Limits of Limit Sets

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School of Mathematics,
Tata Institute of Fundamental Research.

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$=$ Discrete subgroup G of $PSL_2(\mathbb{C})$
$=$ Discrete subgroup G of group of Isometries: $Isom(\mathbb{H}^3)$ i.e. Fundamental group of a hyperbolic manifold $M^3 = \mathbb{H}^3/G$.
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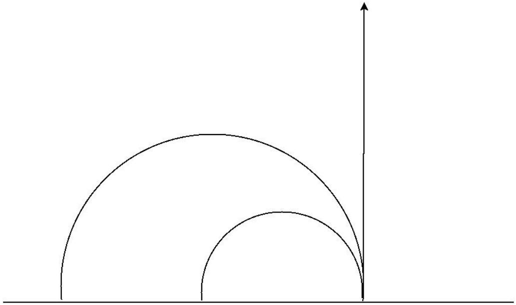
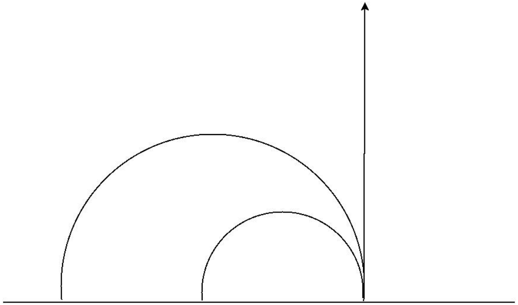


Topology/metric d_v = angle subtended at $v \in \mathbb{H}^3$.

Geodesics are semicircles meeting the boundary at right angles.

Metric = $ds^2 = \frac{dx^2+dy^2+dz^2}{z^2}$ on upper half space.

Metric blows up as on approaches $y = 0$ (resp. $z = 0$).



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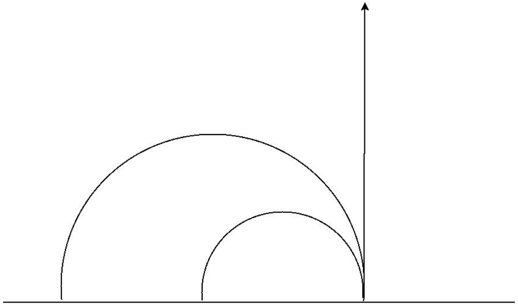

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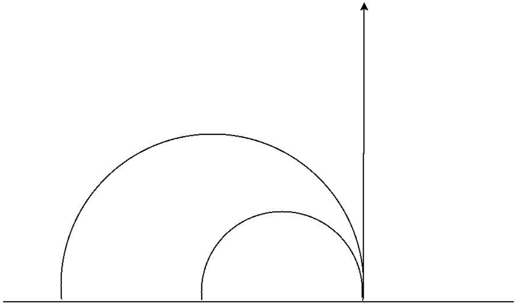

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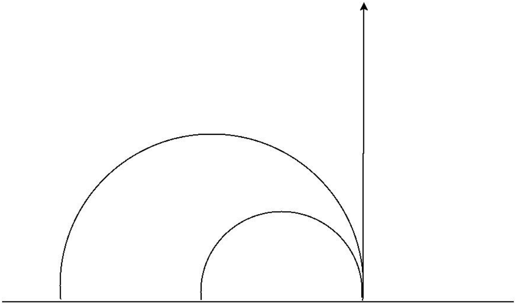

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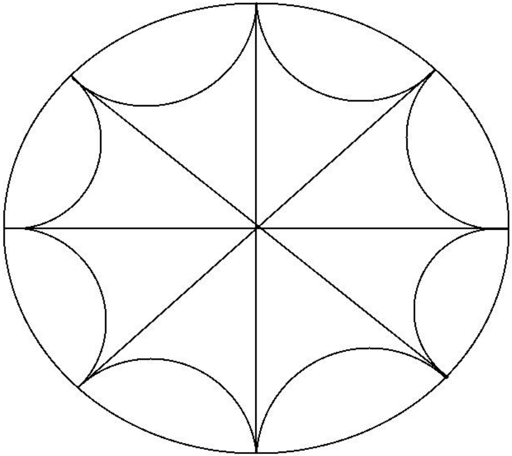

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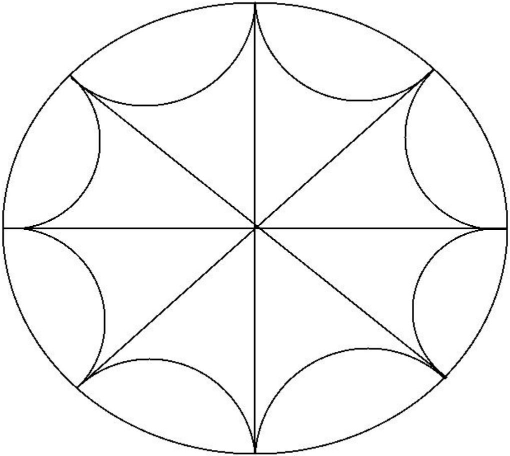
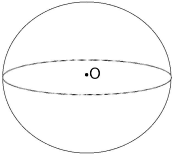
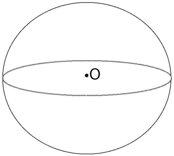




Limit set Λ_G = Set of accumulation points in $\widehat{\mathbb{C}}$ of $G.o$ for some (any) $o \in \mathbb{H}^3$.
Hence for a Fuchsian group (subgroup of $PSL_2(\mathbb{R})$), limit set = round equatorial circle.
Example of an infinite covolume Kleinian group.



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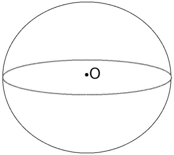

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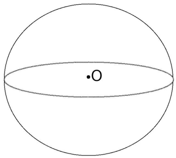

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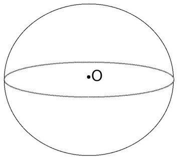

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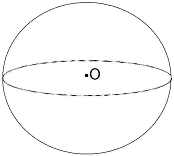

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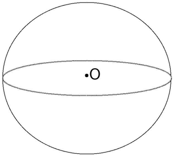

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Limit set is the *locus of chaotic dynamics of the* $\Gamma-$*action on* S^2.
Complement: Two round open discs.
On each, G acts freely (i.e. without fixed points) properly discontinuously, by conformal automorphisms.
Hence quotient is two copies of the 'same' Riemann surface (one dimensional complex analytic manifold.)
$\widehat{\mathbb{C}} \setminus \Lambda_G = \Omega_G$ is called the *domain of discontinuity* D_G of G.

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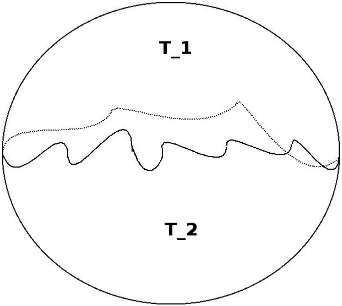
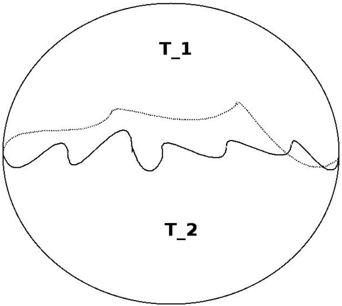


Quasi Fuchsian groups –quasiconformal deformations.
Ahlfors-Bers Simultaneous Uniformization.
Limit set is the image under a quasiconformal map of the round circle.



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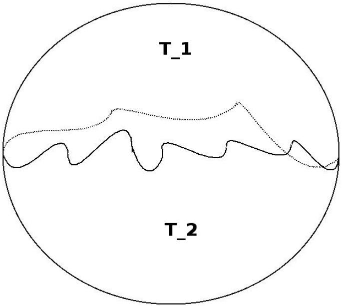

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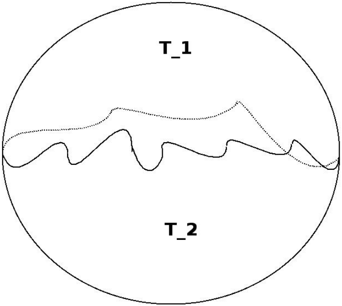

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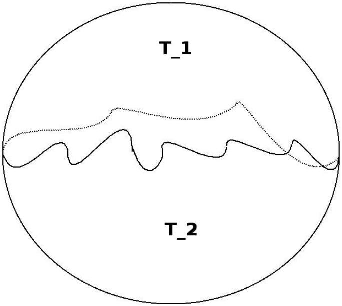

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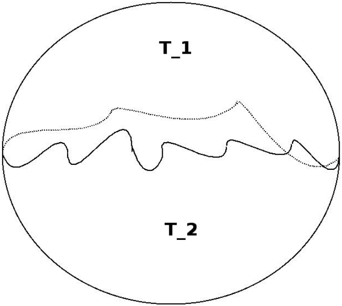

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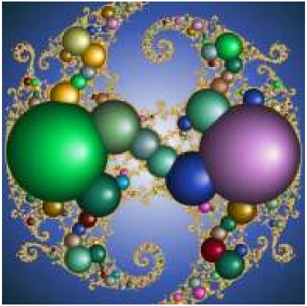

Limits of quasiFuchsian groups:
Thickness of Convex core $CC(M)$ tends to infinity.
2 possibilities: Degenerate only τ_1. Degenerate both τ_1, τ_2.
i.e. $I \to [0, \infty)$ **(simply degenerate)**
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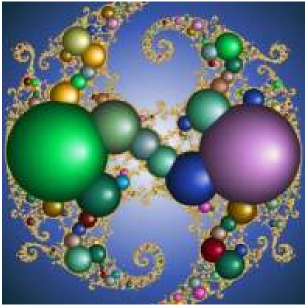

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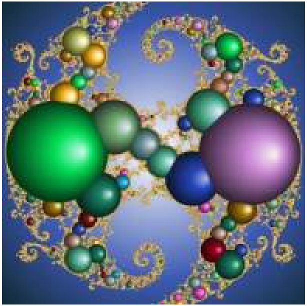

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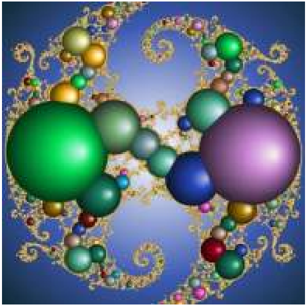

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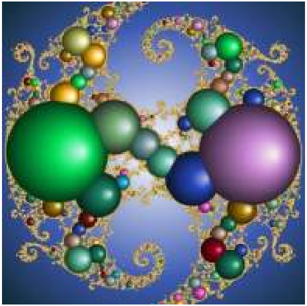

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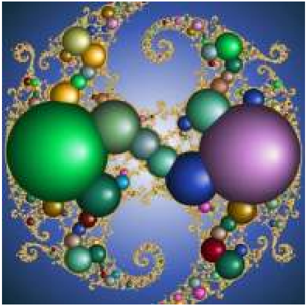

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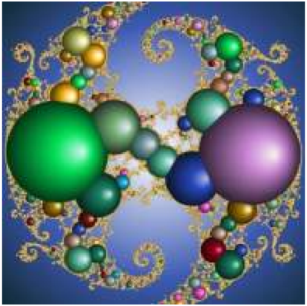

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$M-$ hyperbolic 3-manifold homotopy equivalent to a closed hyperbolic surface S.
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Goal: Given hyperbolic geodesic segment $\lambda \subset \widetilde{S}$ lying outside large ball about $o \in \widetilde{S}$, show that geodesic in $\mathbf{H}^3$ joining endpoints of $\widetilde{i}(\lambda)$ lies outside large ball about $\widetilde{i}(o)$in $\mathbf{H}^3$.

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if for all $g \in G$, $\rho_n(g) \to \rho_\infty(g)$.

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Reformulating Part 2 of Thurston's Question.

- Q1 Does strong convergence of finitely generated Kleinian groups imply uniform convergence of CT-maps?
- Q2 Does algebraic convergence of finitely generated Kleinian groups imply pointwise convergence of CT-maps?

Theorem

[M-, Series]
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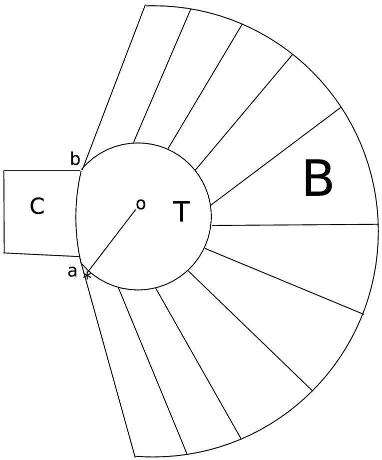
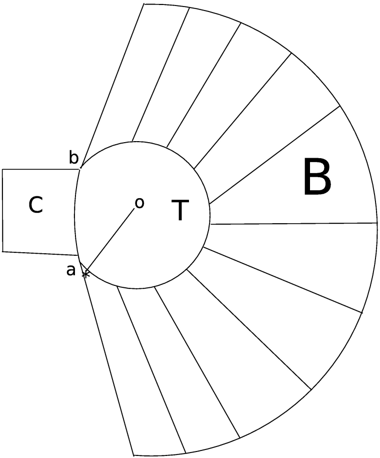


A schematic picture of K built up of B, C and T. The points a, b are the base-points $\phi_n^-(s_0) = o_n^-$ and $\phi_n^+(s_0) = o_n^+$ respectively.



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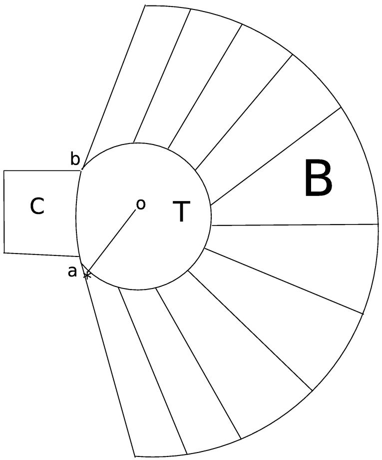

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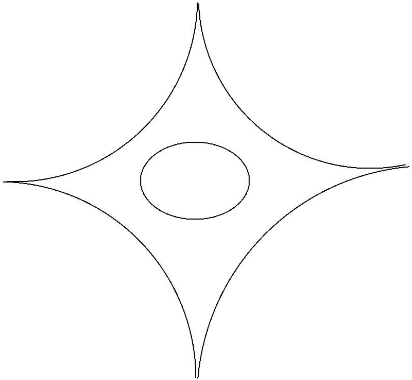
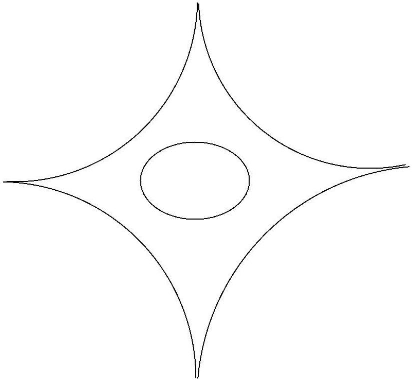


Crown Domain

Points of non-convergence of $\hat{i}_n$: endpoints of lifts of unstable leaves which bound the crown domain of the unstable lamination $|\lambda^u|$ of χ.



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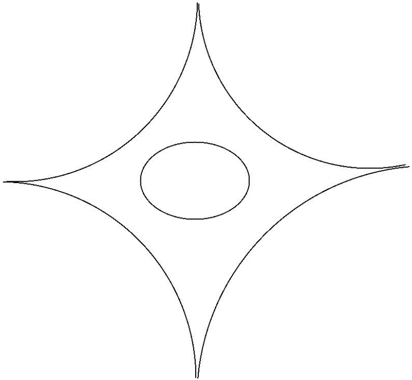

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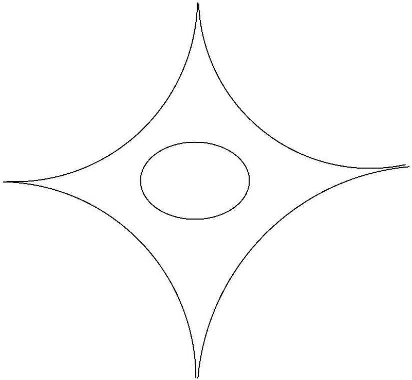

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