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A membrane BF action with dual-pairs

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Plan of the Talk

- Introduction:

  M2-branes and search for a fundamental membrane theory

  $\mathcal{N} = 8$ matter Chern-Simons theory (Bagger-Lambert-Gustavsson)

  $\mathcal{N} = 6$ $U(N) \times U(N)$ matter Chern-Simons (Aharony-Bergman-Jafferis-Maldacena)

- Romans' type IIA supergravity (mass parameter)
- $SU(N)$ membrane BF theory with dual pair of fields
- M2-branes on a smooth C_4/Z_4 space
- Summary

Introduction

- M2-brane is a 2-dimensional extended object living in 11-dimensional space-time. It has 8 transverse spatial

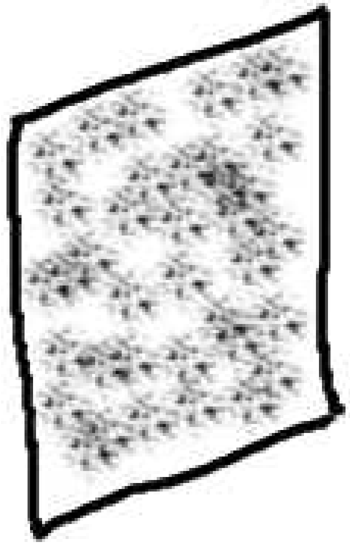

Figure 1: *Supersymmetric membranes live in* $11D$

directions and 2 parallel ones. It is natural to ask what type of matter fields can live on M2-branes and what is their dynamics. What low energy 3-dimensional theory lives on the world volume of M2-branes?

- The close relatives of the M2-branes, the D2-branes, live in $(1+9)$ dimensions. The dynamics of D2-branes is described by $SU(N)$ super-Yang-Mills field theory. The SYMs are well understood from AdS/CFT point of view.

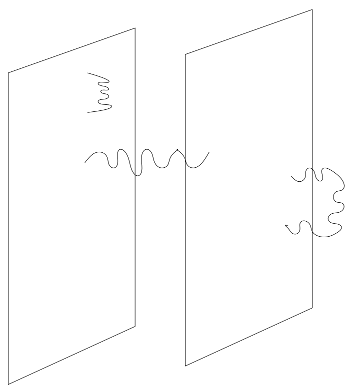

Figure 2: *D2-branes with streched open strings*

Unlike 11-dimensional branes, the D2-branes have open-strings ending on them. The end points of open-strings couple to gauge fields which live on the world volume of D2-branes. The resultant low energy dynamics of N D2-branes is governed by a supersymmetric $SU(N)$ gauge theory

$$S_{D2} = \int d^3x Tr(-\frac{1}{4g_{YM}^2} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2} D_\mu X^i D^\mu X^i$$
$$-\frac{g_{YM}^2}{4}[X^i, X^j][X^j, X^i] + \frac{i}{2}\bar{\Psi}\gamma^\mu D_\mu \Psi + \frac{i}{2}\bar{\Psi}\Gamma_i [X^i, \Psi])$$

Here

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]} - [A_\mu, A_\nu]$$
$$D_\mu X^i = \partial_\mu X^i - [A_\mu, X^i]$$

The real bosonic fields:

$A_\mu; X^1, ..., X^7$ (all in the adjoint of $SU(N)$)

Ψ_α^A (2 compt spinor of $3D$ and 8 compt spinor of $SO(7)$)

- For large N the theory can be described appropriately to live on the boundary of $AdS_4 \times S^6$.
- The mass dimensions:

$$[X^i] = 1/2, \ [A_\mu] = 1, \ [\Psi] = 1, \ [g_{YM}^2] = 1$$

- Notice that the YM coupling is dimensionful in 3D!! So it cannot be a conformal theory. The SYM coupling has a RG flow. The theory has good UV behaviour where it becomes a free theory while in the IR it flows to a strongly coupled fixed point!
- But what is the nature of the theory at this IR fixed point? Could it be described as some kind of a membrane(M) theory?

- While $11D$ SUGRA has $AdS_4 \times S^7$ as a maximal supersymmetric vacua which arises as a near horizon geometry of coincident M2-branes. So it is, generally, expected that a theory of M2-branes should have maximal supersymmetry (16 supercharges), should be conformal and should have $SO(8)$ R-symmetry and should possibly have a gauge symmetry if it is an interacting theory. But what could be the source of these gauge fields in a membrane theory as there are no open-strings in 11D? Are these gauge fields dynamical or are they just some Chern-Simons like ?

Breakthroughs in $3D$ Membrane Theory

Sigma-model action:

Bergshoeff-Sezgin-Townsend (1995)

$$\int d^3x\sqrt{g}\partial_a X^\mu \partial^a X_\mu + \int C_{(3)}$$

which does not have any conformal invariance.

1) $\mathcal{N} = 8$ $SO(8)$ **superconformal matter Chern-Simons BLG theory**

The BLG-model is based on a tri-Lie-algebra

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d, \qquad Tr(T^a T^b) = h^{ab}$$

with structure constants $f^{abcd} = f^{abc}{}_e h^{ed}$ satisfying fundamental Jacobi identity

$$f^{efg}{}_d f^{abc}{}_g = f^{efa}{}_g f^{bcg}{}_d + f^{efb}{}_g f^{cag}{}_d + f^{efc}{}_g f^{abg}{}_d$$

The BL Lagrangian has

i) eight $X^I_a (I = 1, ...8)$ in 8_v of $SO(8)$

ii) eight Ψ^a transforming in the anti-chiral spinor rep of $SO(8)$

iii) Chern-Simons fields A^{ab}_μ (antisymmtric in a, b) which are however non-propagating.

$$\int d^3x \, (-\frac{1}{2} D_\mu X^I_a D^\mu X^I_a - \frac{1}{2} Tr([X^I, X^J, X^K]^2)$$
$$+\frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f^{abcd} A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3} f^{cda}{}_g f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef}\right)$$
$$+\frac{i}{2}\bar{\Psi}^a \gamma^\mu D_\mu \Psi_a + \frac{i}{4}\bar{\Psi}_b \Gamma_{IJ} \Psi_a X^I_b X^J_c f^{abcd})$$

with

$$D_\mu X_a = \partial_\mu X_a - f^{cdb}{}_a A_{\mu cd} X_b$$

Note that CS terms are *twisted* with the structure constant of the algebra.

The $\mathcal{N}=8$ susy tranfs are:

$$\delta X^I_a = i\bar{\epsilon}\Gamma^I\Psi_a$$
$$\delta\Psi_a = \gamma.DX^I_a\Gamma^I\epsilon - \frac{1}{3!}X^I_bX^J_cX^K_d f^{bcd}{}_a\Gamma^{IJK}\epsilon$$
$$\delta(f^{cdb}{}_aA_{\mu cd}) = i\bar{\epsilon}\gamma_\mu\Gamma^I X^I_c\Psi_d f^{cdb}_a$$

Note that $\gamma_{012}\Psi_a = -\Psi_a, \;\; \gamma_{012}\epsilon = \epsilon$. The algebra closes under the gauge transformations:

$$\delta_{gauge}X^I_a = \tilde{\lambda}^b_a X^I_b, \quad \delta_{gauge}\Psi_a = \tilde{\lambda}^b_a\Psi_b,$$
$$\delta_{gauge}\tilde{A}^b_{\mu a} = \tilde{D}_\mu\tilde{\lambda}^b_a$$

where $\tilde{\lambda}^b_a = \lambda_{cd}f^{cdb}{}_a$ and $\tilde{A}^{ab}_\mu = f^{abc}{}_dA^d_{\mu c}$.

Only when $SO(4)$**:** However, with these strict requirements such a formulation of membrane theory is only known to exist for $SO(4) \sim SU(2)\times SU(2)$ gauge group, in which case the

$$f^{abc}{}_{d} = f\epsilon^{abcd}, \quad Tr(T^a T^b) = \delta^{ab}$$

Nevertheless, it is interesting to learn that such a formulation in 3D could be invented altogether!

Moduli Space:

Mukhi et. al., Distler et. al.; Lambert & Tong

We solve

$$f^{abcd} X_a^I X_b^J X_c^K = 0$$

It comes out to be $\frac{R^8 \times R^8}{Z_2 \times Z_2}$.

Thus instead of M2-branes on R^8, the theory is found to describe *two* M2-branes on an orbifold space!

What about the theory if there are more than *two* M2-branes on a flat space?

However, for *noncompact* 3-algebras with

$$h^{ab} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & \delta \end{pmatrix}$$

there does exist an $SU(N)$ generalisation BLG theory with Lorentzian extension of the tri-algebra. However, such L-BLG theories involve ghost d.o.f. which can successfully be taken care of.

See Benvenuti et. al; Gomis et. al

2) $\mathcal{N} = 6$ superconformal matter-Chern-Simons theory (ABJM)

- *There is no requirement of a tri-algebra to start with*!
- The gauge group is $U(N)_k \times U(N)_{-k}$.
- Theory is classified as the theoey of N M2-branes on noncompact orbifold space C_4/Z_k.

All 8 scalars are arranged into pairs of bi-fundamental complex scalars Z_A and W^A $(A = 1, 2)$.

A pair of CS gauge fields A_μ and B_μ are in the adjoints of respective $SU(N)$'s.

Various indices are as:

$$Z^A \equiv (Z^A)_{\alpha\dot{\alpha}}, \quad W^A \equiv (W^A)_{\dot{\alpha}\alpha}, \quad A_\mu \equiv (A^a_\mu)T^a, \ldots$$

The full ABJM action is

$$\int d^3x \; Tr(-|D_\mu Z_A|^2 - |D_\mu W_A|^2) + V(Z, W)$$
$$+\frac{k}{4\pi}\epsilon^{\mu\nu\lambda} tr((A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda) - (B_\mu \partial_\nu B_\lambda + \frac{2i}{3} B_\mu B_\nu B_\lambda))$$

with

$$D_\mu Z^A = \partial_\mu Z^A + i(A_\mu Z^A - Z^A B_\mu)$$

- R-symmetry is $SU(4) \times U(1)_b$, but not $SO(8)$. $SU(4) \sim SO(6)$ for (Z_1, Z_2, W_1^*, W_2^*) and $U(1)_b$ baryon number symmetry.
- $\mathcal{N} = 6$ unless $k = 1, 2$.
- Note that gauge fields are purely *topological*. However, they do give rise to an interacting scalar field theory

through the couplings to matter fields.

- For large N, k but fixed

$$N/k \equiv \lambda$$

limit the theory does compactify to yield super-Yang-Mills theory in $3D$ with weak g_{YM}.

- The moduli space of the theory is

$$\frac{(C_4/Z_k)^N}{S^N}$$

This describes N M2-branes on C_4/Z_k transverse space.

Romans' type IIA SUGRA

Bergshoeff, Roo, Green, Papadopoulos, Townsend (1996)

The massive type IIA theory is

$$\sqrt{-g}\left(e^{-2\phi}[R+4|d\phi|^2-\frac{1}{2}|H_3|^2]-F_2^2-F_4^2-\frac{m^2}{2}\right)$$
$$+\epsilon[dCdCB+mdCB^3+\ldots] \qquad (1)$$

where

$$F_2 \sim dA+mB,\ F_4 \sim dC+AdB+mBB$$

The action is completely fixed by the requirement of Stueckelberg invariance $\delta A=-m\Lambda$ and SUSY.

The dilaton couples to mass parameter as $e^{5\phi/2}m^2$ as is typical in the RR sector. In fact the constant value of m breaks the discrete invariance under which all RR-fields reverse their signs. This symmetry could be restored if m

is lifted to a scalar field $M(x)$ and which in turn can be dualised to a 10-form $F_{10} = dA_9$. In this way m can be related to the expectation value $< F_{10} >$.

One introduces a coupling in the action

$$\int \epsilon M dA_9$$

So A_9 also acts as a lagrange multiplier such that its equation is

$$dM = 0$$

It means that in a vacuum $M = m$. While the field equation of M simply implies

$$\frac{dS(M)}{dM} = -\epsilon dA_9$$

which is a Hodge-dual relation between M and A_9 in 10D. So the net content of the theory is essentially unchanged!

Interestingly one also obtains $D8$-branes as $1/2$ BPS solutions.

Dual pairs and $SU(N)$ B-F theory

de-Wit-Nicolai-Samtlben $3D$ duality: The propagating vector fields in $3D$ carry one d.o.f.. It is a familiar kind of Poincare duality between vector field and a scalar field ($\frac{1}{2!}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} = \partial^{\mu}\phi$). In a more general non-Abelian situation we will have

$$\frac{1}{2!g}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} = D^{\mu}\phi - gB^{\mu}$$

We can easily see that

$$-\frac{1}{4g_{YM}^2}Tr(F_{\mu\nu})^2 \equiv -\frac{1}{2}Tr(D^{\mu}\phi - g_{YM}B^{\mu})^2 + Tr(\frac{1}{2}\epsilon^{\mu\nu\lambda}B_{\mu}F_{\nu\lambda})$$

Actually after the dNS transformation the SYM Lagrangian takes the form of a matter B-F Lagrangian

$$S_{BF} = \int Tr(-\frac{1}{2}(D^\mu \phi - g_{YM} B^\mu)^2 + \frac{1}{2}\epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda}$$
$$- \frac{1}{2}(D_\mu X^i)^2 - \frac{g_{YM}^2}{4}(X^{ij})^2$$
$$+ \frac{i}{2}\bar{\Psi}\gamma^\mu D_\mu \Psi + \frac{i}{2}\bar{\Psi}\Gamma_i [X^i, \Psi])$$

where we can now identify $\phi \equiv X^8$. With this we can define an $SO(8)$ vector:

$$X^I \equiv (X^i, X^8) \quad (1 \leq I \leq 8)$$

Also define a coupling constant 8-vector:

$$g^I = (\vec{0}, g_{YM})$$

With this one can write $SO(8)$ covariant BF Lagrangian

$$S_{BF} = \int d^3x \, Tr(-\frac{1}{2}(D^\mu X^I - g^I B^\mu)^2 + \frac{1}{2}\epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda}$$
$$- U(g^I, X^I) + \frac{i}{2}\bar{\Psi}\gamma^\mu D_\mu \Psi + \frac{i}{2} g^I \bar{\Psi}\Gamma_{IJ}[X^J, \Psi])$$

where the potential is

$$U = \frac{1}{2.3!}(V_{IJK})^2$$

with the

$$V_{IJK} = g_{[I} X_{JK]} \equiv g_I X_{JK} + \ \mathit{cyclic\ permutations}$$

The antisymmetrization of V_{IJK} should not be confused with any tri-algebra like object as in BLG. However, the anti-symmetrization is more like NS-NS 3-form $H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$.

The above BF action has an $SO(8)$ covariance provided

$$g^I \to \Omega^I_J g^J, \qquad \Omega \in SO(8)$$

transform along with various fields under $SO(8)$ rotations. Thus, the theory has $SO(8)$ invariance but its action is transitive on couplings.

The case of BF action: A suitable step would be like that in the Romans' theory in ten dimensions.

$$m \to M(x)$$

$$M(x) \sim *_{10} dA_{(9)}$$

- So we first define corresponding 8 scalar fields $\eta^I(x)$ such that

$$g^I = \ < \eta^I(x) >, \quad g^I g^I = (g_{YM})^2$$

- In the next step we also introduce 2-form potentials

$C^I_{(2)}$, also in the 8_v,

$$\eta^I(x) \sim *_3 dC^I_{(2)}$$

- We must make sure that the vacua are such that η^I's are constant.

  This can be done simply by introducing a new topological term in the $SO(8)$ covariant BF action

$$-\int C^I_{(2)} \wedge d\eta^I \tag{2}$$

  which is $SO(8)$ invariant and has the gauge invariance:

$$C^I_{(2)} \to C^I_{(2)} + d\alpha^I_{(1)} \tag{3}$$

  Thus the complete membrane action can be written as

The complete Membrane BF action

$$S_{MBF} = \int d^3x\, Tr(-\frac{1}{2}(D^\mu X^I - \eta^I B^\mu)^2 + \frac{1}{2}\epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda}$$
$$-\frac{1}{2.3!}(V_{IJK})^2) - \frac{1}{2}\epsilon^{\mu\nu\lambda} C^I_{\mu\nu}\partial_\lambda \eta^I + S_{fermions}$$

where

$$D^\mu X^I = \partial^\mu X^I - [A_\mu, X^I], \qquad V_{IJK} = \eta_{[I} X_{JK]}\ .$$

The equations of motion are now augmented with two new set of equations.

C^I equation:

$$\partial_\lambda \eta^I = 0$$

η^I equation:

$$Tr((D^\mu X^I - \eta^I B^\mu)B_\mu - \frac{1}{2}V^{IJK} X_{JK}) + \frac{1}{2}\epsilon^{\mu\nu\lambda}\partial_\mu C^I_{\nu\lambda} = 0$$

The C^I-equation implies that all η^I are constant. The second equation only relates η^I with its dual tensor field $C^I_{\nu\lambda}$ and should be taken as the duality relation.

– The rest of the field equations remain unchanged. So the net content of the theory remains intact.

– There are no free parameters in the theory.

– The MBF action also has scale invariance.

– The supersymmetry presumably can also be made manifest.

Thus, in bringing the BF theory to the MBF form we have actually introduced dual pairs of fields $(C^I_{(2)}, \eta^I)$.

Moduli Space: We need to solve

$$\partial_{\eta^I} U(\eta, X) - \frac{1}{2!}\epsilon^{\mu\nu\lambda}\partial_\mu C^I_{\nu\lambda} = 0$$
$$\rightarrow \eta^{[I} Tr(X^{JK]} X_{JK}) - \epsilon^{\mu\nu\lambda}\partial_\mu C^I_{\nu\lambda} = 0$$
$$\partial_{X^I} U(\eta, X) = 0 \qquad (4)$$

These equations have interesting possibilities.

Case-1: We take first $C^I_{\mu\nu} = constt.$ Since the $\eta^I(x) = g^I$, we need to have

$$X^{IJ} = [X^I, X^J] = 0. \qquad (5)$$

This can happen when all

$$X^I = diag(a^I_1, ..., a^I_N)$$

That means all M2-branes are coincident. Hence the moduli space is exactly that of N coincident M2-branes on noncompact R^8 space.

However, the special case can also arise when

$$\eta^8 = g_{YM}, \; \eta^i = 0 \; .$$

This will then require

$$X^{ij} = 0 \; .$$

In the simplest case

$$X^i = diag(a_1^i, ..., a_N^i)$$

but X^8 can still be nontrivial but constant. These presumably will be the desired Goldstone modes corresponding to the spontaneously broken $SO(8)$ invariance. This corresponds to the moduli space of N D2-branes on R^7.

For both of the above solutions

$$V_{IJK} = 0$$

So these would make the maximally supersymmetric vacua in MBF theory.

Case-2: Another rather interesting case is of 3D domain-walls. Let us take the tensor components C^I_{01} to be linearly dependent on one of the space coordinates, x_2 (say), then

$$dC^I \sim m^I dx^0 \wedge dx^1 \wedge dx^2 \tag{6}$$

is nontrivial, the m^I being the slope parameters. The two such phases with different slopes can be separated via domain-walls which are just the line defects in 2-dimensional plane. In this situation, we shall have g^I and m^I related via

$$\frac{1}{2} g^{[I} Tr(X^{JK]} X_{JK}) - m^I = 0 \tag{7}$$

This will describe a noncommuting (*fuzzy*) configuration of membranes. However, we are not sure if any nontrivial fuzzy configuration can be found in which will satisfy e.o.m. In any case, it will be interesting to find such a solution.

Quantisation:

The dNS equation for X^I

$$\frac{1}{2!}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} = (D^\mu X^I - \eta^I B^\mu)\eta^I$$

and the equation for η^I can be combined to give

$$Tr(\frac{1}{2!}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}B_\mu - U) + \frac{1}{2}\eta^I\epsilon^{\mu\nu\lambda}\partial_\mu C^I_{\nu\lambda} = 0$$

In the vacuum where $U = 0$, it has interesting implications.

Euclidean monopole configuration: Consider the situation where $F_{ij} \neq 0$ in the region V^3 with a boundary $\partial V^3 \sim S^2$, we will have

$$Tr\frac{1}{2\pi}\int B \wedge F = p(N) \in \mathbf{Z} \ .$$

Then specially taking $\eta^8 = g_{YM}$, $\eta^i = 0$, above equation leads us to the quantization

$$-\frac{1}{2\pi}\int dC_{(2)} = \sqrt{R}k \in \mathbf{Z}\ .$$

$$g_{YM} \equiv \frac{p(N)}{\sqrt{R}k}$$

Here parameter R has length dimensions.

Summary

– We have taken an approach where we augmented the B-F theory with scalars and dual 2-rank tensor fields, namely $(\eta^I,\ C^I_{(2)})$. This leads us to a membrane B-F theory which has $SU(N)$ gauge symmetry and has $SO(8)$ invariance as well as the scale invariance.

– There are no free parameters in the action. The theory does not have ghost degrees of freedom and also has no tri-algebra structure. So in that aspect this $SU(N)$ theory is distinct from the L-BLG theory.

– The theory presumably also has maximal supersymmetry as it is simply the topological extension of the $3D$ super Yang-Mills theory?

– Interestingly, the moduli space comes out to be that of N coincident M2-branes on transverse R^8.

– It does mean that Yang-Mills coupling in a given topological vacuum is controlled by the ratio of $p(N)$ and k. Thus by having large k limit we can accomodate a weak Yang-Mills coupling. This appears almost analogous to the large k limit in ABJM model.

Gravity dual of ABJM: AdS_4/CFT_3

Consider the following solution of 11-dimensional SUGRA:

$$
\begin{aligned}
ds^2 &= \frac{R^2}{4} ds^2_{AdS_4} + R^2 ds^2_{S^7/Z_k} \\
F_{(4)} &\sim N' Vol(AdS) \\
ds^2_{S^7/Z_k} &= \frac{1}{k^2}(d\theta + k\, A_1)^2 + ds^2_{CP_3}
\end{aligned}
$$

where $R = (2^5 \pi^2 N k)^{1/6} l_p$. This is a background which can be obtained in the near horizon region of N M2-branes on a C_4/Z_k transverse space.

The space C_4/Z_k is defined via the following action on the 4 complex (or 8 real) Z_i coordinates

$$
Z_i = e^{\frac{2\pi i}{k}} Z_i
$$

So the isometry of the background is $SU(4) \times U(1)$.

This is precisely the R-symmetry in the ABJM theory. The ABJM CS matter field theory lives on the boundary of $AdS_4 \times S^7/Z_k$.

While 11D SUGRA description is valid so long as radii R as well as $R/k \gg l_p$. Which means when $N/k^5 \gg 1$. When $N/k^5 < 1$ the type IIA desription will make more sense. The 't Hooft coupling is given by $\lambda \equiv N/k$. Note that taking the limit

$$N, k \to \infty, \qquad N/k = fixed$$

makes the description in terms of interacting $SU(N)$ Yang-Mills. The gauge fields become dynamical. It leads to the compactification in the 11-dimensional theory to give type IIA string description of D2-branes in $AdS_4 \times CP_3$. These type II descriptions are valid when

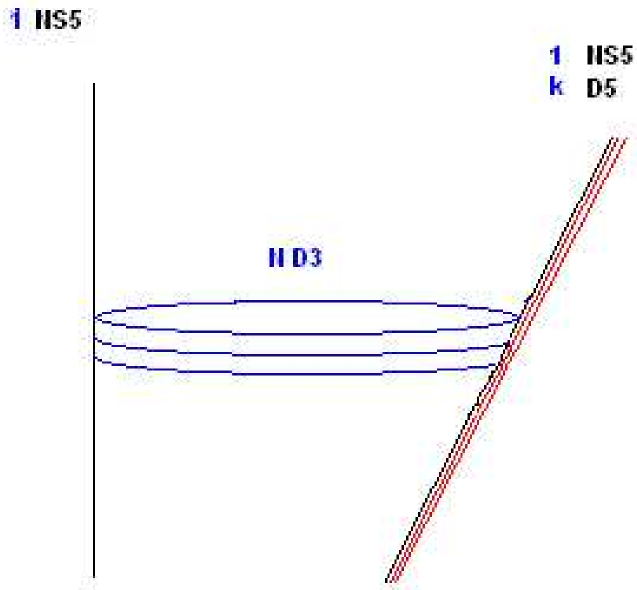

Figure 3: *D3-branes suspended between (NS5,D5) brane pairs*

$$\lambda \gg 1, \ (N/k^5) \ll 1.$$

The super-Yang-mills coupling constant is

$$g_{YM}^2 \equiv < e^{\phi} >= (\frac{R}{k})^{3/2} \sim (\frac{N}{k^5})^{\frac{1}{4}} \sim \frac{1}{N}(\frac{N}{k})^{\frac{5}{4}}$$

Corresponding configuration in type IIB is (NS5,D5)-D3-(NS5,D5)' configuration in which N 3-branes are suspended in between (1,0) and (1,k) (NS5,D5) bound states.

M2-branes on a resolved C_4/Z_4

The flat metric on C_4/Z_k

$$ds^2_{C_4/Z_k} = dr^2 + \frac{r^2}{k^2}(dz + kA)^2 + r^2 ds^2_{CP_3}$$

where $r^2 = y^m y^m$. The Fubini-Study metric on unit size CP_3 space is

$$ds^2_{CP_3} = d\xi^2 + \cos^2\xi \sin^2\xi \ (\tilde{\psi})^2 + \frac{\cos^2\xi}{4}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{\sin^2\xi}{4}(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) \qquad (8)$$

where $0 \leq \xi < \frac{\pi}{2},\ 0 \leq z < 2\pi,\ 0 \leq \theta_i < \pi,\ 0 \leq \phi_i < 2\pi$. The $\tilde{\psi}$ and the 1-form along the Hopf fibre z are

$$\tilde{\psi} \equiv d\psi + \frac{\cos\theta_1}{2}d\phi_1 - \frac{\cos\theta_2}{2}d\phi_2$$

$$A_{(1)} = \frac{1}{2}((\cos^2\xi - \sin^2\xi)d\psi + \cos^2\xi\cos\theta_1 d\phi_1$$
$$+\sin^2\xi\cos\theta_2 d\phi_2)$$

The space is an ALE but has (orbifold) singularity at $r = 0$ for all $k \geq 2$.

The M2-brane solution on this transverse space is given by

$$ds_{11}^2 = h^{-\frac{2}{3}}(-dx_0^2 + dx_1^2 + dx_2^2) + h^{\frac{1}{3}}ds^2_{C_4/Z_k}$$
$$F_{(4)} = d(h^{-1}) \wedge dx^0 \wedge dx^1 \wedge dx^2$$

where the harmonic function is

$$h(r) = 1 + \frac{2^5\pi^2 Nkl_p^6}{r^6}.$$

The near horizon limit ($r \to l_p^2 U,\ l_p \to 0$) gives M2-

branes on $AdS_4 \times S_7/Z_k$

$$ds_{11}^2 \sim R^2(U^4(-dx_0^2 + dx_1^2 + dx_2^2) + \frac{dU^2}{U^2}) + R^2 ds_{S_7/Z_k}^2$$

$$F_{(4)} \sim 6R^3\, vol(AdS_4)$$

where $(R/l_p)^2 = (2^5\pi^2 Nk)^{1/3}$.

The number of M2-branes is taken as Nk, so that the flux through S^7/Z_k remains integral of N. The doubling of supersymmetries in the near horizon region suggests that the AdS_4 geometry will preserves 24 supersymmetries.

Resolution of the singularity: Specially C_4/Z_4

C_4/Z_k is taken as Eguchi-Hanson type

$$ds^2_{C_4/Z_k} = \frac{dr^2}{f(r)} + \frac{r^2}{k^2} f(r)(dz + kA)^2 + r^2 ds^2_{CP_3}$$

The Ricci flatness gives

$$f(r) = (1 - r_0^8/r^8)$$

and the ranges are fixed as

$$r_0 \leq r \leq \infty, \quad 0 \leq z \leq 2\pi.$$

To know if the metric is regular near $r = r_0$ region, we define a local coordinate patch

$$r^2(1 - r_0^8/r^8) = (k\rho)^2$$

The $r = r_0$ neighborhood geometry then becomes

$$ds^2 \simeq \frac{k^2}{16} d\rho^2 + \rho^2(dz + kA)^2 + r_0^2 ds^2_{CP_3}$$

So the metric can be regular when $k = 4$ but not otherwise. Only then it will have a smooth $R^2 \times CP_3$ geometry.

M2-brane solution:

Correspondingly the (delocalised) $4N$ M2-brane background on resolved C_4/Z_4 space can be obtained by solving

$$\partial_r(r^7 f \partial_r h) = 0$$

The solution is a new harmonic function

$$h(r) = 1 + \frac{Q}{4r_0^6}\left(\arctan(\frac{r^2}{r_0^2}) - \frac{1}{2}\log\frac{(r^2 - r_0^2)}{(r^2 + r_0^2)}\right)$$

Near $r = r_0$ it behaves

$$h \simeq \frac{Q}{8r_0^6}\log(\frac{r_0^2}{2\rho^2}).$$

So the solution diverges but logarithmically. While for $r \gg r_0$ it becomes

$$h \sim 1 + \frac{Q}{6r^6} + \frac{Qr_0^4}{10r^{10}} + \cdots$$

Here

$$Q/6 \equiv 2^5\pi^2 4Nl_p^6.$$

The near horizon decoupled geometry is

$$ds_{11}^2 \simeq l_p^2 h^{\frac{1}{3}} U^2 [\frac{(-dx_0^2 + dx_1^2 + dx_2^2)}{U^2 h} + \frac{dU^2}{U^2 f} + \frac{f}{16}(dz + 4A)^2 + ds_{CP_3}^2]$$

$$F_{(4)} = l_p^3 d(h^{-1}) \wedge dx^0 \wedge dx^1 \wedge dx^2$$

and

$$h(U) \simeq \frac{Q_0}{4U_0^6}\left(\frac{1}{2}\log\frac{(U^2 + U_0^2)}{(U^2 - U_0^2)} + \arctan(\frac{U^2}{U_0^2})\right)$$

$$f = 1 - \frac{U_0^8}{U^8}$$

Here we identify $\frac{Q_0}{6} \equiv 2^5\pi^2(4N) \gg 1$ so that the overall curvature of the spacetime is small in the Planck units. It is obviously a difformation of the $AdS_4 \times S^7/Z_4$ near $U \sim U_0$.

While for $U \gg U_0$ the geometry becomes exactly $AdS_4 \times S^7/Z_4$

$$ds_{11}^2 \sim l_p^2 (Q_0/6)^{\frac{1}{3}} [\frac{U^4(-dx_0^2 + dx_1^2 + dx_2^2)}{(Q_0/6)} + \frac{dU^2}{U^2} + \frac{1}{16}(dz + 4A)^2 + ds_{CP_3}^2]$$

$$F_{(4)} \equiv l_p^3 \sqrt{6Q_0}\, vol(AdS_4)$$

which corresponds to $4N$ M2-branes on C_4/Z_4.

Summary:2

– We have constructed M2-brane solutions on a resolved C_4/Z_4 8-manifold. In the IR region the near horizon geomtry is a deformation of the $AdS_4 \times S^7/Z_4$ spacetime of the M2-branes. While in the UV region the geometry becomes precisely $AdS_4 \times S^7/Z_4$ and therefore corresponding memebrane theory is the ABJM matter Chern-Simons theory.

– It will be interesting to find a regular brane configuration of M2-branes on resolved C_4/Z_4. This can be obtained by localising M2-branes on the resolution instead of homogeneously distributing them, as in the works of Klebanov-Murugan. It will definitely help us in understanding the ABJM theory in the IR regime.

- There is likely possibility that the IR limit of the ABJM theory is maximally supersymmetric $SU(N)$ MBF theory.