

# PHYSICS FROM NEUTRINO FACTORIES

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We review the physics potential of future neutrino factories, given the current status of knowledge about neutrino mixing parameters.

**Key Words:** Long Baseline Experiments; Neutrino Factories; Neutrino Oscillation Parameters; CP Violation; Magic baseline

## 1 The Major Results So Far

This article is based on a set of excellent reviews<sup>1</sup> in this field, and is an attempt to elucidate the physics potential of neutrino beams from future possible neutrino factories, in contrast to that of beams or super-beams from conventional accelerators. While neutrino factories are still somewhat far off in the future, there already exist long baseline experiments with beams from accelerators in FermiLab and KEK. The latter, the so-called K2K experiment, has already yielded data. For more details on long baseline experiments with conventional beams, see the review by S. Uma Sankar in this volume.

Neutrino physics has become more of a precision science, nearly four decades after its inception. The major results in this area, from various experiments around the world<sup>2</sup>, have contributed to provide overwhelming evidence that neutrino flavours mix, viz., that the propagating neutrino mass eigenstates are not the same as the interacting flavour eigenstates, and that they *oscillate*. A requirement of neutrino flavour oscillations is that neutrinos should have mass (more precisely, that they should have different masses). Hence, measurements of neutrino fluxes from various sources indicate that neutrinos are massive, thus leading to the first evidence for new physics beyond the Standard Model of particle physics.

Neutrinos have been observed from the Sun. They have also been observed from cosmic ray interactions with the atmosphere (the so-called atmospheric neutrinos) and from supernovae. Apart from these natural sources, neutrinos from laboratory sources such

as from reactors and accelerators have also been studied. Results have been obtained from a world-wide community of neutrino physicists, in labs such as the Kamiokande, Super-Kamiokande in Japan, Sudbury Neutrino Observatory in Canada, CHOOZ in France, and LSND in the U.S.

Together, the data yield clear and compelling evidence for neutrino flavour oscillations and hence of neutrino mass. They also indicate the presence of at least three active neutrino flavours or generations (already known from Z-width data). While the mixings between the (12) and (23) generations seem to be large, and indeed maximal in the latter case, the (13) mixing seems small. Hence the mixing decouples into two sets of 2-flavour mixings, thus leading to a virtual decoupling between the mixing angles measured in solar and atmospheric neutrino experiments.

**Table I**

*The table shows the presently known values (or limits) on the neutrino oscillation parameters.*

parameter	central value
$\delta_{21}$	$6.9 \times 10^{-5} \text{ eV}^2$
$\sin^2 2\theta_{12}$	0.82
$ \delta_{32}  \sim  \delta_{31} $	$2.6 \times 10^{-3} \text{ eV}^2$
$\sin^2 2\theta_{23}$	1.0
$\sin^2 2\theta_{13}$	$< 0.13$

The central allowed values of the known parameters (mass squared differences  $\delta_{ij} \equiv m_i^2 - m_j^2$  and mixing angles  $\theta_{ij}$ ) that define the oscillations are listed in Table I. Solar neutrino data and the KamLAND reactor experiment helped determine the parameters in the (12) sector while atmospheric neutrino data was used to determine the (23) parameters. The reactor experiment CHOOZ has given the single limit on the (13)

mixing angle while the LSND accelerator experiment indicates that there may be more than three neutrino flavours, one of them possibly sterile.

## 2 Issues to be Addressed in the Future

This brings us to the questions still to be answered. The first question is of course whether what is observed is indeed evidence for neutrino oscillation. To establish this beyond doubt, it is necessary to “see” the oscillation pattern, that is, not just a depletion but also an enhancement of the signal. While this has not yet been directly seen, a combination of the SNO charged-current (CC) and neutral-current (NC) data strongly favours the oscillation scenario. Assuming that this is true, it remains to fix the oscillation parameters precisely.

The major issue is then whether  $\theta_{13}$  is zero or not. For instance, only when  $\theta_{13}$  is not zero can one ask questions related to CP violation in the neutrino sector. Since the CP violation phase  $\delta_{\text{CP}}$  occurs with  $\sin\theta_{13}$ , CP violating effects are expected to be small; hence their measurement is definitely a challenge. Theoretically, also, it is interesting to ask whether CP violation in the leptonic sector can contribute to baryogenesis via leptogenesis.

While it is established that  $m_2 > m_1$ , the sign of  $\delta_{32}$  is not yet known. The theoretical motivation for determining this is in determining the mass hierarchy in the leptonic sector. More technically, actual construction of neutrino mass/mixing models requires knowing whether  $\theta_{23}$  is exactly maximal or just nearly so. Finally, there is still some room to ask questions about sterile neutrinos, neutrino decay, CPT violation, etc.

Notice that issues related to neutrino oscillation have been highlighted here, as setting the motivation for building neutrino factories. There is a long list of questions still awaiting answer in neutrino physics that do not necessarily involve oscillation phenomena: an example is whether neutrinos are Dirac or Majorana particles, and hence whether they can participate in neutrino-less double beta decay. Another issue that cannot be addressed by future neutrino factories is the actual scale of neutrino masses; these can only be determined by direct experiments such as the measurement of the end-point of the tritium beta decay spectrum. Many of these issues have been dealt with in other chapters of this review. We however would like

to highlight our belief that *physics at a neutrino factory is interesting only if  $\sin^2 2\theta_{13} \neq 0$  and relevant only if it is not too small.*

## 3 Neutrino Masses and Mixings: A Short Review

Let  $\nu_\alpha$  refer to the flavour eigenstates,  $\alpha = e, \mu, \tau$ . These are related to the mass eigenstates,  $\nu_j, j = 1, 2, 3$ , through the  $3 \times 3$  unitary matrix,

$$\nu_\alpha = \sum_j U_{\alpha j} \nu_j.$$

The mixing matrix is parametrised in terms of the mixing angles  $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$ , and the CP violating phase  $\delta$ , as

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

Here  $c_{12}$  and  $s_{12}$  refer to  $\cos\theta_{12}$  and  $\sin\theta_{12}$ , etc. Also, note that the CP violating phase always occurs with  $s_{13}$ , as has been highlighted above.

The propagation of neutrinos of energy  $E_\nu$  in matter is determined by

$$i \frac{d\nu_\alpha}{dt} = \sum_\beta \left( \sum_j U_{\alpha j} U_{\beta j}^* \frac{m_j^2}{2E_\nu} + \frac{A}{2E_\nu} \delta_{\alpha e} \delta_{\beta e} \right) \nu_\beta,$$

where  $A/(2E_\nu)$  refers to the amplitude for coherent forward CC scattering in electronic matter, i.e.,  $\nu_e e$  scattering. The matter dependent term is given by,

$$A = 2\sqrt{2}G_F Y_e \rho E_\nu \sim 1.52 \times 10^{-4} Y_e \rho (E_\nu/\text{GeV}) \text{eV}^2.$$

Here  $\rho$  is the density of Earth matter in gm/cc and  $Y_e$  is the electron fraction in the matter, while  $G_F$  is the usual Fermi coupling constant. Using the fact that the (31) mass-squared difference is larger in magnitude than the (21) mass-squared difference (see Table I), we can rearrange the expression to give

$$i \frac{d\nu_\alpha}{dt} = \sum_\beta \frac{1}{2E_\nu} \left( \delta_{31} U_{\alpha 3} U_{\beta 3}^* + \delta_{21} U_{\alpha 2} U_{\beta 2}^* + A \delta_{\alpha e} \delta_{\beta e} \right) \nu_\beta,$$

where an irrelevant diagonal piece has been discarded.

The results can be expressed in terms of transition probabilities of the various flavour eigenstates after propagating a distance  $ct = L$ . In the absence of

matter, the leading order probabilities are,

$$\begin{aligned} P_{e\mu} &= s_{23}^2 \sin^2 2\theta_{13} \sin^2 (\delta_{32}L/(4E)) , \\ P_{e\tau} &= c_{23}^2 \sin^2 2\theta_{13} \sin^2 (\delta_{32}L/(4E)) , \dots(1) \\ P_{\mu\tau} &= c_{13}^4 \sin^2 2\theta_{13} \sin^2 (\delta_{32}L/(4E)) . \end{aligned}$$

Clearly, these expressions are independent of  $\delta_{21}$ , the sign of  $\delta_{32}$ , and the CP violating phase  $\delta$ . At next-to-leading order in  $\alpha = \delta_{21}/\delta_{31}$  and  $\theta_{13}$ , both  $\delta_{21}$  and  $\delta_{CP}$  appear while the sign of  $\delta_{32}$  becomes relevant only when matter effects are included.

In the absence of matter, an eight-fold degeneracy was noted<sup>3</sup>, arising from

1. sign degeneracy in  $|\delta_{32}|$ , that is, in  $\delta_{32} \rightarrow -\delta_{32}$ .
2.  $\theta_{23} \rightarrow \pi/2 - \theta_{23}$ ; a change that can be compensated by a corresponding change in the *unknown*  $\sin^2 2\theta_{13}$ ,
3. Changes in  $\delta_{CP}$  being compensated by changes in  $\sin^2 2\theta_{13}$ . These parameters are strongly correlated, as can be seen from the definition of the mixing matrix above. In fact, it turns out that it is always possible to find a solution by changing  $\sin^2 2\theta_{13}$  in such a way as to always retain  $\delta_{CP} = 0$ . Hence this degeneracy mixes the CP conserving and CP violating sectors.

On including matter effects, some of these degeneracies can be lifted. The probabilities can be expanded in  $\alpha$  and  $\sin^2 2\theta_{13}$  to give<sup>4</sup>,

$$\begin{aligned} P_{e\mu} &\sim \sin^2 2\theta_{13} s_{23}^2 \frac{\sin^2(1-\hat{A})\Delta}{(1-\hat{A})^2} \\ &\pm \alpha \sin 2\theta_{13} \xi \sin \delta_{CP} \sin \Delta \frac{\sin \hat{A} \Delta \sin(1-\hat{A})\Delta}{\hat{A} (1-\hat{A})} \\ &+ \alpha \sin 2\theta_{13} \xi \cos \delta_{CP} \cos \Delta \frac{\sin \hat{A} \Delta \sin(1-\hat{A})\Delta}{\hat{A} (1-\hat{A})} \\ &+ \alpha^2 c_{23}^2 \sin^2 2\theta_{12} \frac{\sin^2 \hat{A} \Delta}{\hat{A}^2} . \dots(2) \end{aligned}$$

We have dropped the subscript on the neutrino energy for clarity, and have used  $\Delta = \delta_{31}L/(4E)$ ,  $\xi = c_{13} \sin 2\theta_{12} \sin 2\theta_{13}$ ,  $\hat{A} = \pm(2\sqrt{2}G_F Y_e \rho E)/\delta_{31}$  with plus (minus) sign for neutrinos (antineutrinos), so that

$$\hat{A}\Delta = \pm \frac{\sqrt{2}}{2} G_F Y_e \rho L ,$$

independent of the neutrino energy  $E$  and proportional to the path length  $L$ .

With this rearrangement, several features of the mixing stand out:

1. The MSW resonance condition is  $\hat{A} = 1$ . Near resonance, the *first term* is enhanced by matter effects. This is also therefore most sensitive to the sign of  $\delta_{31}$  through the sign of  $\hat{A}$ .
2. CP violation occurs only in the order  $\mathcal{O}(\alpha)$  terms, viz., the *second and third terms* and is small, being not only suppressed by  $\alpha$  but also by  $\sin 2\theta_{13}$ .
3. The *fourth term* is independent of  $\theta_{13}$  and  $\delta_{CP}$  and is sensitive to the value of  $\delta_{12}$ , should the former two be zero.
4. Note that most degeneracies arise for large  $\alpha$ .

### CP Independence and the Magic Baseline

We have seen that the sign of  $\delta_{31}$  is determined essentially by the first term in eq.2. The degeneracy that affects the precision with which this can be determined is one where the CP phase can be changed to compensate for this effect. A solution<sup>5</sup> is to search for regions where the CP-dependent terms drop out, so that this ‘‘compensation’’ cannot happen. It is clear from eq.2 that this happens when

$$\sin \hat{A}\Delta = 0 .$$

The first non-trivial solution to this condition is

$$\sqrt{2}G_F Y_e \rho L = 2\pi .$$

For  $Y_e \sim 1/2$ , this energy independent condition leads to  $L \sim 32,726/\rho(\text{gm/cc})$  in km. For a constant Earth matter density of  $\rho \sim 4.3 \text{ gm/cc}$ , this gives  $L \sim 7630$  km, that reduces somewhat to  $L \sim 7250$  km when a detailed Earth density profile from the Preliminary Reference Earth Model (PREM) model is used. Notice that for this baseline, the neutrinos barely graze the Earth’s core and hence density variations are not expected to be large.

As a consequence, for such a baseline,

1. There is no degeneracy from the sign of  $\delta_{31}$ .
2. All terms containing  $\delta_{CP}$  vanish so there is no CP dependence.

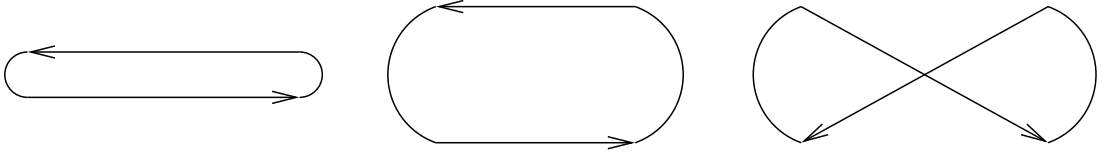


Fig. 1 Possible schemes for a muon storage ring. On the left, almost 50% of the muons will decay in each straight section; in the middle, a more modest approach with rounded sides of the same length as the straight sections allows for about 25% of the muons to decay in each straight section, which may be about 37 m long for 20 GeV muons. On the right, a bow-tie configuration can allow for large dip angles.

3. All correction terms to order  $\mathcal{O}(\alpha^2)$  vanish, hence allowing for a clean measurement of  $\sin \theta_{13}$ .
4. Since  $L$  is large, it is expected that statistics will be poor; hence it is necessary to have powerful beams. Sizable event samples will therefore only be obtained with beams from future neutrino factories.
5. It is possible to combine results from such a baseline with another baseline, say at  $L \sim 3000$  km, to get a handle on the CP phase  $\delta$ .

#### 4 Neutrino Beams

Conventional beams can be upgraded to superbeams by increasing the power of the initial proton beams to megawatt power. Such beams are obtained by producing pions and kaons from the initial protons, which then decay in a long decay channel, thus generating the neutrino beam. While the dominant  $\pi^+$  gives a  $\nu_\mu$ , for instance,  $K^+$  will give a  $\nu_e$  so that there will always be a contamination at the level of a percent or so in the flavour content of the neutrino beam. This is, provided, of course, that  $\mu$  decay is also suppressed. Apart from the wrong-flavour contamination, the flux of the  $\nu_e$  and  $\nu_\mu$  is not precisely known. Furthermore, at high energies,  $\nu_\tau$  contamination of the beam may limit future searches for  $\nu_\mu \rightarrow \nu_\tau$  oscillation.

In a muon storage ring, many of these drawbacks are overcome. Of course, the primary source of the

muons is still a proton beam, with meson production target and collection system, and a meson decay channel, but here it is the muons that are circulating in the ring. Such a ring is still in the design stage; indeed, some designs may require muon cooling as well.

Work on the feasibility of such a muon storage ring and a neutrino factory is in progress at CERN in Europe, in the US, and in Japan<sup>6</sup>. Most of the details in this review are taken from detailed studies conducted by these research groups. The key idea is that the “ring” will have long straight sections where the muon can decay. This will help obtain a narrow, focused beam. Possible schemes are shown in Fig.1 and possible baselines are listed in Table II.

A neutrino beam from such a neutrino factory with a muon storage ring will therefore contain equal amounts of  $e$  and  $\mu$  flavours. For example, a  $\mu^-$  beam will give equal amounts of  $\nu_\mu$  and  $\bar{\nu}_e$  in the neutrino beam. Also, the fluxes of each flavour are precisely calculable<sup>8</sup>. The angular resolution of the beam is dominated by divergence due to decay kinematics. An angular precision of about  $20 \mu r$  is feasible with a spatial resolution of about 500 microns, so that a beam size of about 1 km is achieved on the far side, at the detector. Also, a compact size (about 150 m circumference for a 20 GeV muon beam) implies that the whole instrument can be tilted with respect to the horizontal, an essential requirement for long baseline beams.

The main advantage of knowing the composition of beams from neutrino factories is that a primary signal for neutrino oscillation is the discovery of “wrong-

**Table II**

*Some possible baselines (in km) and source/detector combinations for neutrino factory experiments. Table from ref.[7]. Also included are the baselines to two possible sites in Rammam and PUSHEP in India.*

FermiLab to	Brookhaven to	JHF to	CERN to
Soudan (730)	Cornell (350)	Super-K (295)	Frejus (150)
Homestake (1290)	Soudan (1720)	Seoul (1200)	Gran Sasso (730)
San Jacinto (2640)	Homestake (2540)	Beijing (2100)	
SLAC (2900)		PUSHEP (6595)	PUSHEP (7145)
		Rammam (4900)	Rammam (6900)

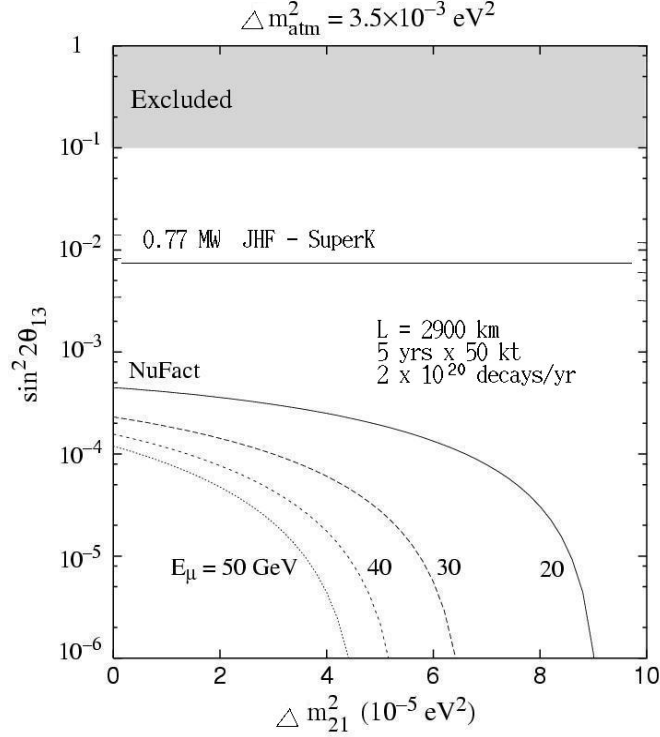


Fig. 2 Sensitivity to  $\sin^2 2\theta_{13}$  as a function of the (12) mass-squared difference,  $\delta_{21}$ . The currently favoured region is the right half of the  $x$ -axis. Figure from ref.[10]. For details, see the text.

sign” muons. For example, if the original beam composition is  $\nu_e$  and  $\bar{\nu}_\mu$ , then the CC interaction of the latter in the detector gives  $\mu^+$ . However, the oscillation of  $\nu_e$  as  $\nu_e \rightarrow \nu_\mu$  results in a CC interaction with  $\mu^-$  being produced. Hence, charge identification is a primary requirement of any far-end detector. Of course, a near-detector is essential for flux normalisation as well. The detector must be sensitive to electrons or muons or both.

In the  $\nu_\mu$  beam, the event rate is proportional to  $E^3$  and so is very much higher than from superbeams. The flux energy distribution has a sharp cut-off at  $E_\mu$ . The absence of a high-energy tail (as seen in conventional superbeams) implies that the background of “fake” events due to high energy NC  $\pi^0$  events is avoided. For  $\nu_e \rightarrow \nu_\mu$ , the wrong-sign muon background is less than few parts  $10^{-4}$  in contrast to conventional beams where it may be as high as one in hundred.

The physics reach of a neutrino beam from either right- or wrong-sign muons and electrons is to determine the sign of  $\delta_{31}$ , the magnitude of the across-generation mixing parameter,  $\sin \theta_{13}$ , and the CP phase  $\delta$ . In short, it has the potential to determine all oscillation parameters to great accuracy.

## 5 Numerical Simulations with Neutrino Factory Beams

We now show results obtained by various groups working on the physics potential of neutrino factories. A compilation of various working group reports and technical status is available at the neutrino oscillation industry web-site<sup>9</sup>. The sensitivity of a certain beam-baseline combination can be defined differently. For instance, the sensitivity to  $\sin^2 2\theta_{13}$  can be defined as the largest value of  $\sin^2 2\theta_{13}$  that fits the true value of  $\sin^2 2\theta_{13} = 0$ ; it can also be defined as that value which gives at least 10 wrong-sign muon events per year. We shall not specify the definition used in each instance but refer the interested reader to the original papers.

### Sensitivity to $\sin^2 2\theta_{13}$

Starting with  $\nu_e$ , an oscillation to  $\nu_\mu$  is sensitive to the (13) mixing angle through the measurement of wrong-sign muons. (See eq.2). There is no dependence on the sign of  $\delta_{31}$ . All the plots shown are taken from ref.[10], where it has been assumed that  $\sin^2 \theta_{23}$  and  $|\delta_{31}|$  are known to within 10%. Fig.2 shows the sensitivity to  $\sin^2 2\theta_{13}$  from neutrino factory beams of

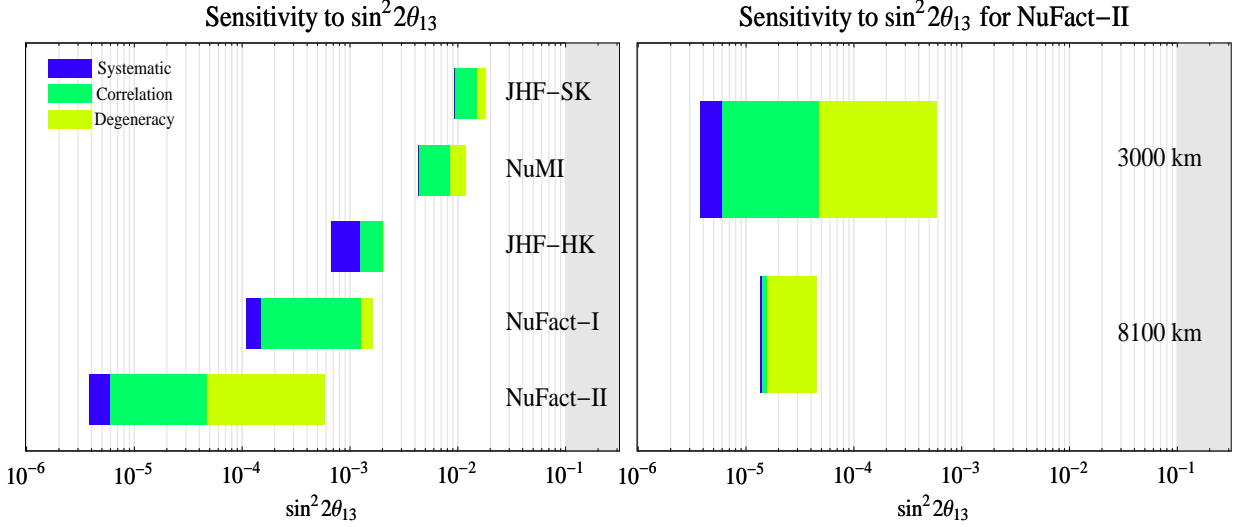


Fig. 3 Effects of degeneracies on extraction of  $\sin^2 2\theta_{13}$ . On the left is the effect on data from superbeams in comparison with those from nu-factories with  $L = 3000$  km. On the right are data from nu-factories with different baselines. It is seen that degeneracies are lifted for large  $L$ . The effects of systematics and correlations are also shown. Figure from ref.[14].

different energies (from 20–50 GeV) as a function of the (12) mass-squared difference,  $\delta_{21}$ , for a baseline of  $L = 2900$  km.

The calculation assumes  $2 \times 10^{20}$  decays per year, and data collection using a 50 kton detector over a 5 year period. The reach of a conventional superbeam with a 22.5 kton detector (JHF to Super-Kamiokande) is also shown, along with the current excluded region from CHOOZ<sup>11</sup>. Note that this uses a central value of  $|\delta_{32}| = 3.5 \times 10^{-3} \text{eV}^2$ , which has been revised to a lower value due to the re-evaluation of the Super-K data<sup>12</sup>; due to this the CHOOZ limits have also slightly worsened.

The effects of degeneracies on the measurement of the (13) mixing angle are shown in Fig.3. The left edge of each band is the statistical limit to the sensitivity, which is defined as obtaining at least 10 wrong sign muons per year. Systematics, correlational errors, and degeneracies successively reduce the sensitivity to the right edge of the band for each experiment.

While systematic errors are under control both for superbeam and nu-factory beams, correlations are significant in both, with degeneracies sharply limiting the reach of  $\sin^2 2\theta_{13}$  to the  $10^{-3}$  region for both superbeams and nu-factories, and improving to a little better than  $10^{-4}$  for very long baseline ( $L = 8100$  km) nu-factories. In fact, we will see later that this also limits the sensitivity to the sign of  $\delta_{31}$ .

### Sensitivity to the Sign of $\delta_{31}$

Matter effects are different for particles and an-

tiparticles (the matter-dependent term  $A$  changes its sign in the two cases). For example, the leading order term in the  $\nu_e \rightarrow \nu_\mu$  transition probability can be expressed in terms of matter-dependent mixing angles and mass-squared differences as,

$$P_{e\mu} = \sin^2 2\theta_{13}^m \sin^2 \theta_{23} \sin^2 \Delta_{31}^m,$$

while that for the corresponding transition between antineutrinos is given by  $P_{\bar{e}\bar{\mu}}$  as above, with  $A \rightarrow -A$  in the expression for  $\sin \theta_{13}^m$ . Hence, if  $\delta_{31} > 0$ , then the resonance condition  $A \sim \delta_{31}$  enhances  $P_{e\mu}$ , while, if  $\delta_{31} < 0$ , then the resonance condition  $-A \sim \delta_{31}$  enhances  $P_{\bar{e}\bar{\mu}}$ . This means that sensitivity to the sign of  $\delta_{31}$  can be obtained only if the nu-factory is run with both  $\mu^+$  and  $\mu^-$  in the storage ring, so as to get neutrinos and antineutrinos of the same flavour. Then the ratio of wrong sign muons with  $\bar{\nu}_e$  in the beam to those with  $\nu_e$  in the beam is greater or less than a half (the ratio of the antineutrino to neutrino cross-section), depending on whether whether  $\delta_{31}$  is less than or greater than zero. The enhancement (or suppression) of this ratio increases with baseline, as can be seen from Fig.4.

Note that the magnitudes of both mass-squared differences have been used as input here, although the dominant contribution to this effect is from the first term in eq.2, which is independent of  $\delta_{21}$ . The relative contributions of the second and third terms of eq.2 will increase substantially if  $\alpha$  is as large as  $\sin 2\theta_{13}$ , but the event rates will be severely depleted if  $\sin 2\theta_{13}$  is too small. Hence, there is a close correspondence

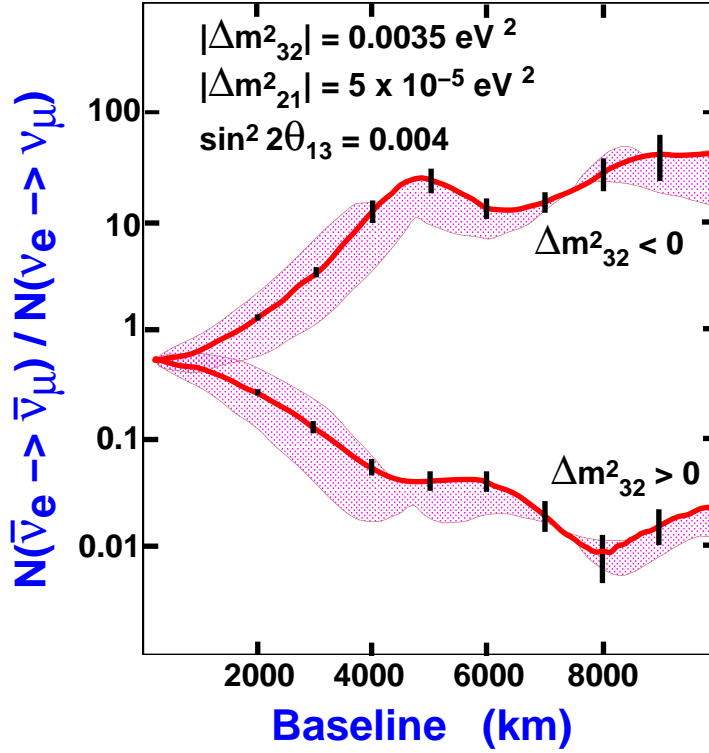


Fig. 4 Sensitivity to the sign of  $\delta_{32}$  (or equivalently  $\delta_{31}$ ) as a function of the baseline. Also shown are the bands from varying the CP phase maximally from  $-\pi/2$  to  $\pi/2$ . The error bars shown correspond to  $5 \times 10^{22}$  decays-kton and are plotted for the solid line with CP phase  $\delta = 0$ . For details, see the text. Figure from ref.[13].

between sensitivity to the sign of  $\delta_{31}$  and the value of  $\sin^2 2\theta_{13}$ . This is shown in Fig.5. It is seen that sensitivity to the sign of  $\delta_{31}$  is constrained by the magnitude of the (13) mixing angle; if  $\sin^2 2\theta_{13} < 10^{-4}$  or so, the sign of this mass-squared difference cannot be determined.

### Sensitivity to the CP Phase $\delta$

As stated earlier, the CP phase  $\delta$  occurs always with  $\sin \theta_{13}$ ; hence CP effects are expected to be subdominant. That the phase  $\delta$  is the source of CP violation is easy to see by expressing the transition probability as

$$P_{\alpha\beta} = P_{\alpha\beta}^{\text{CPEven}} + P_{\alpha\beta}^{\text{CPOdd}}.$$

Then

$$P_{\alpha\beta}^{\text{CPEven}} = P_{\bar{\alpha}\bar{\beta}}^{\text{CPEven}} \\ = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re} \left( U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \right) \sin^2 \frac{\delta_{ij} L}{4E}, \dots (3)$$

$$P_{\alpha\beta}^{\text{CPOdd}} = -P_{\bar{\alpha}\bar{\beta}}^{\text{CPOdd}} \\ = 2 \sum_{i>j} \text{Im} \left( U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \right) \sin \frac{\delta_{ij} L}{2E} \dots (4)$$

Hence the phase  $\delta$  contributes to the imaginary part of the matrix element, which then contributes to the CP-odd part. Because of this, the CP violating terms show up in the particle-antiparticle asymmetry, i.e., in the same ratio of wrong sign events from  $\bar{\nu}_e$  and  $\nu_e$  beams respectively. The effect of CP violation is not large, as can be seen from Fig.4. Again, the effect of CP violation is only visible if the (13) angle is not too small and not too large. The value used in the figure,  $\sin^2 2\theta_{13} = 0.004$  is near the limit of sensitivity of CP measurements; if it is greater than about 0.001 or so, the sign of  $\delta_{31}$  has the largest effect on the ratio plotted. As already mentioned, sensitivity to CP effects can be enhanced by using two baselines, one of them

### Comparison with Reactor Experiments

Reactor experiments typically have short baselines and hence no matter effects. Hence they are insensitive to the sign of  $\delta_{31}$  and the CP phase  $\delta$ . The main advantage is that there are no degeneracies<sup>15</sup>. Hence they provide a precision measurement of the (13) mixing angle. For more details on neutrino ex-

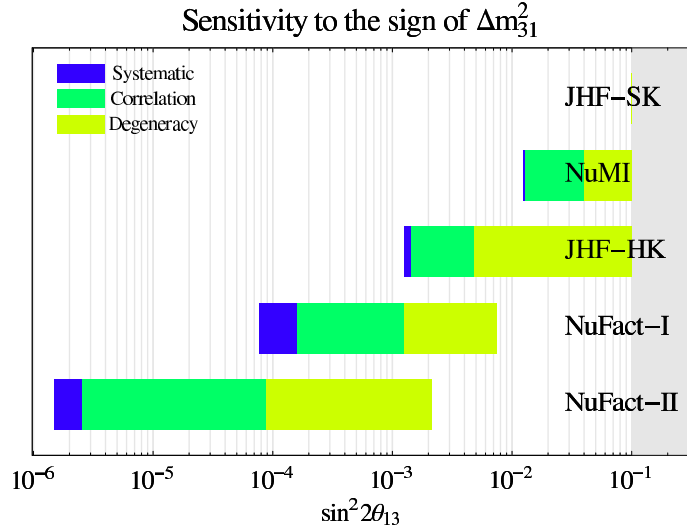


Fig. 5 Effect of systematics, correlations and degeneracies in the minimum value of  $\sin^2 2\theta_{13}$  for which the sign of  $\delta_{31}$  can be determined by superbeams or nu-factory beams. Figure from ref.[14].

periments at reactors, see the article by C V K Baba in this volume. Detailed studies at future reactor experiments (with a near detector for flux normalisation) have been done<sup>15</sup>. It is probable that a combination of superbeams and reactor experiments can give improved bounds on the sign of  $\delta_{31}$  and  $\delta_{CP}$  in a reasonably short time.

### Comparison with Beta Beams

The technology requirement for building neutrino factories is rather stringent. In the meanwhile, a new idea in the shape of beta-beams is being discussed. Here, a radioactive nucleus ( $e^-$  or  $e^+$  emitter) is accelerated so that the produced neutrinos are collimated and energetic. The energies involved are of the order of 100s of MeVs. The physics reach seems to be competitive with that of neutrino factories, when the results are combined with those from superbeams. The main thrust of such experiments will be searches for leptonic CP violation since the original beams are very pure. For details, see ref.[16].

### Outlook

It is clear that precision measurements on the neutrino oscillation parameters can only be made with neutrino factory beams. While a lot of work is still going on in

this field, it appears as though a combination of two baselines (at 3000 and 7200 km) will be best-suited to determine the maximal number of parameters with the maximum sensitivity. It may be pointed out here that the latter magic baseline is very close to the CERN-PUSHEP (India) and not too far off the JHF-PUSHEP (India) distance. Hence the possibility of having an India-based detector as a far-end to a magic (or near magic) long baseline experiment should be seriously discussed.

Of course, it must be kept in mind that there are a lot of non-oscillation physics possibilities at a neutrino factory where  $10^6$ – $10^7$  neutrino events are available per kg per year. The possible experiments include precision neutrino cross-section measurements, structure function measurements, including  $\alpha_s$  from non-singlet structure functions, nuclear effects such as shadowing, determination of individual parton densities, spin-dependent structure functions, tagged single charm meson/baryon production, electro-weak tests, exotic searches, and the entire gamut of tests of the standard model, not to forget physics with the primary proton beam. It appears that a wealth of physics opportunities is in store at neutrino factories; their potential is just being tapped and the technical requirements and feasibility issues are yet to be completely studied and understood.



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