon. 1305.2919

Novel Phase transitions between FSC and hot flat space.

•Analogues of Hawking-Page, but now between time independent and time dependent solutions.

Free Energies:

$$\boxed{F_{\mathrm{HFS}} = T\,\Gamma_{\mathrm{HFS}} = -\frac{1}{8G} \qquad F_{\mathrm{FSC}} = T\,\Gamma_{\mathrm{FSC}} = -\frac{r_+^2}{8G}}$$

Main conclusion: there is a phase transition between HFS and FSC.

$r_+ > 1$: FSC is the dominant saddle.

$r_+ < 1$: HFS is the dominant saddle.

$r_+ = 1$: FSC and HFS coexist.

The phase transition occurs at the critical temperature

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}\,.$$

Successes of (our version of) Flat Holography

AB, R Basu, D Grumiller, M Riegler 1410.4089

Matching Entanglement Entropy

•The infinite dimensional symmetry of the GCA allows us to compute Entanglement Entropy exactly in GCFTs.

• In 2D GCFTs, the calculations proceed in a way similar to the Cardy-Calebrese analysis for 2D CFTs. We use Ward identities of GCFT EM tensor with twist fields to obtain an analytic expression for EE in these field theories.

$$S = (c_L/6) \log (x/a) + (c_M/6) (t/x).$$

•This answer can be obtained holographically by either a computation of geodesics from the null boundary of flat space, or also by a computation of Wilson lines in the equivalent Chern-Simons formulation of 3D Gravity.

•Current investigations: matching with thermal entropy of the FSC in the large temperature limit. More general answers from the geodesic analysis. Quantum quenches in GFCTs and their bulk counterparts.

TENSIONLESS STRINGS AND GCA

- Tensionless strings: Why bother?

Tensionless strings have been studied since Schild in 1977.

- Limit expected to probe string theory at *very high energies*.
- Supposed to uncover a sector with larger symmetry.
  - String theory $\Rightarrow$ infinite tower of massive particles of arbitrary spin.
  - In this limit all of them become massless.
  - Expect *higher spin symmetry structures* to arise.
- Of interest to the recent higher spin dualities.
  [Klebanov-Polyakov ’02, Sezgin-Sundell ’02, Gaberdiel-Gopakumar ’10]

Folklore: Tensionless Type IIB strings on $AdS_5 \otimes S^5$ $\Rightarrow$ higher-spin gauge theory.
[Witten ’01, Sundborg ’01, ...]

Aim(1): Understand string theory in this “ultra-stringy” regime.

Aim(2): Make connection between tensionless strings and higher spins concrete.

Lacking: An organising principle (like 2d CFT for string theory). We aim to rectify this.

TENSIONLESS STRINGS AND GCA

- Classical Closed Tensionless strings

Start with Nambu-Goto action

$$S = -T \int d^2\xi \sqrt{-\det \gamma_{\alpha\beta}}$$

To take the tensionless limit, first switch to Hamiltonian framework.

Generalised momenta: $P_m = T\sqrt{-\gamma}\gamma^{0\alpha}\partial_\alpha X_m$.

Constraints: $P^2 + T^2\gamma\gamma^{00} = 0, \;\; P_m \partial_\alpha X^m = 0$.

Hamiltonian of the system: $\mathcal{H} = \lambda(P^2 + T^2\gamma\gamma^{00}) + \rho^\alpha P_m \partial_\alpha X^m$.

Action after integrating out momenta:

$$S = \frac{1}{2}\int d^2\xi \, \frac{1}{2\lambda}\left[\dot{X}^2 - 2\rho^\alpha \dot{X}^m \partial_\alpha X_m + \rho^a \rho^b \partial_b X^m \partial_a X_m - 4\lambda^2 T^2 \gamma\gamma^{00}\right]$$

Identifying $g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 + 4\lambda^2 T^2 \end{pmatrix}$

Action takes the familiar Weyl-invariant form

$$S = -\frac{T}{2}\int d^2\xi \sqrt{-g}g^{\alpha\beta}\partial_\alpha X^m \partial_\beta X^n \eta_{mn}.$$

TENSIONLESS STRINGS AND GCA

Tensionless limit can be taken at various steps.

Metric density $T\sqrt{-g}g^{\alpha\beta}$ degenerates and is replaced by a rank-1 matrix $V^\alpha V^\beta$ where V^α is a vector density

$$V^\alpha \equiv \frac{1}{\sqrt{2}\lambda}(1, \rho^a) \tag{5}$$

Action in $T \to 0$ limit

$$S = \int d^2\xi \;\, V^\alpha V^\beta \partial_\alpha X^m \partial_\beta X^n \eta_{mn}. \tag{6}$$

Tensionless action is invariant under world-sheet diffeomorphisms.

Fixing gauge: "Conformal" gauge: $V^\alpha = (v, 0)$ (v: constant).

Tensile: Residual symmetry after fixing conformal gauge = Vir $\otimes$ Vir.
Central to understanding string theory.

Tensionless: Similar residual symmetry left over after gauge fixing.

TENSIONLESS STRINGS AND GCA

Tensionless residual symmetries:

$$\delta\xi^\alpha = \lambda^\alpha,\ \lambda^\alpha = (f'(\sigma)\tau + g(\sigma), f(\sigma)) \qquad \text{where} \quad f, g = f(\sigma), g(\sigma)$$

Define: $L(f) = f'(\sigma)\tau\partial_\tau + f(\sigma)\partial_\sigma, \quad M(g) = g(\sigma)\partial_\tau\,.$

Expand: $f = \sum a_n e^{in\sigma}, g = \sum b_n e^{in\sigma}$

$$\begin{aligned} L(f) &= \sum_n a_n e^{in\sigma}(\partial_\sigma + in\tau\partial_\tau) = -i\sum_n a_n L_n \\ M(g) &= \sum_n b_n e^{in\sigma}\partial_\tau = -i\sum_n b_n M_n\,. \end{aligned}$$

Symmetry algebra in terms of Fourier modes:

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + C_1 m(m^2-1)\delta_{m+n,0}, \quad [M_m, M_n] = 0. \\ [L_m, M_n] &= (m-n)M_{m+n} + C_2 m(m^2-1)\delta_{m+n,0}\,. \end{aligned}$$

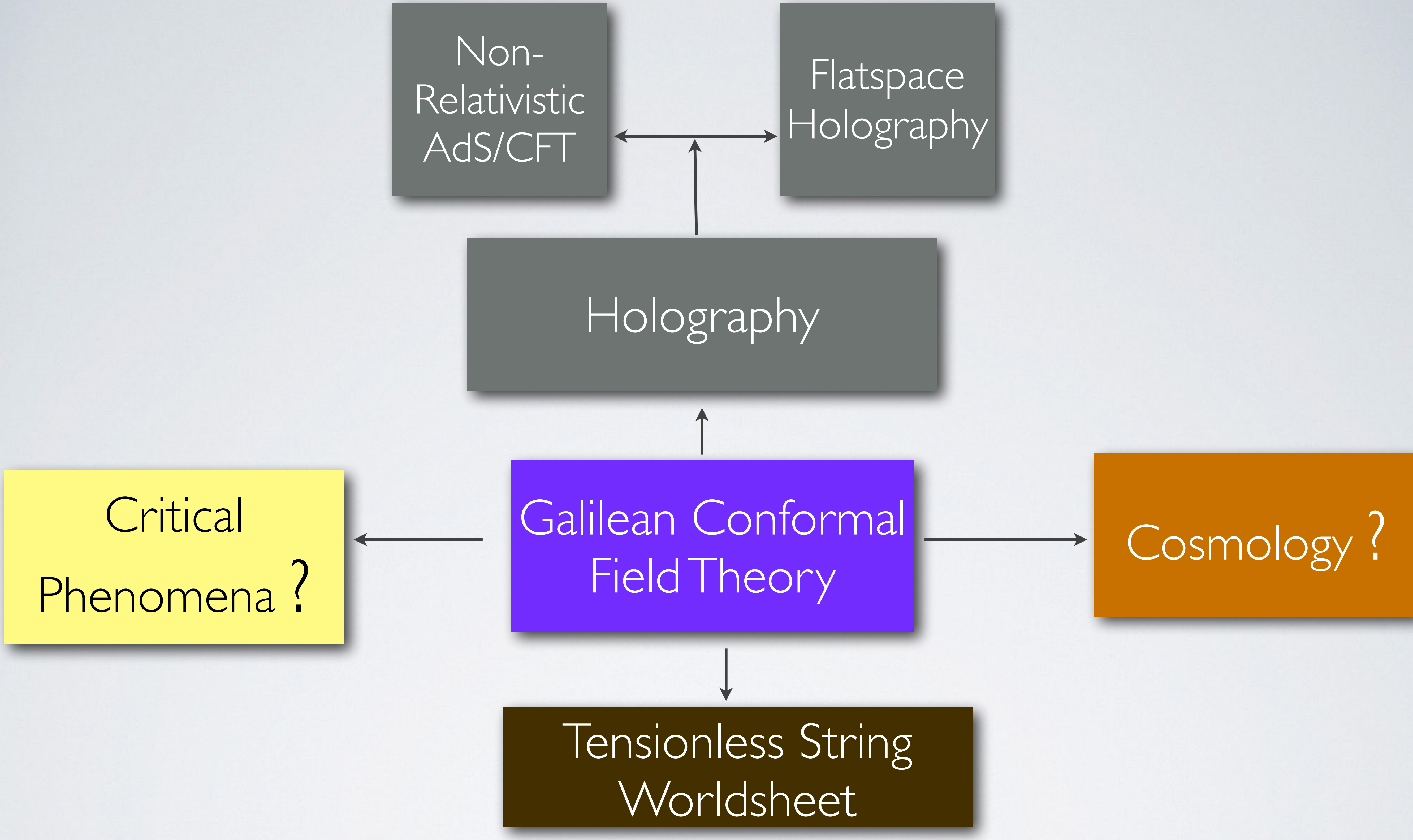
TENSIONLESS STRINGS AND GCA

- We have understood how the tensionless limit works as a contraction of the world sheet symmetries [AB 2013].

- The theory of tensionless strings can be made more systematic and be built in lines of usual string theory by using 2D GCFT techniques instead of 2D CFT techniques.

- A new way to look at old problems. Perhaps a way to eradicate the confusions and help understand the relations to higher spin theories more concretely.

- <u>Current work</u> [*with S. Chakrabortty and P. Parekh*]: oscillator construction of Tensionless string and EM tensor in keeping with GCA, constructing the spectrum from GCFT techniques. Encouraging initial results.



Many miles to go before we sleep ....