

Interfacing analytical and numerical relativity in modeling binary black hole coalescences

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- **Understanding gravity in the weak and strong regime**
e.g., comparing with post-Newtonian theory; grasping the transition inspiral to merger to ringdown
- **Detecting gravitational waves and extracting unique information**
e.g., building analytic templates
- **Making astrophysical predictions**
e.g., recoil velocity of merging black holes; how supermassive black holes formed

Modeling the long inspiral phase using PN theory

[Blanchet, Damour, Iyer, Faye, Deruelle; Wagoner, Will, Wiseman, Kidder, ...]

- **In general relativity radiation-reaction effects appear at order $\sim v^5/c^5$ beyond the Newtonian force law**

$$m \frac{d\mathbf{v}}{dt} = \mathbf{F}_{\text{Newt}} + \cdots + \left(\frac{v}{c}\right)^5 \mathbf{F}_{\text{RR}}$$

- **Throughout the inspiral $T_{\text{RR}} \gg T_{\text{orb}} \Rightarrow$ natural *adiabatic parameter***

$$\frac{\dot{\omega}}{\omega^2} = \mathcal{O}\left[\left(\frac{v}{c}\right)^5\right]$$

- **PN expansion: formal expansion in $1/c$ when $c \rightarrow +\infty$**
- **For compact bodies, such as neutron stars and black holes,**

$$\frac{v^2}{c^2} \sim \frac{Gm}{c^2 r} \sim \frac{R_S}{r} \ll 1$$

Waveforms in the adiabatic approximation

- **Inspiral as an adiabatic sequence of circular orbits:**

$$h(t) \propto \ddot{Q} \propto \frac{v^2}{c^2} \cos 2\varphi \propto \left(\frac{GM\omega}{c^3}\right)^{2/3} \cos 2\varphi$$

- **Energy-balance equation:** $\frac{dE(v)}{dt} = -F(v)$

$E(v)$ → center-of-mass energy $F(v)$ → gravitational-wave energy flux

$E(v)$ and $F(v)$ known as a PN expansion in $v/c = (GM\omega/c^3)^{1/3}$

$$\Rightarrow \dot{\omega} = -\frac{F(\omega)}{[dE(\omega)/d\omega]} \quad \Rightarrow \quad \varphi_{\text{GW}}(t) = 2\varphi(t) = 1/\pi \int \omega dt$$

Effective-one-body and Padé resummation

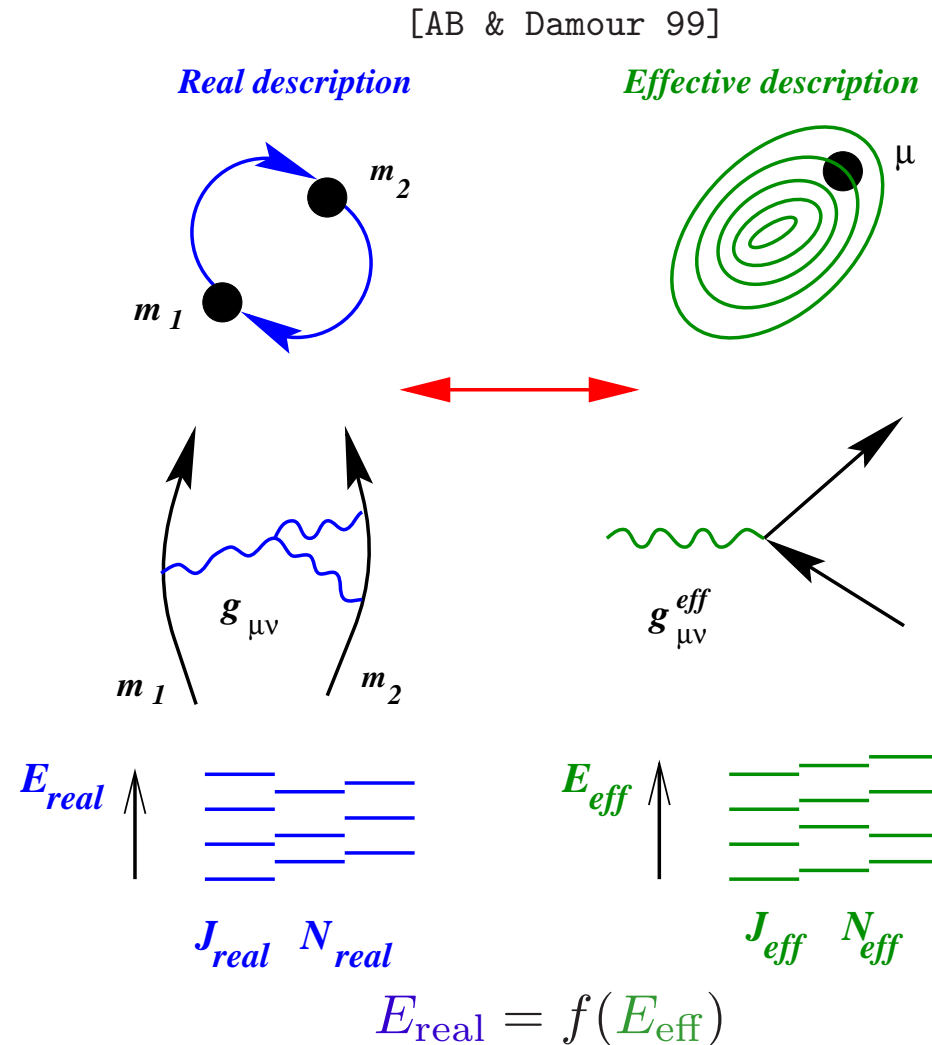
- Resum so that *known* test mass limit results are recovered
- Resum the PN expansion assuming that the equal-mass limit is a η -deformation of the test-mass limit

$$\eta = m_1 m_2 / M^2$$

$$0 \leq \eta \leq 1/4$$

- Padé resummation of the energy flux F

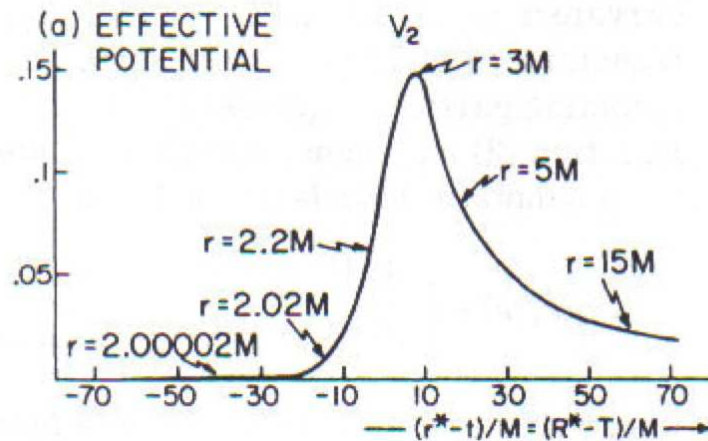
[Damour, Iyer & Sathyaprakash 97]



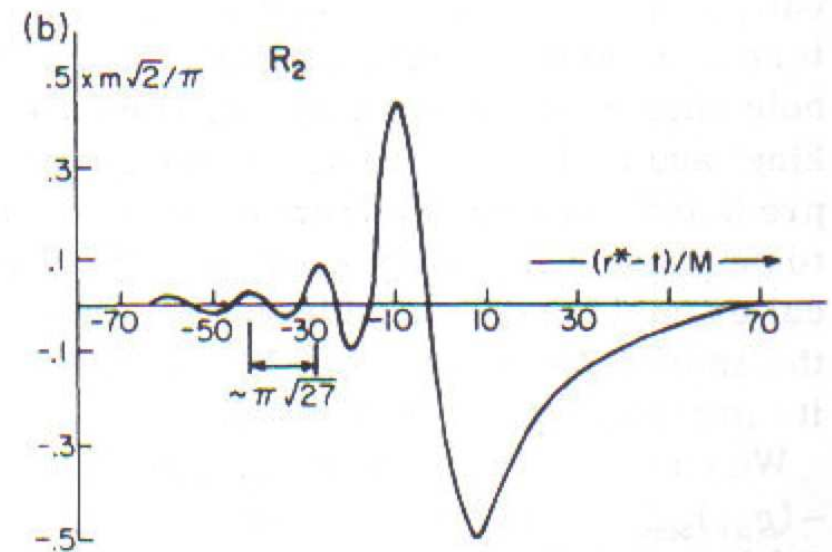
Features of the GW signal emitted by a test-particle falling radially in a Schwarzschild black hole

$$\frac{d^2}{dr_*^2} Z_l + (V_l - \omega^2) Z_l = \mathcal{S}_l$$

$Z_l \rightarrow$ perturbation $\mathcal{S}_l \rightarrow$ source



Outgoing field

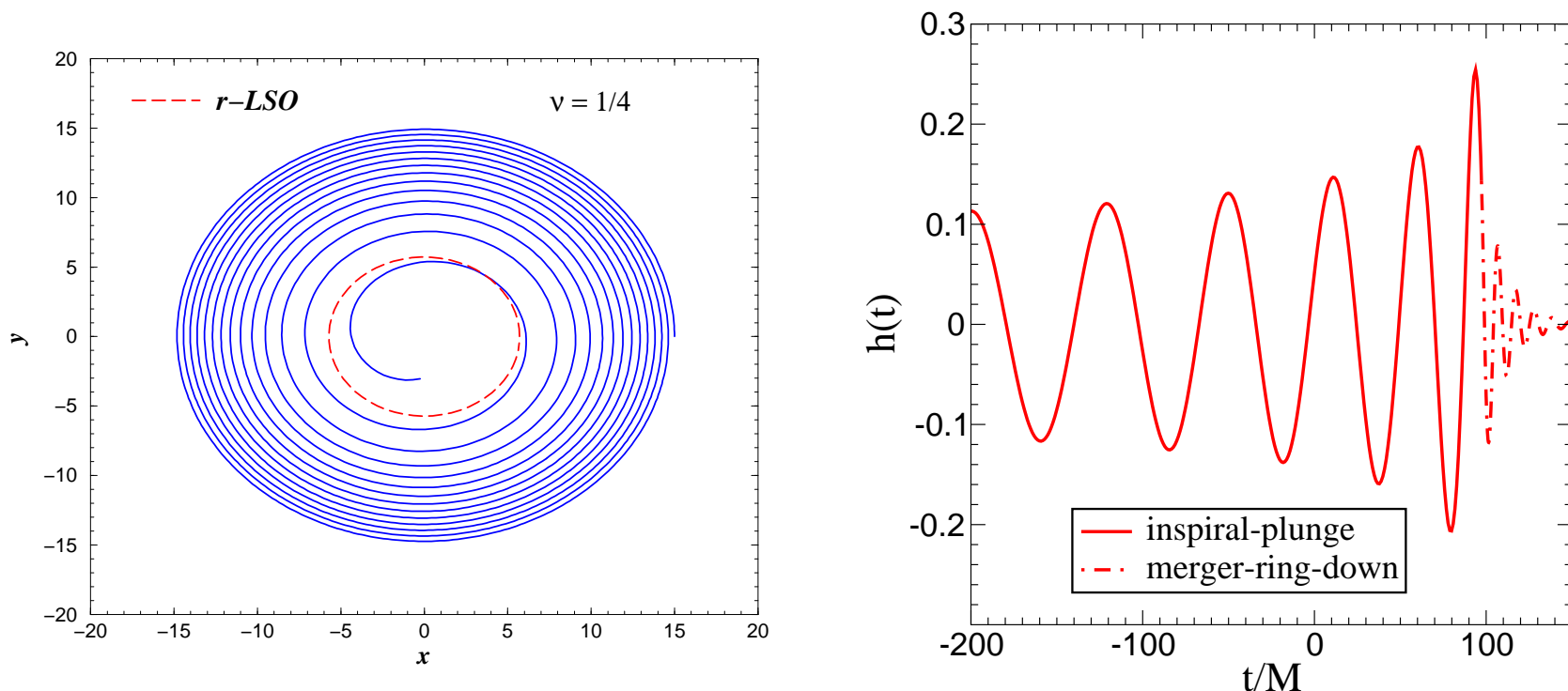


... part of the energy produced in the strong-burst region is stored in the resonant cavity of the geometry, and then slowly released in ringdown modes.

[Press 71; Davis, Ruffini, Press & Price 71; Davis, Ruffini & Tionmo 72]

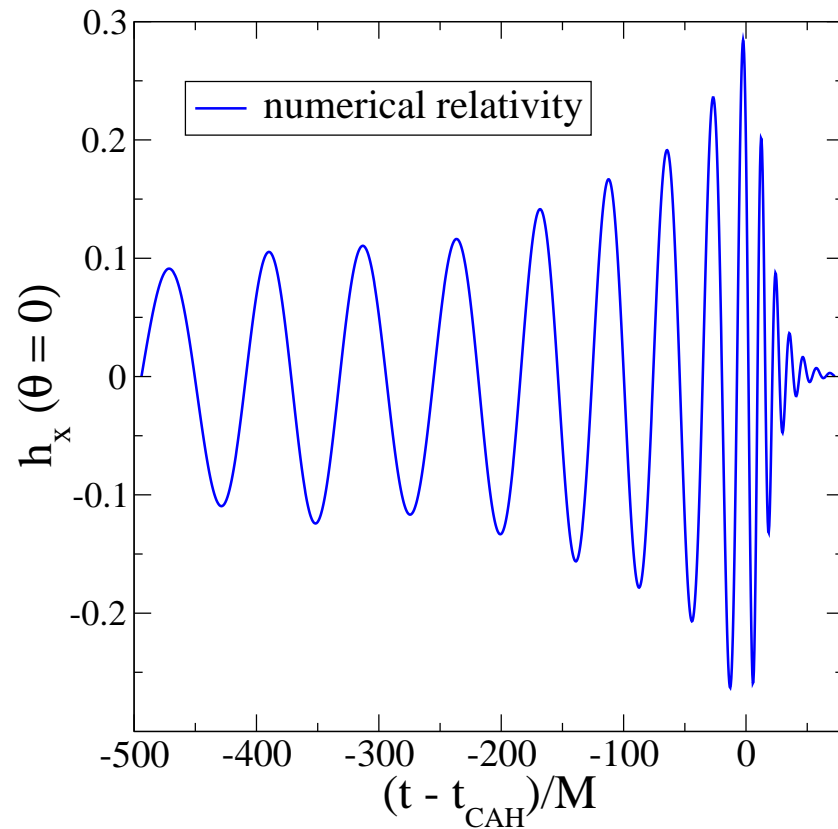
Full waveform as predicted by the EOB-*Padé* model

- The plunge (~ 1.5 GW cycles) is a smooth continuation of the inspiral phase
- The transition merger to ringdown was assumed *very short*
- One single QNM matched using $M_{\text{BH}} = E_{\text{LR}} = 0.976 M$, $a_{\text{BH}} = \mathbf{J}_{\text{LR}}/E_{\text{LR}}^2 = 0.77$

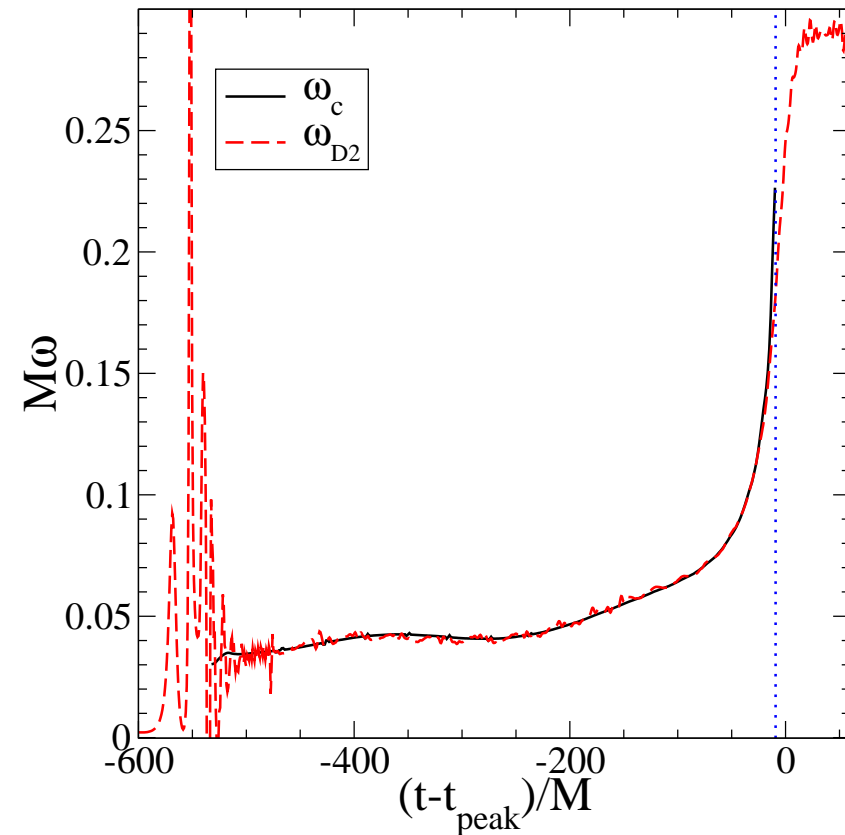


[AB & Damour 99, 00; Damour, Jaranowski & Schafer 00; Damour 01; AB, Chen & Damour 06]

Numerical simulations of equal-mass binary: *one* dominant frequency



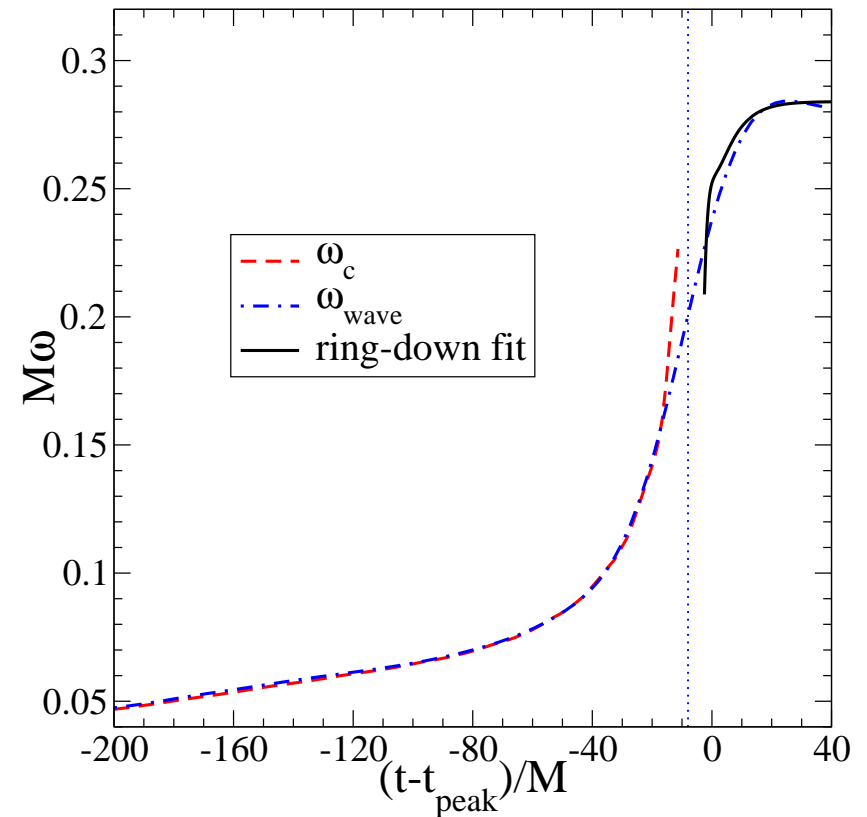
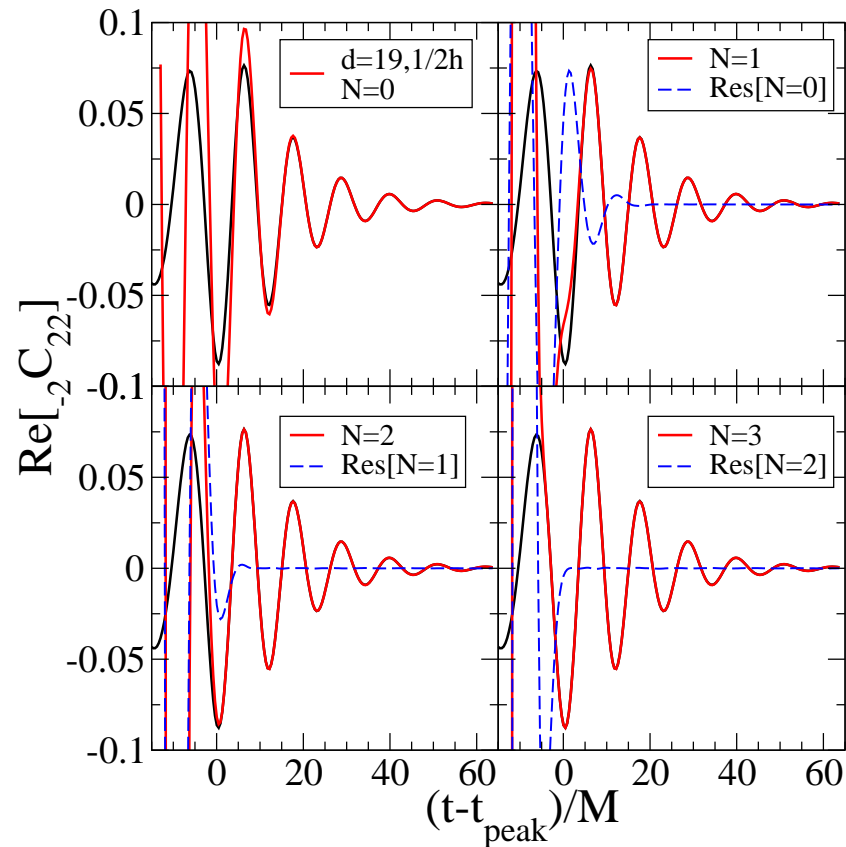
[AB, Cook & Pretorius 06]



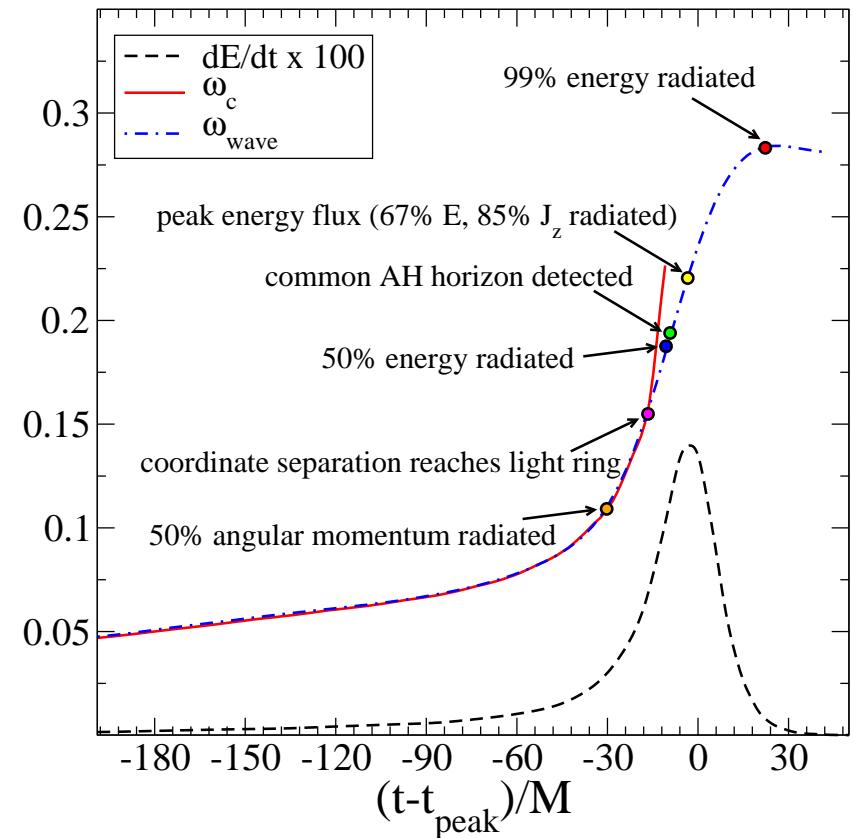
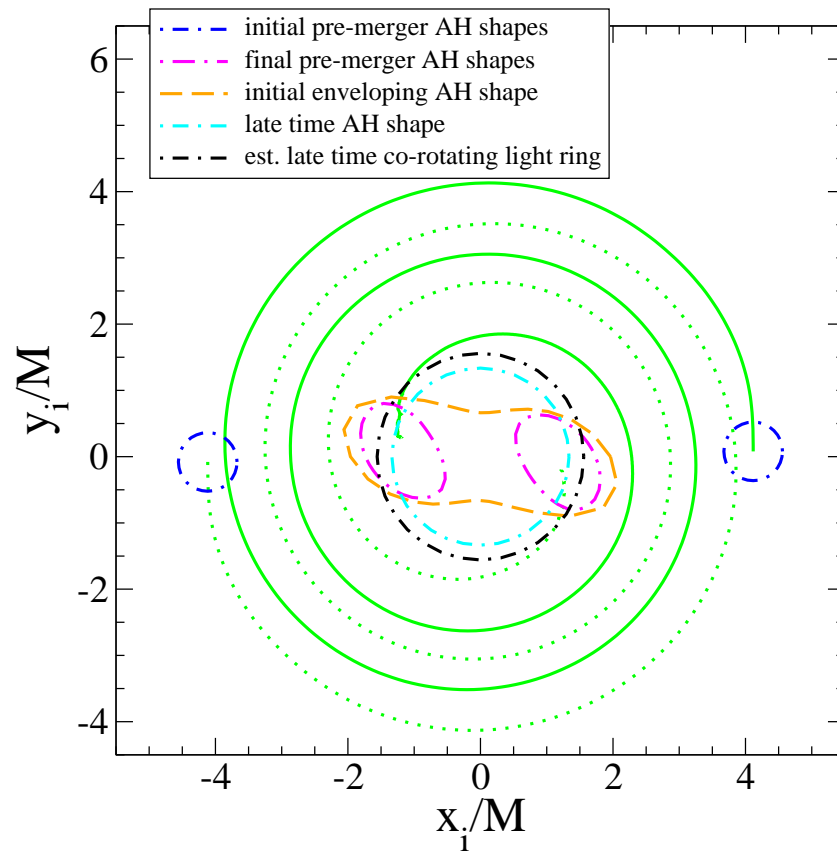
- $\omega_c \Leftarrow$ from the coordinate separation
- $\omega_{D2} = -\frac{1}{2}\text{Im} \left[\frac{\dot{C}_{22}}{C_{22}} \right] \Leftarrow$ from the wave

When the ringdown phase starts. Higher overtones.

[AB, Cook & Pretorius 06; see also Berti et al. 07]



The (plunge and) merger



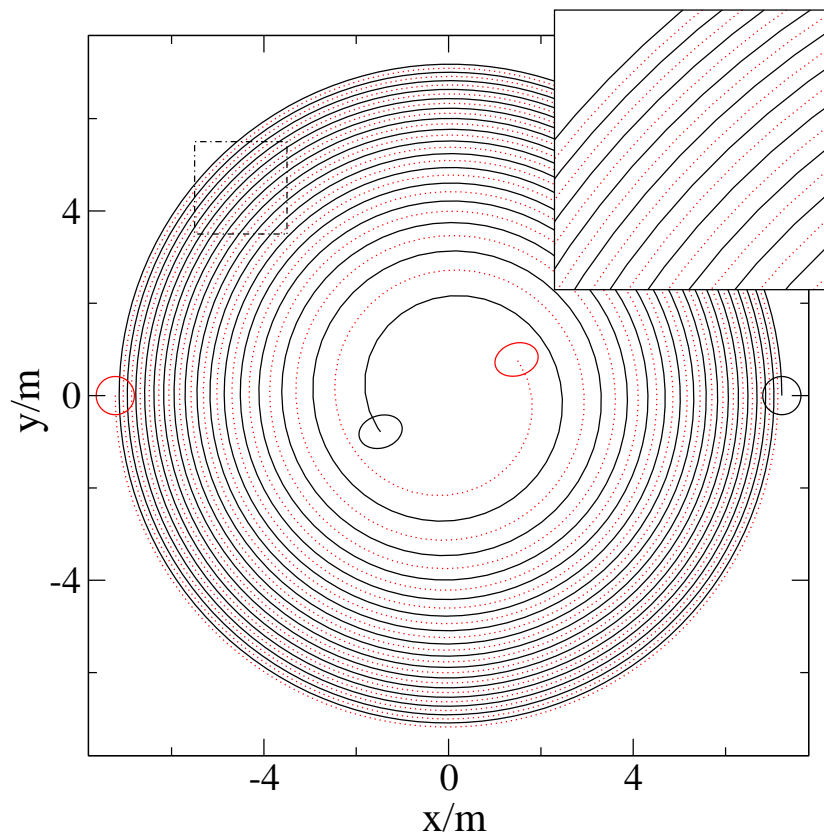
- *Short transition merger–ringdown*

- **Energy and angular-momentum quickly released during merger**

[AB, Cook & Pretorius 06]

Extremely accurate NR simulation using spectral methods

- **Equal-mass non-spinning black-hole binary** Caltech-Cornell collaboration



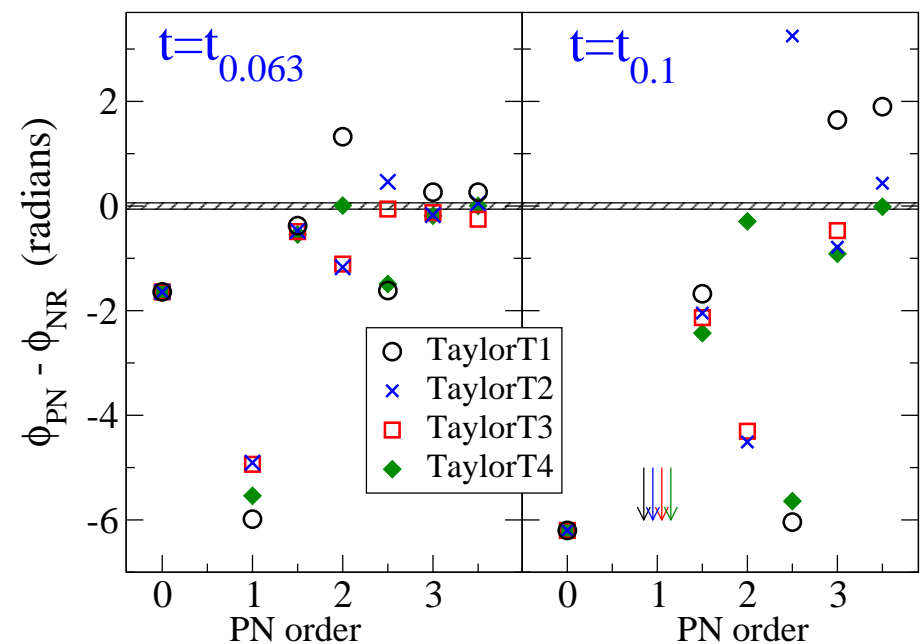
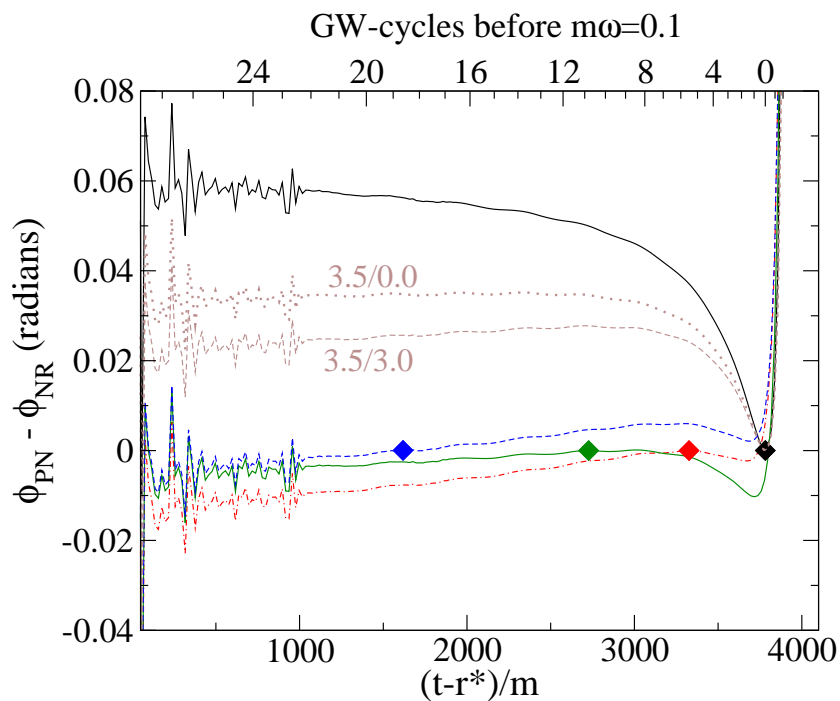
- During the first 15 GW cycles all PN models agree with NR within 0.05 rad
- Different PN models differ by the way of solving:

$$\dot{\omega} = -\frac{F(\omega)}{[dE(\omega)/d\omega]}$$

Comparison PN-adiabatic models and extremely accurate numerical simulations

- **Equal-mass non-spinning black-hole binary** Caltech-Cornell collaboration
- **Later on the PN models accumulate a dephasing of few rads, except for one model**

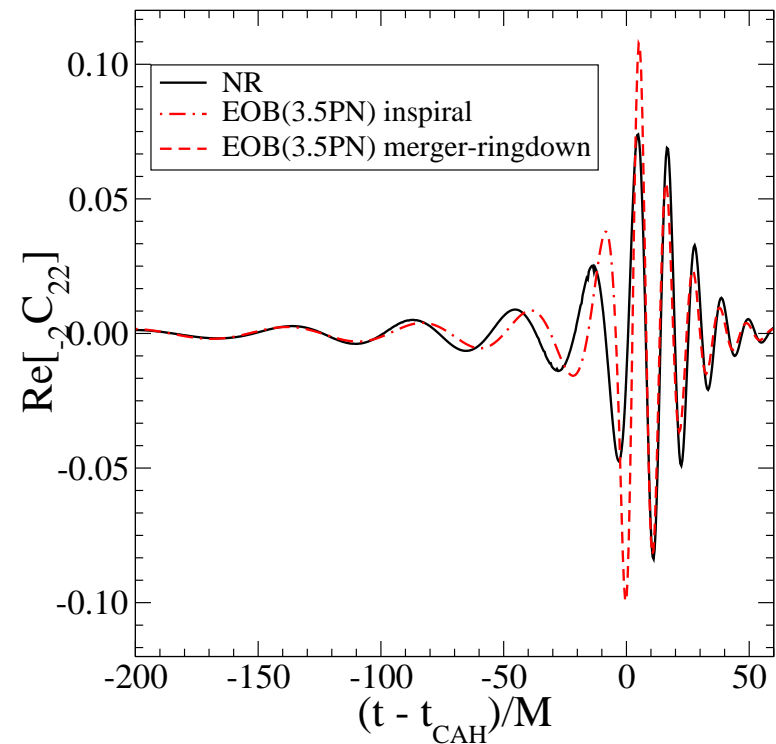
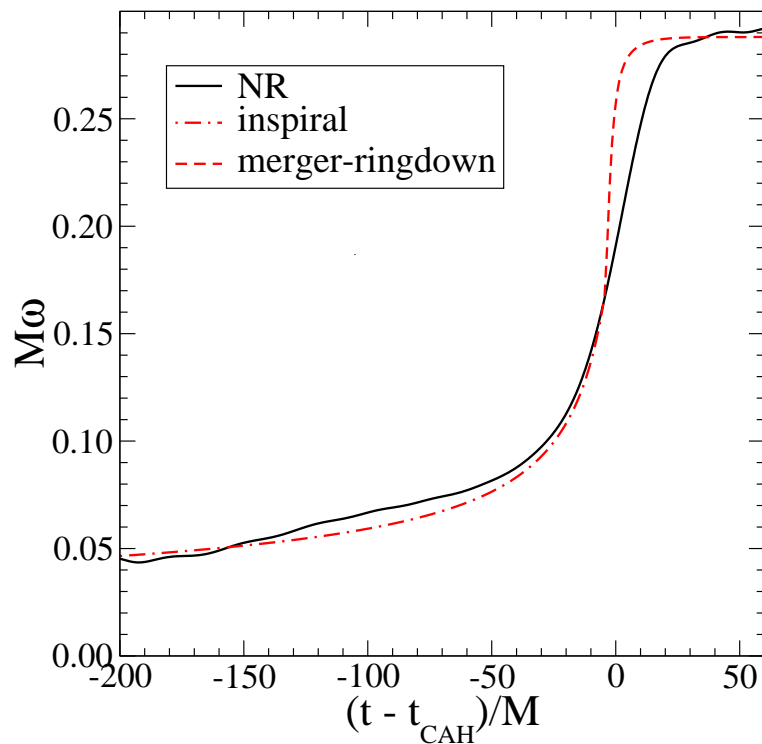
[see also Nasa-Goddard 07; Jena 07]



Comparing NR and EOB waveforms: *effectualness*

[AB, Cook & Pretorius 06; see also Pan, AB & NASA-Goddard 07]

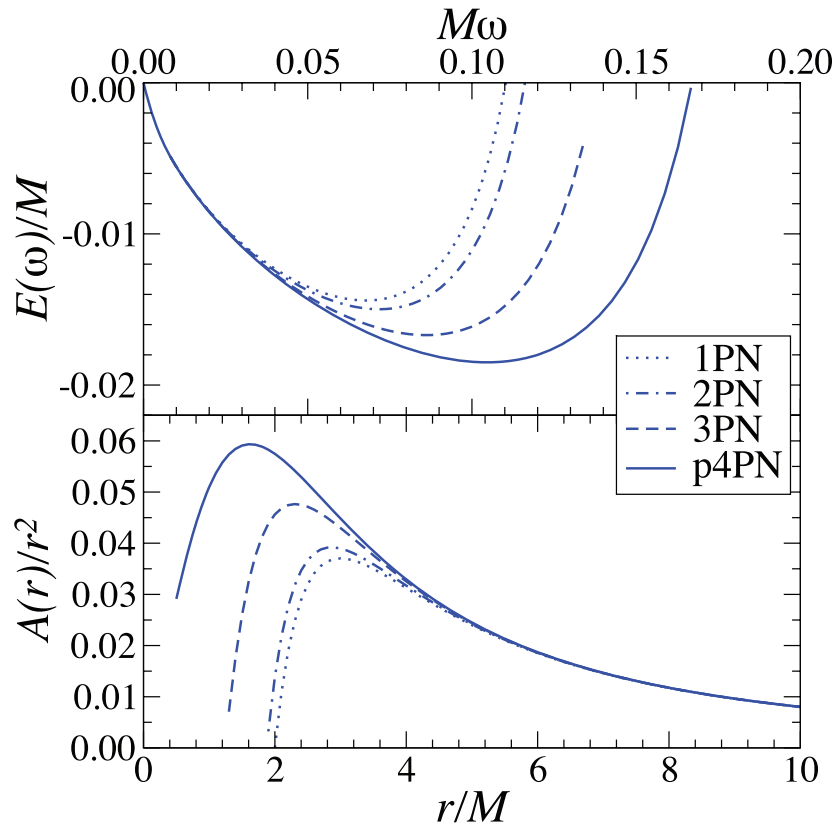
- **Fundamental QNM mode and two overtones included**



- **overlap $\gtrsim 0.97$ maximizing on binary parameters, time-of-arrival, initial phase**

Improving EOB model using NR as *guide*

[AB, Pan & NASA-Goddard 07]



[Damour, Iyer, Jaranowski & Sathyaprakash 03]

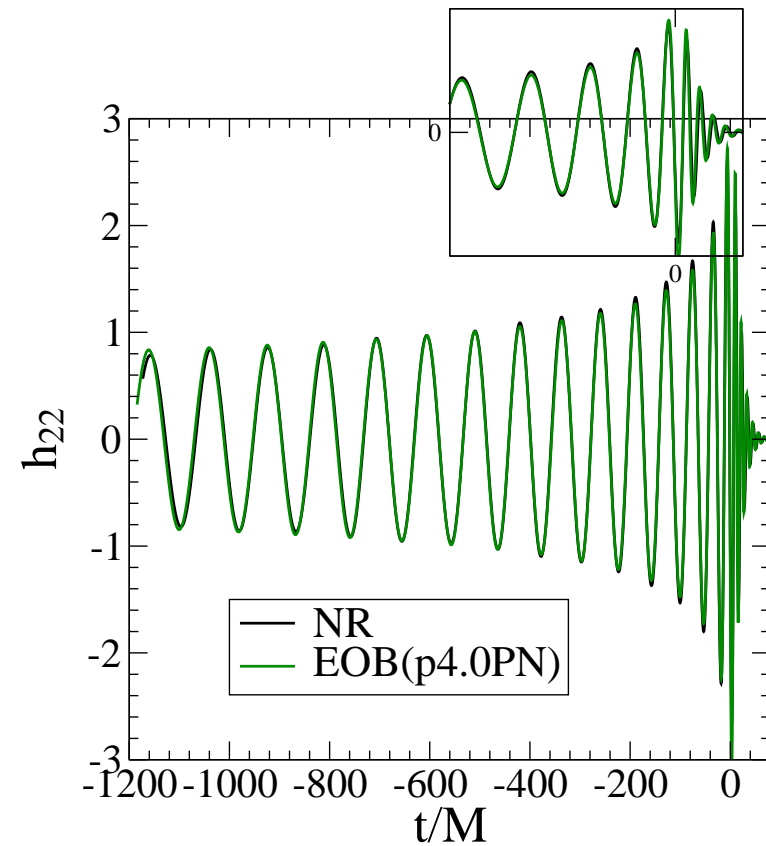
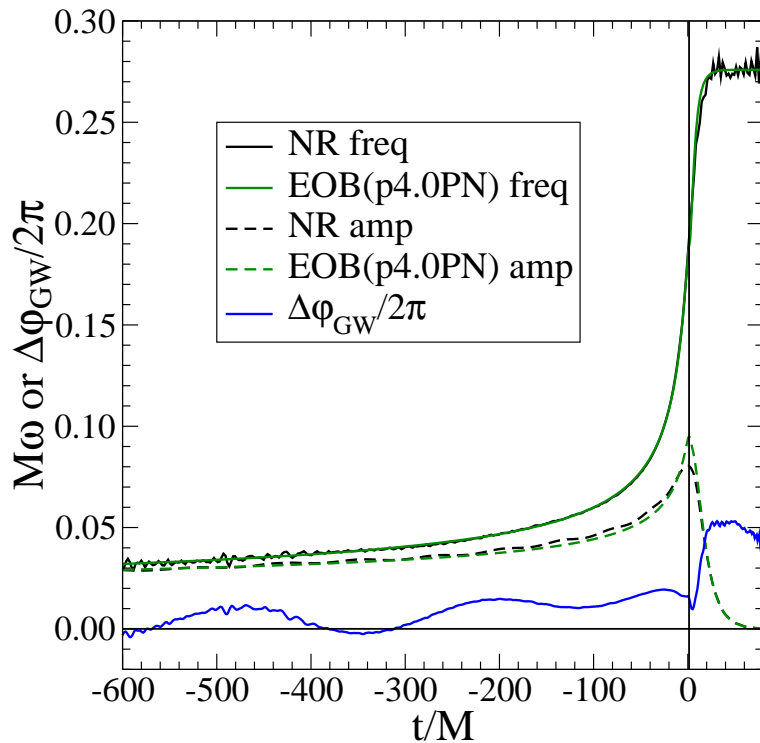
- $A^{\text{p4PN}}(r) = A^{\text{3PN}}(r) + \frac{\lambda\eta}{r^5}$, $\lambda = 60$
- **Apply Padé resummation to ensure presence of LSO and light ring**
- **Analytic inspiral/ringdown matching point**
 $M \omega_{\text{match}} = 0.133 + 0.183 \eta + 0.161 \eta^2$
- **QNM frequency and decay time depend only on M_{BH}/M and a_f/M_{BH}**

$$\frac{M_{\text{BH}}}{M} = 1 + (\sqrt{8/9} - 1) \eta - 0.498 \eta^2$$

$$\frac{a_f}{M_{\text{BH}}} = \sqrt{12} \eta - 2.90 \eta^2$$

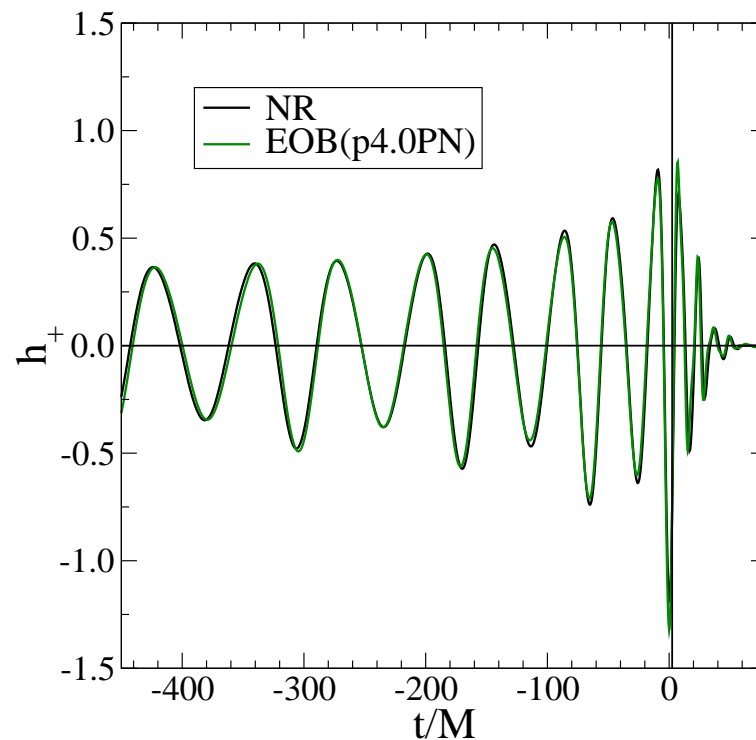
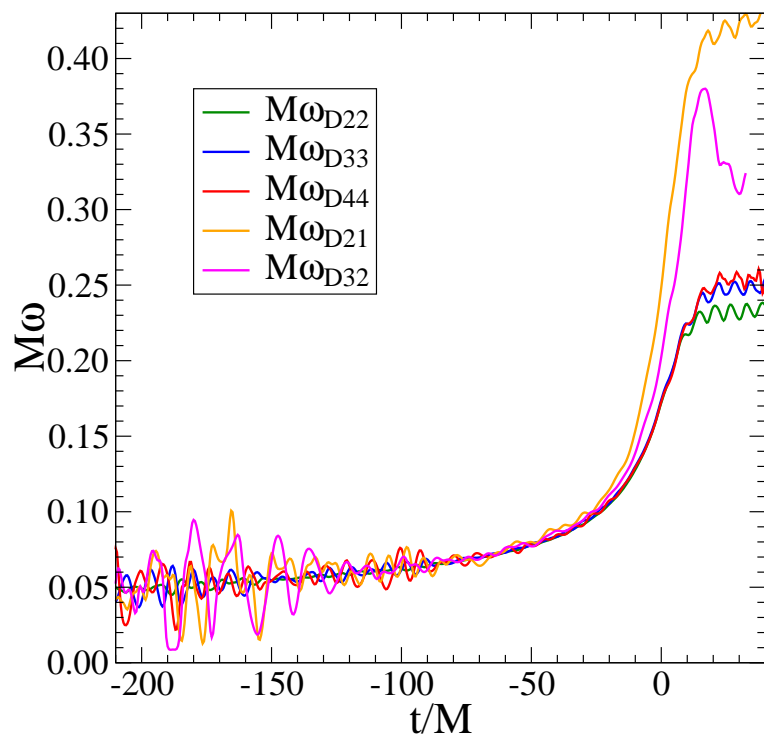
NR and EOB waveforms for equal-mass binary: *faithfulness*

- **Phase difference in GW cycles of $\sim 5\%$** [AB, Pan & NASA-Goddard 07]
- **overlap $\gtrsim 0.98$ maximizing *only* on time-of-arrival and initial phase**



NR and EOB waveforms for unequal-mass binary: *faithfulness*

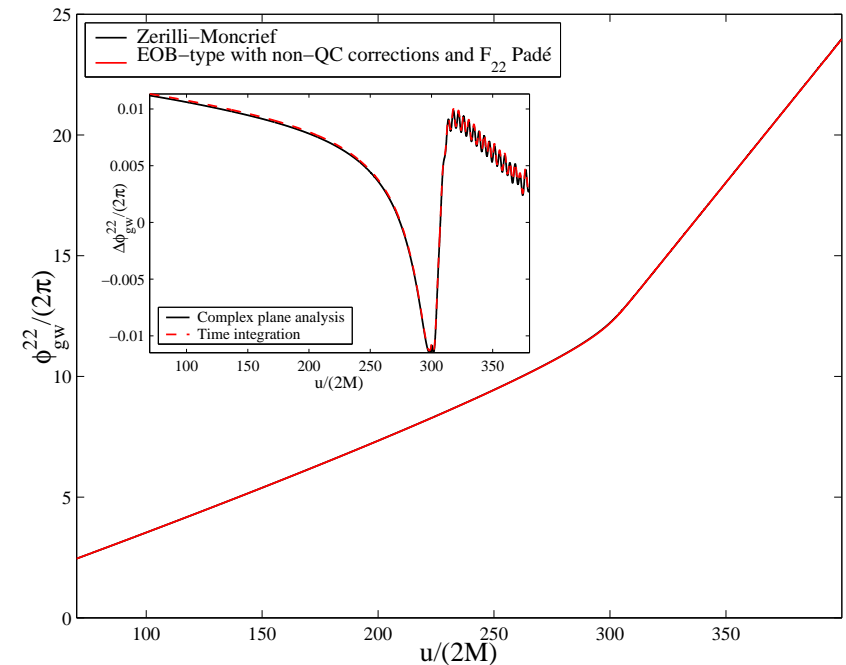
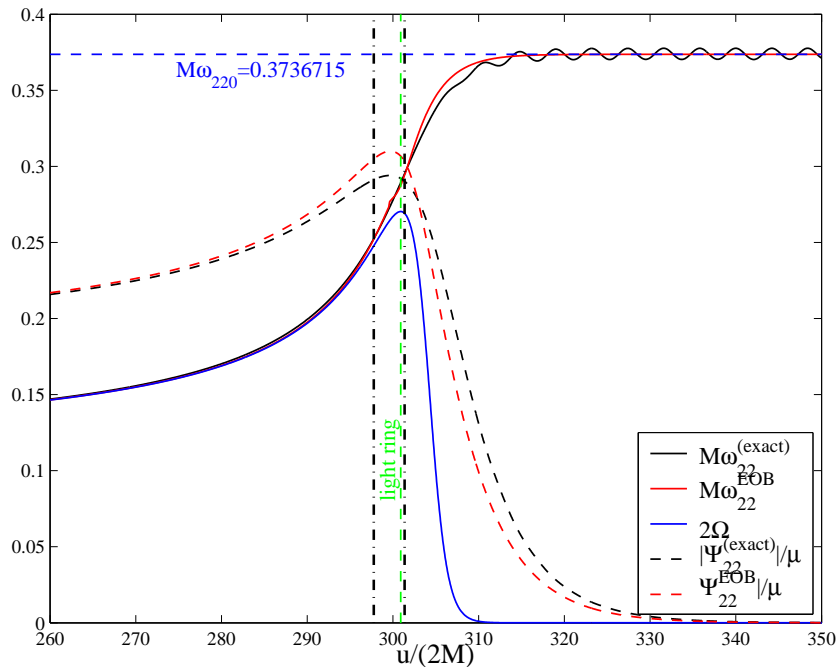
- Phase difference in GW cycles of $\sim 8\%$ [AB, Pan & NASA-Goddard 07]
- overlap $\gtrsim 0.98$ maximizing *only* on time-of-arrival and initial phase



[talk by Ajith on frequency-domain template family for inspiral-merger-ringdown]

Comparison Regge-Wheeler-Zerilli and EOB in the test-mass limit

[Damour, Nagar & Tartaglia 06; Damour & Nagar 07]

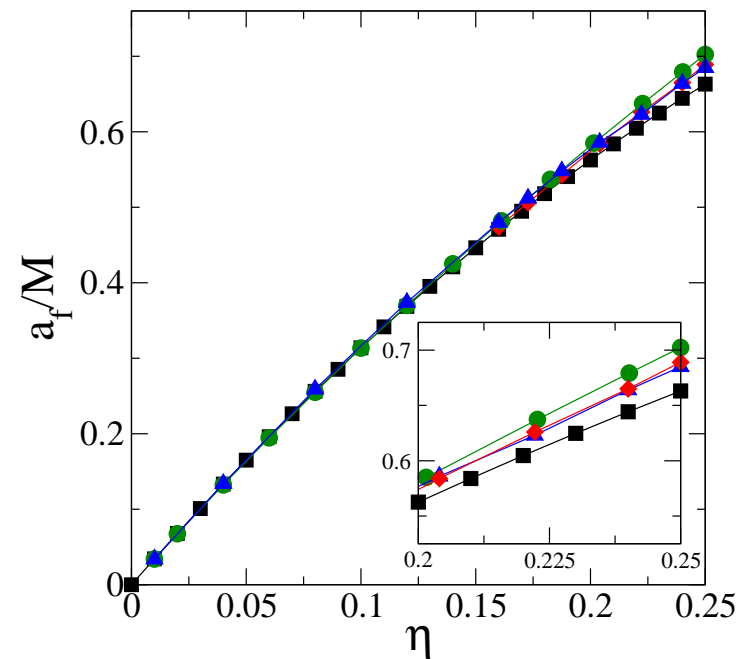
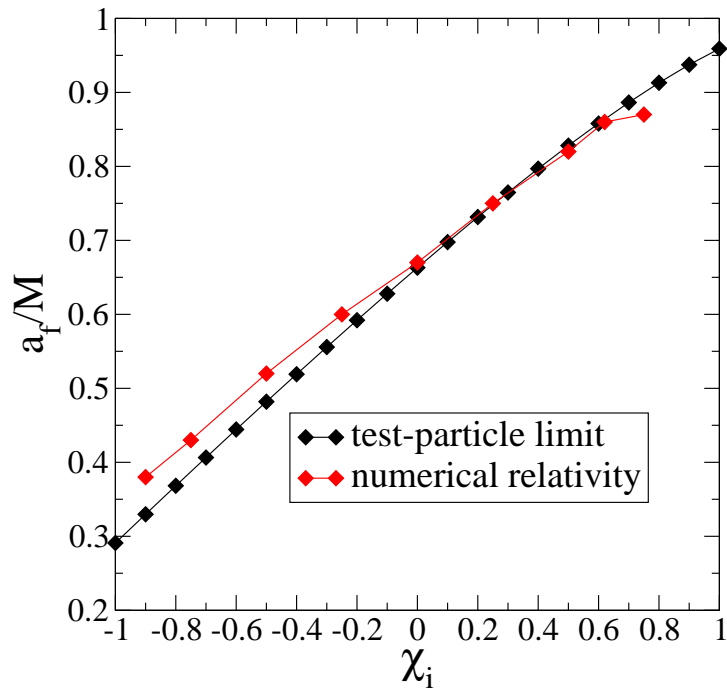


- **Several improvements: resummed higher-order amplitude corrections; deviations from quasi-circular motion; matching inspiral to ringdown on a *comb* instead of a point**

What is the final black hole spin and mass?

[Berti et al. 07; Damour & Nagar 07; AB et al. 07; Pollney et al. 07; Boyle et al. 07; Sperhake et al. 07]

- $\frac{a_f}{M} = \frac{L_{\text{orb}}^{\text{ISCO}}(a_f, \eta)}{M^2} + \frac{S_1}{M^2} + \frac{S_2}{M^2} \quad \Leftarrow \text{using Kerr spacetime! [AB, Kidder \& Lehner 07]}$
- **Good estimations also for precessing, spinning binaries**

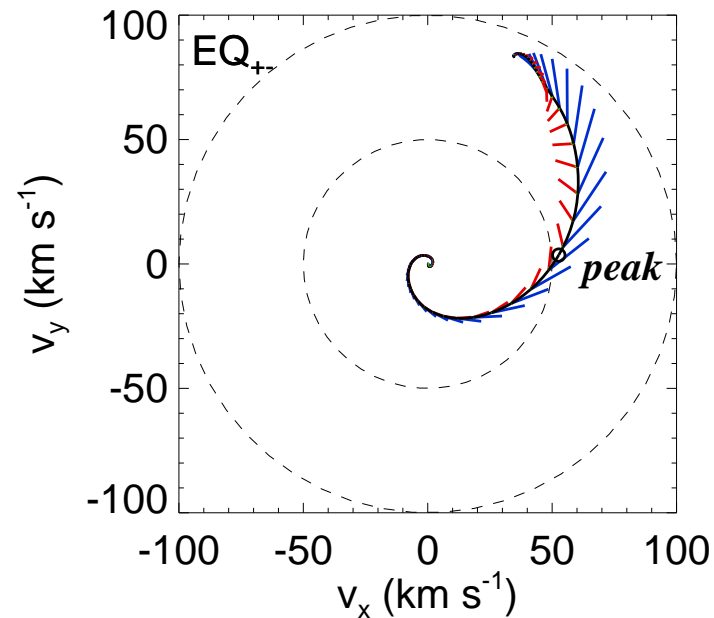
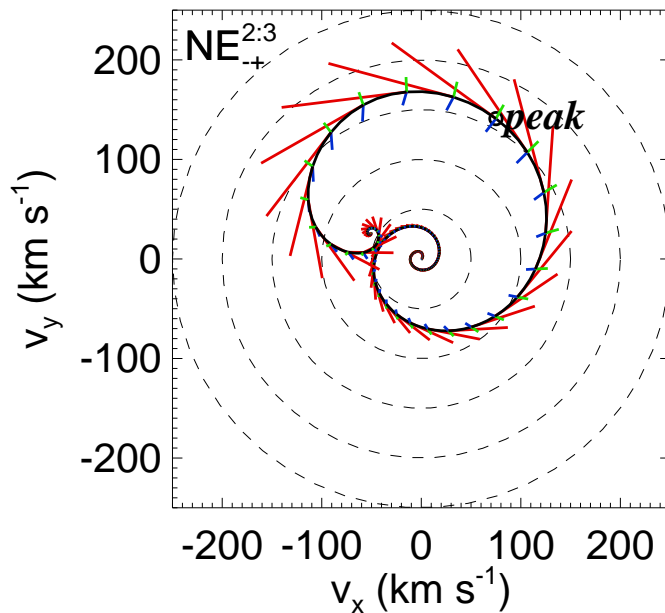


Anatomy of the kick (inspiral–merger–ringdown)

- **Analytic predictions for the kick** [Bekenstein; Fitchett & Detweiler; Kidder; Blanchet et al.]

[Damour & Gopakumar 06; Schnittman, AB & NASA-Goddard 07]

$$\mathbf{V}_{\text{kick}} \simeq \int \left[\hat{\mathbf{V}} \cdot \frac{d\mathbf{P}}{dt}(I^{22} S^{21}) + \hat{\mathbf{V}} \cdot \frac{d\mathbf{P}}{dt}(I^{22} I^{33}) + \hat{\mathbf{V}} \cdot \frac{d\mathbf{P}}{dt}(I^{33} I^{44}) \right] dt$$

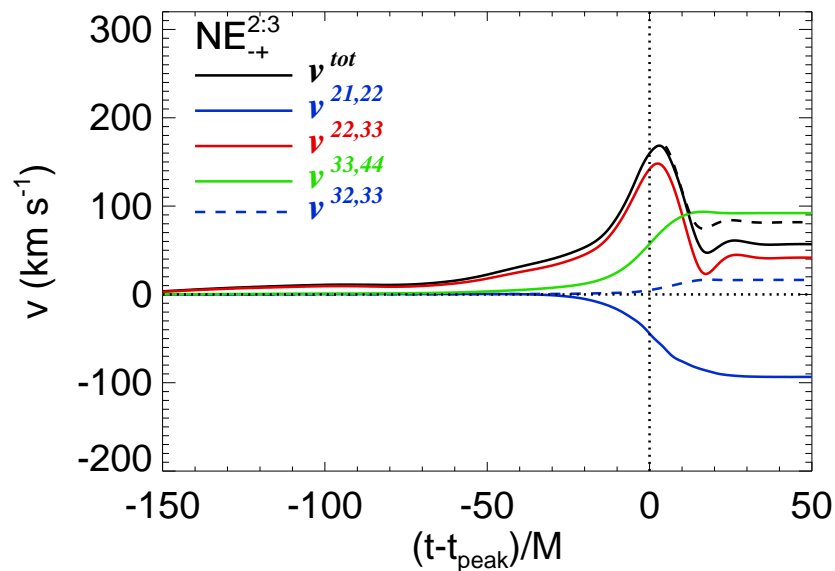


Anatomy of the kick and anti-kick

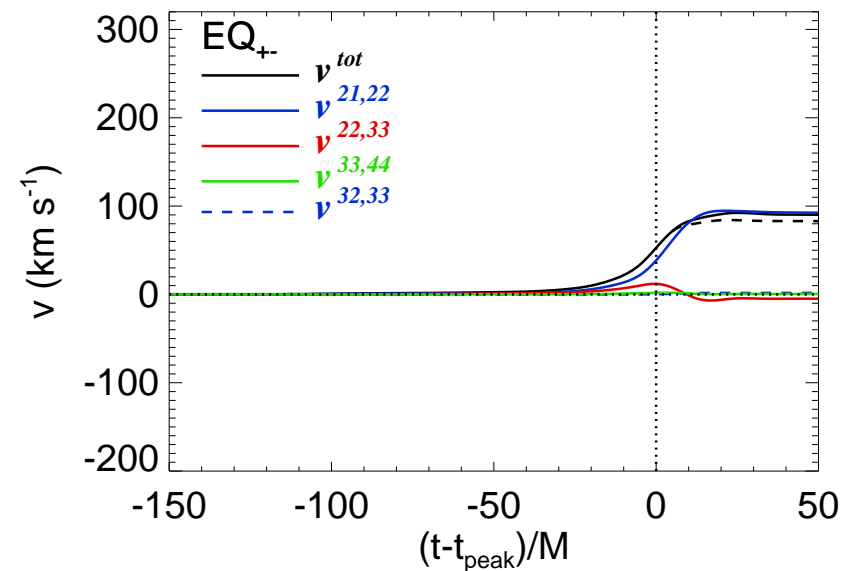
[Schnittman, AB & NASA-Goddard 07]

- Magnitude of anti-kick depends on QNM-frequencies associated to dominant modes

$I^{22} I^{33*}$: $(\omega_{33}^{\text{QNM}} - \omega_{22}^{\text{QNM}})$ is large
 \Rightarrow spiral back inward

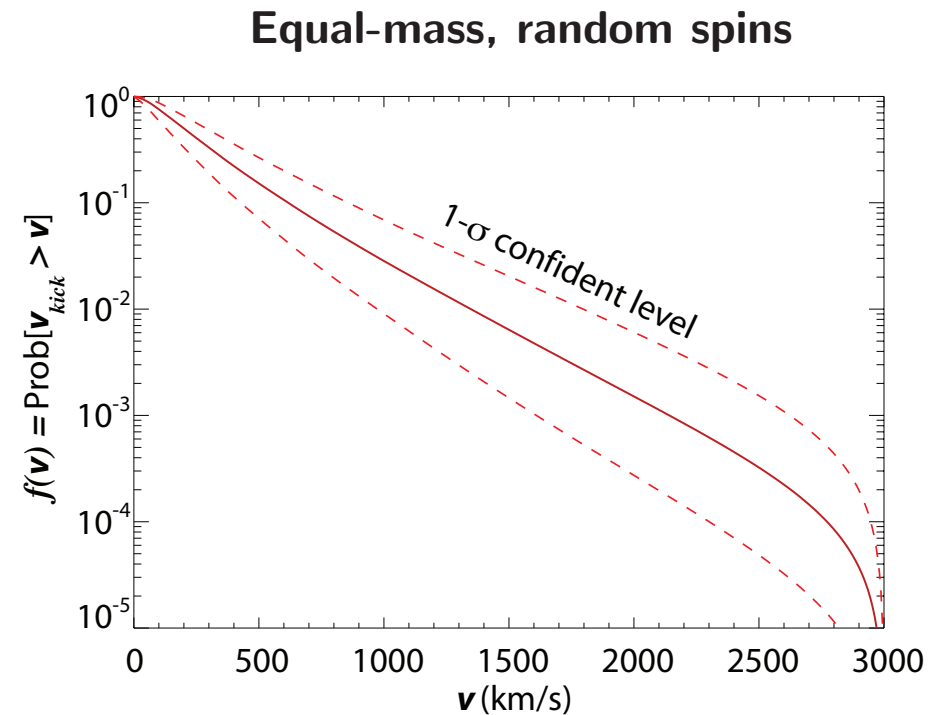
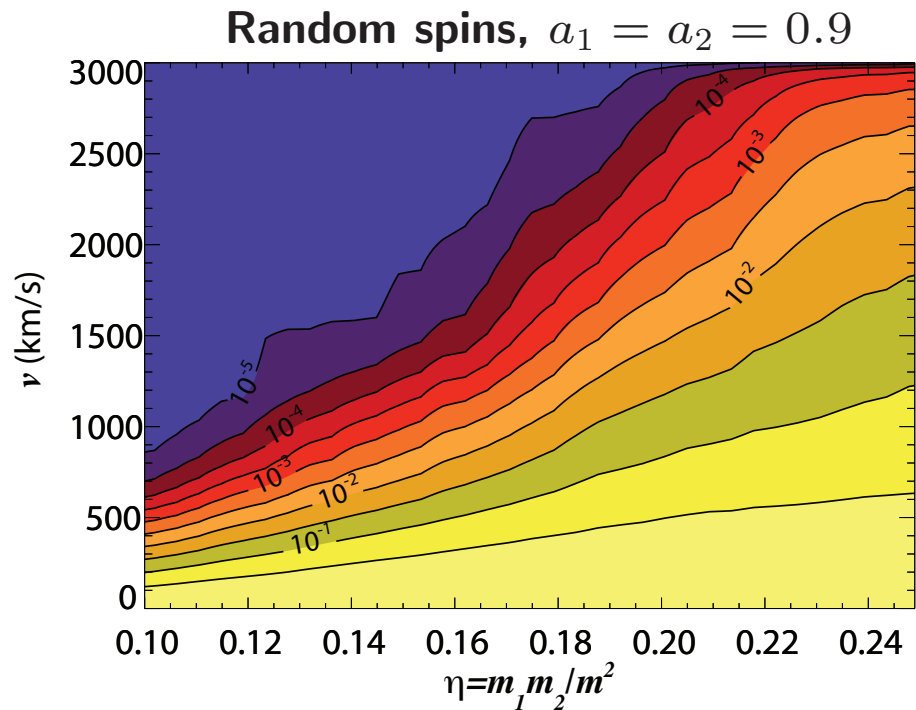


$I^{22*} S^{21}$: $(\omega_{21}^{\text{QNM}} - \omega_{22}^{\text{QNM}})$ is small
 \Rightarrow drifts off



Cumulative probability distribution for recoil velocities using EOB approach

[Schnittman & AB 07]



$$f_{v_{\text{kick}} > 500} = 0.12^{+0.06}_{-0.05}$$

$$f_{v_{\text{kick}} > 1000} = 0.027^{+0.021}_{-0.014}$$

Conclusions

- **Intriguing (anticipated) *simplicity* of (non-spinning) binary coalescence: details of merger hidden behind the curvature potential barrier.**
- **Consistency between PN calculations through 3PN order and numerical simulations**
- **Several progresses in estimating the final black-hole mass and spin**
- **Guided by NR simulations and by PN theory (at earlier times), notably the EOB model, we have a first example of analytical model for inspiral, merger, and ringdown *to be further improved* and extended to longer and accurate simulations.**
- **Gravitational recoil determined mainly by merger-ringdown phases.**
- **Improvement of analytic modeling to reduce uncertainties in Monte Carlo simulations of recoil velocity distribution.**