

# Status of Numerical Relativity: From my personal point of view

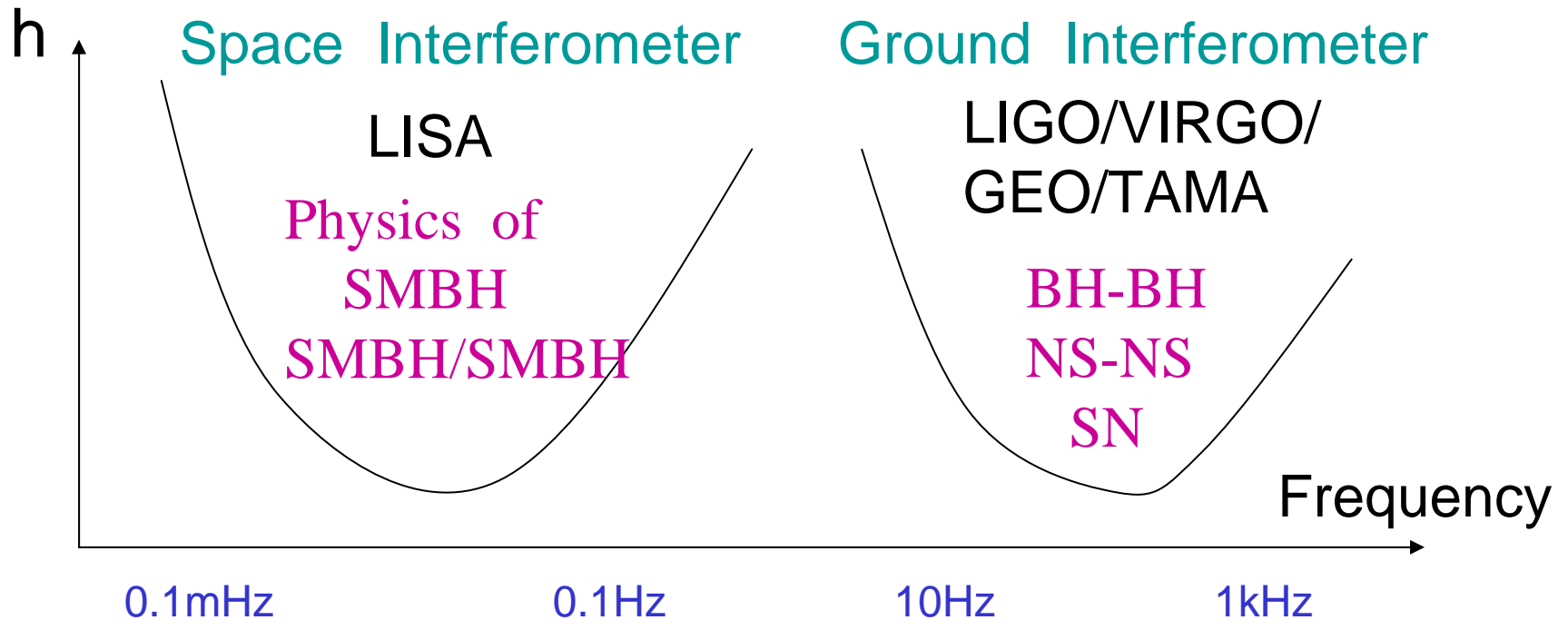
Masaru Shibata (U. Tokyo)

- 1 Introduction
- 2 General issues in numerical relativity
- 3 Current status of implementation
- 4 Some of our latest numerical results:  
NS-NS merger & Stellar core collapse
- 5 Summary & perspective

# 1: Introduction: Roles in NR

A To predict gravitational waveforms:

Two types of gravitational-wave detectors work now or soon.

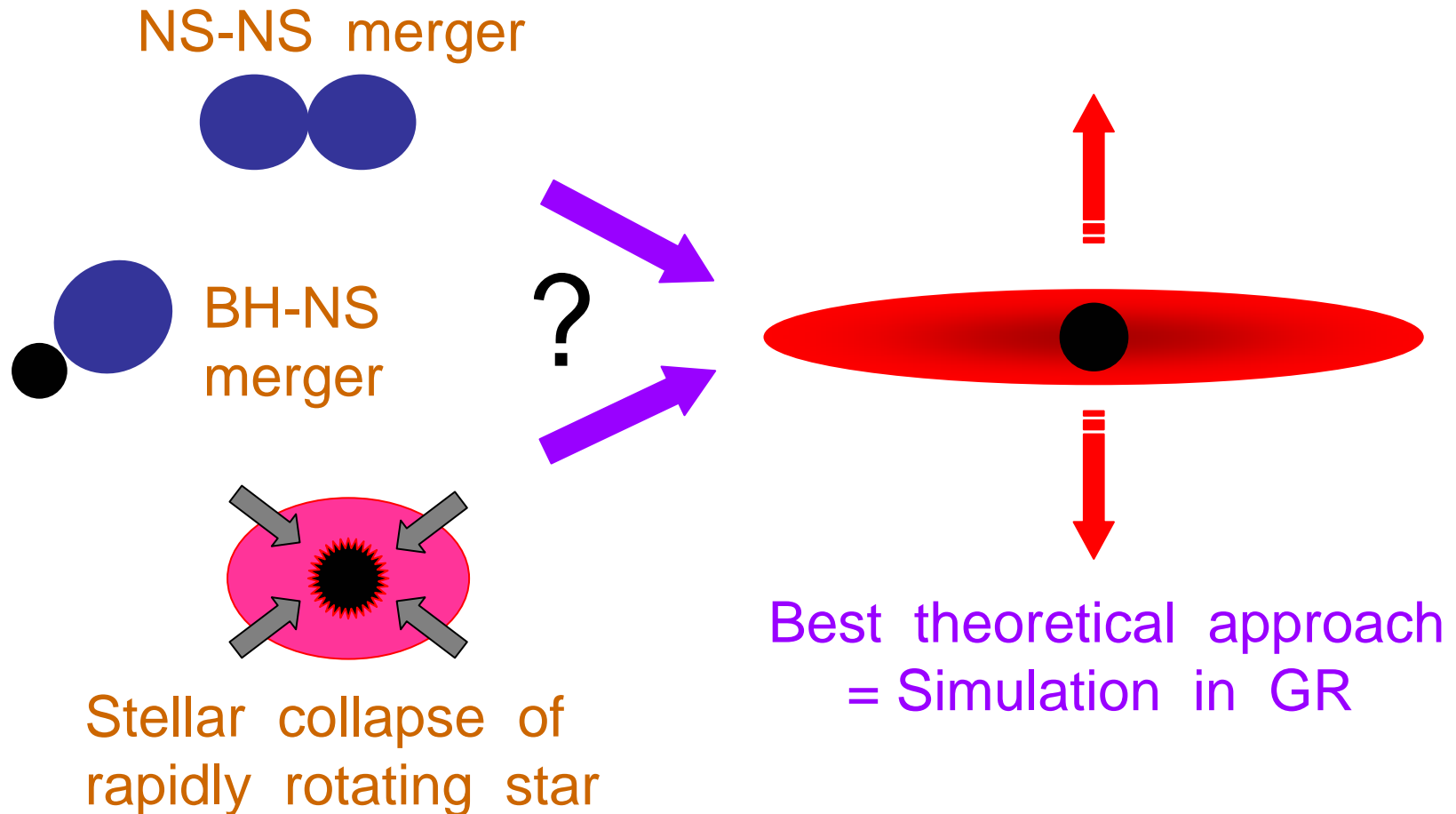


Templates (for compact binaries, core collapse, etc) should be prepared

## B To simulate Astrophysical Phenomena

e.g. Central engine of GRBs

= Stellar-mass black hole + disks (Probably)



## C To discover new phenomena in GR

In the past 20 years, community has discovered

e.g.,

1: Critical phenomena (Choptuik)

2: Toroidal black hole (Shapiro-Teukolsky)

3: Naked singularity formation (Nakamura, S-T)

# GR phenomena to be simulated ASAP

- NS-NS / BH-NS /BH-BH mergers  
(Promising GW sources/GRB)
- Stellar collapse of massive star to a NS/BH  
(Promising GW sources/GRB)
- Nonaxisymmetric dynamical instabilities of rotating NSs  
(Promising GW sources)
- ....

In general, 3D simulations are necessary

## 2 Issues: Necessary elements for GR simulations

- Einstein's evolution equations solver
- GR Hydrodynamic equations solver
- Appropriate gauge conditions (coordinate conditions)
- Realistic initial conditions in GR
- Gravitational wave extraction techniques
- Apparent horizon (hopefully Event horizon) finder
- Special techniques for handling BHs / BH excision
- Micro physics (EOS, neutrino processes, B-field ...)
- Powerful supercomputers

RED = Indispensable elements

# 3: Current Status: Achievements in the past decade

Here, focus on progress in main elements:

- Einstein evolution equation solver in 3D
- GR Hydro equation solver
- Appropriate gauge conditions in 3D
- Supercomputers

# Progress I

- Formulations for Einstein's evolution equation

Many people 10 yrs ago believed the standard ADM formalism works well.

**BUT:**

Unconstrained  
free evolution

Standard ADM

Variables in standard  
ADM formalism:

$$\gamma_{ij}, K_{ij}$$

12 components



Numerical simulation  
becomes **unstable**  
even in the evolution of  
linear GW  
(Nakamura 87, Shibata 95,  
Baumgarte-Shapiro (99))

Due to constraint violation instabilities

- New formulations for Einstein's evolution eqs :

(i) BSSN formalism

Nakamura (87), Shibata-Nakamura (95),  
Baumgarte-Shapiro (99).....

Choose variables:

$$\phi \equiv \frac{1}{12} \ln(\det(\gamma))$$

$$\tilde{\gamma}_{ij} \equiv e^{-4\phi} \gamma_{ij}$$

$$K \equiv K_k^k$$

$$\tilde{A}_{ij} \equiv e^{-4\phi} \left( K_{ij} - \frac{1}{3} \gamma_{ij} K \right)$$

$$F_i \equiv \delta^{jk} \partial_j \tilde{\gamma}_{ik}$$

17 components

An Important step

Rewrite ADM equations using

$$\left\{ \begin{array}{l} \text{constraint equations} \\ \det(\tilde{\gamma}_{ij}) = 1 \end{array} \right\}$$

Unconstrained  
free evolution

Stable numerical simulation

(So far no problem in the  
absence of black holes)

- New formulations for Einstein's evolution eqs. :  
(ii) Hyperbolic formulations

Bona-Masso (92) ..... many references .....

Kidder-Scheel-Teukolsky (KST) (01)

$$\partial_t g^{ij} + \partial_k Q^{kij} = \underline{F^{ij}}(g, Q, \dots)$$

No derivatives

30~40 variables are defined

Advantage for imposing boundary conds. at BH

→ Perhaps, robust for BH spacetimes

But, no success in 2BH merger so far.

(Something is short of. Need additional ideas.)

# Progress II

- GR Hydro scheme

Trend until the middle of 1990

⇒ Add artificial viscosity to capture shocks

(Wilson 1980, Centrella 1983, Hawley et al. 1984,

Stark-Piran 1985, Evans 1986, Nakamura 1993, Shibata 1999)

Schematically,

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial (\rho v_i v^j + P \gamma_i^j)}{\partial x^j} = \underline{[Viscous term]_i} + \dots$$

Very phenomenological;  
Not very physical

**Drawback** : Strong shocks cannot be captured accurately.

**Concern** : We do not know if it always gives the correct answer for any problems ???

- Hydro scheme: Current trend

High-resolution shock-capturing scheme

= Solve equations using characteristics

(+ Piecewise-Parabolic interpolation

+ Approximate Riemann solver) : very physical !

No artificial  
viscosity

Developed by Valencia (Ibanez, Marti, Font, ...)

& Munich (Mueller ...) groups in 1990s.

Now used by many groups (including myself)

- Strong shocks & oscillations of stars are computed accurately

- Physical Scheme → No concern on the outputs

⇒ This is currently the best choice for simulations of

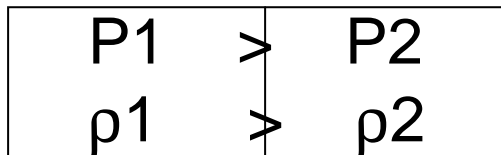
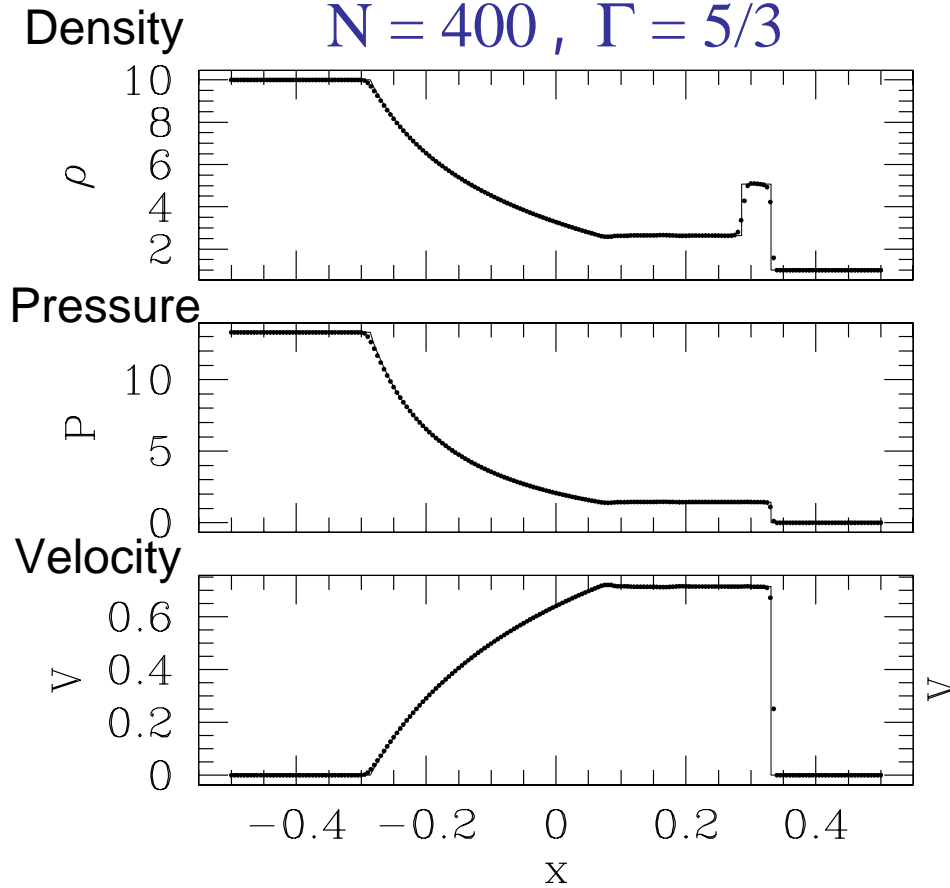
- Stellar core collapse

- NS-NS merger

# Standard tests for hydro code in special relativity

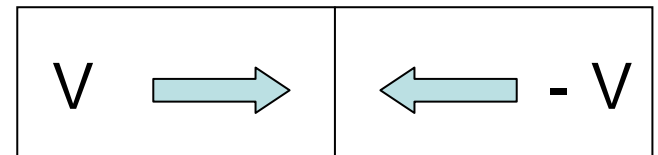
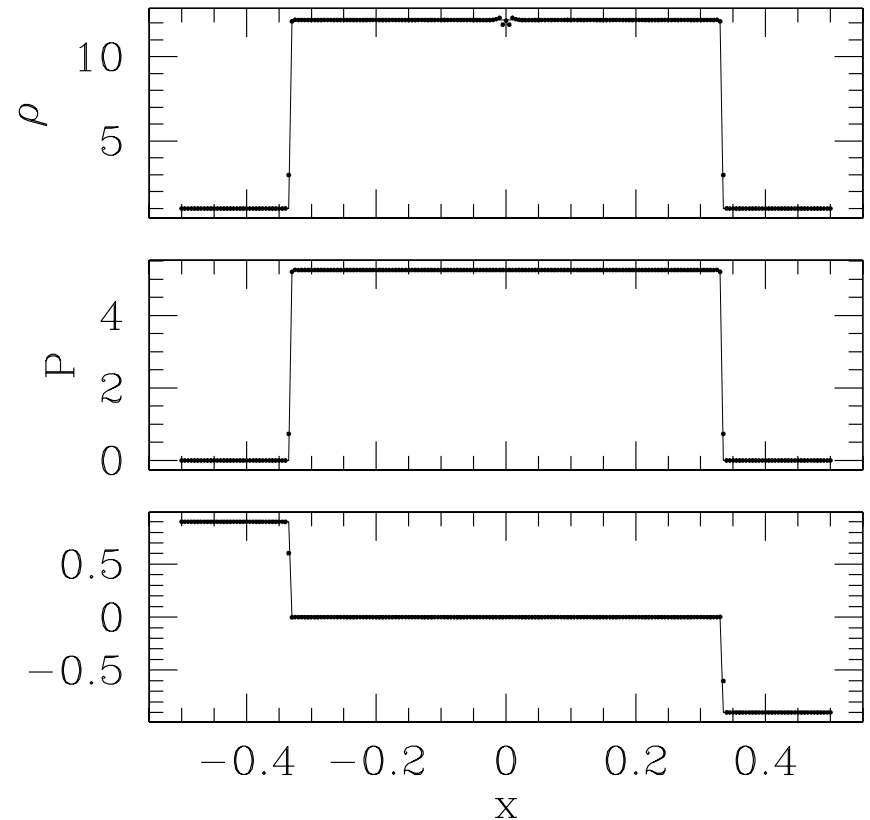
## Riemann Shock Tube

$N = 400, \Gamma = 5/3$



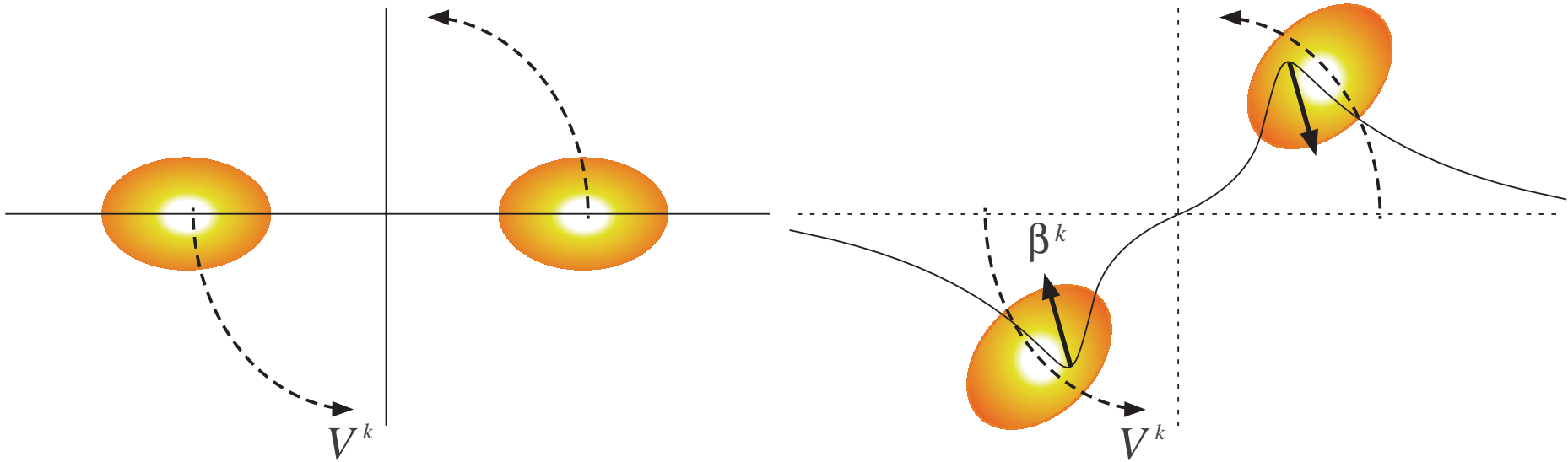
## $V = 0.9c$ . Wall Shock

$N = 400, \Gamma = 4/3$

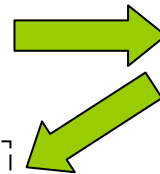


# Progress III

- Choice of appropriate spatial gauge condition :



Frame dragging

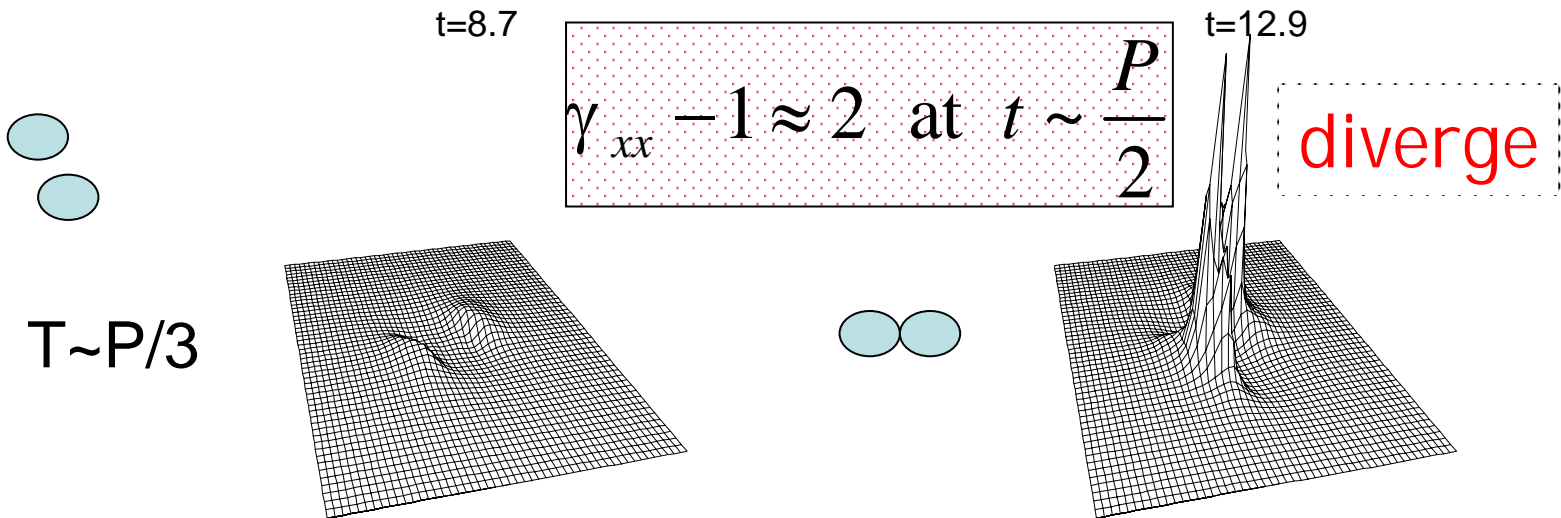
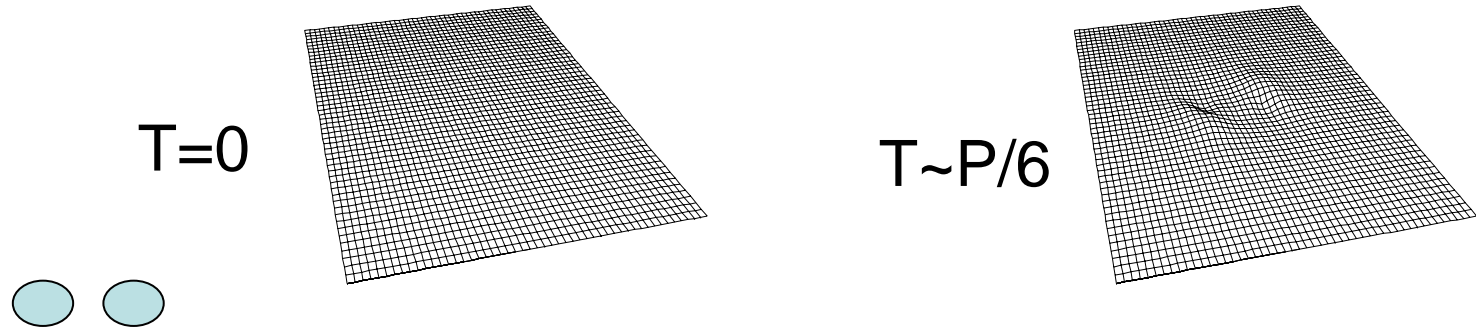


Coordinate distortion

Could increase the magnitude of unphysical parts of metric

We need to suppress it for a long-term evolution.

$\gamma_{xx}$  on the equatorial plane  
with zero shift vector



Distortion monotonically increases to crash

**Previous belief:** Minimal distortion gauge  
(Smarr & York 1978)

Require that an action which denotes the global magnitude of the coordinate distortion is minimized.



$$\text{MD gauge : } \Delta\beta^k + \frac{1}{3}D^k D_j \beta^j = S^k$$

Physically good.  
But, computationally  
time-consuming

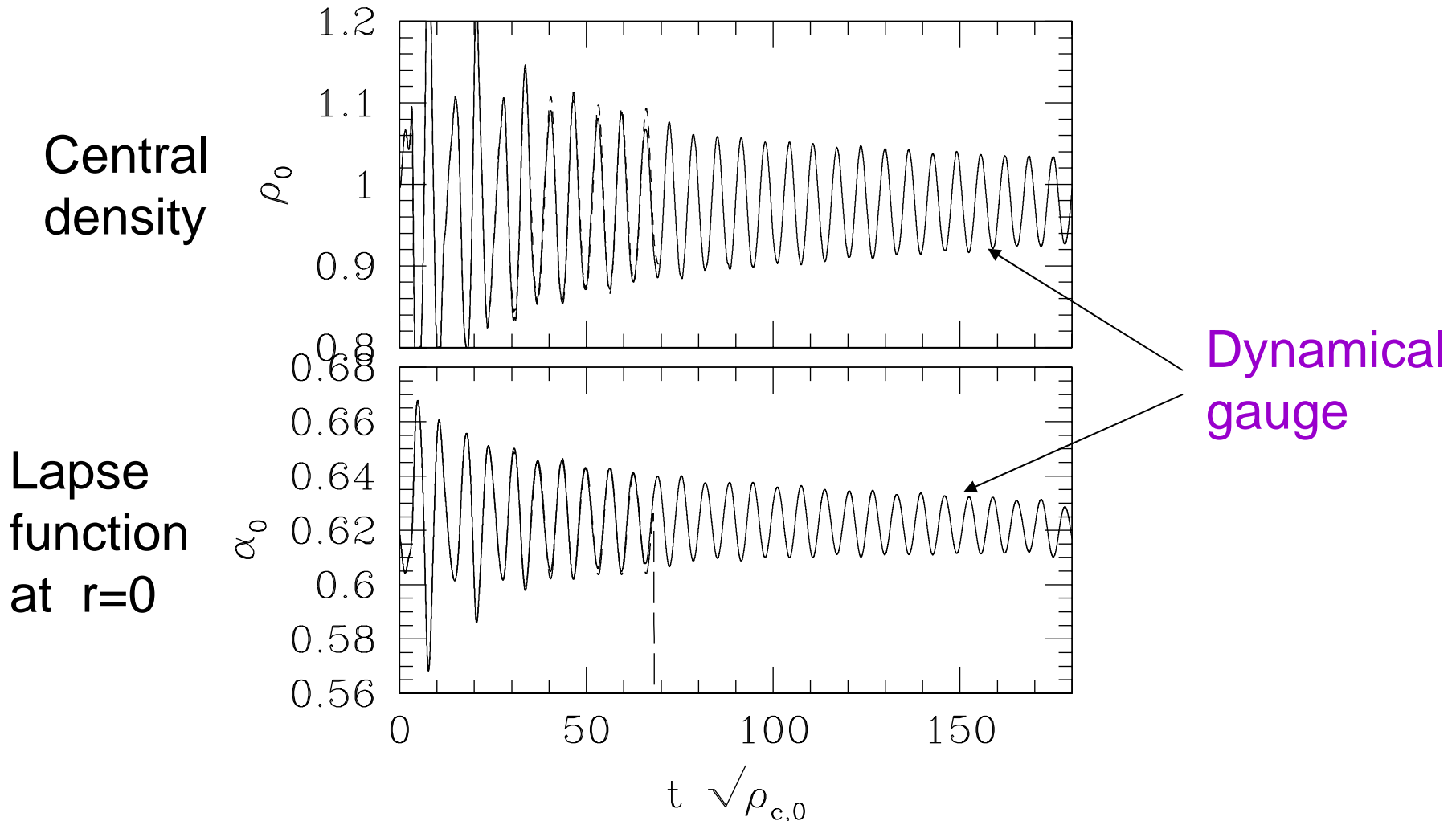
**New Trend: Dynamical gauge** (Alcubierre et al 2000,  
Lindblom & Scheel 2003, Shibata 2003 .....

Schematic form :

$$\ddot{\beta}^l \approx \Delta\beta^l + \frac{1}{3}D^l D_j \beta^j - S^l$$

Save CPU time  
significantly !!  
Recent numerical  
experiments show  
it works well !!

# Evolution of compact, rapidly rotating & oscillating NS in a dynamical gauge



Stable evolution for  $> 30$  oscillation ( $\sim$  rotation) periods.

# Progress IV

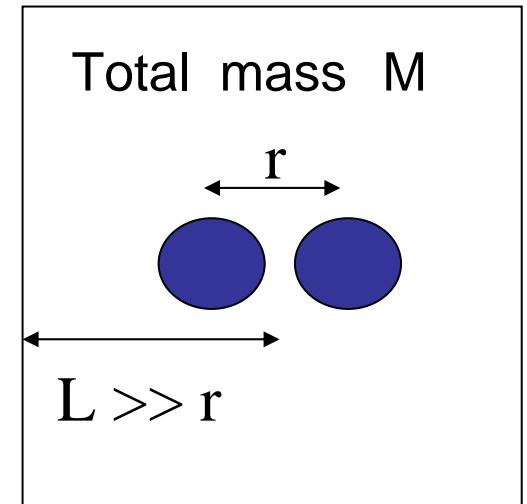
## Computational resources

Minimum required grid number for extraction of gravitational waveforms

$$\lambda_{GW} \leq \lambda_{ISCO} \approx 58 \left( \frac{GM}{c^2} \right) \left( \frac{rc^2}{7GM} \right)^{3/2}$$

Require  $L \geq \lambda_{GW}$  &  $\Delta x \leq 0.2 \left( \frac{GM}{c^2} \right)$

$$\Rightarrow \frac{L}{\Delta x} \geq 290 \left( \frac{rc^2}{7GM} \right)^{3/2} \quad \& \quad N \geq 580 \left( \frac{rc^2}{7GM} \right)^{3/2}$$



Minimum grid number required (in uniform grid):  
~ 600 \* 600 \* 300 (equatorial symmetry is assumed)  
 $\Rightarrow$  Memory required ~ 200 GBytes (~200 variables)

# An example of current supercomputer

## FUJITSU FACOM VPP5000 at NAOJ

- Vector-Parallel Machine (60 vector PEs)
- Maximum memory  $\rightarrow$  0.96TBytes
- Maximum speed  $\rightarrow$  0.58TFlops
- Our typical run with 32PEs

Typical current  
memory & speed

$633 * 633 * 317$  grid points = 240 Gbytes memory  
(in my code)

About 20,000 time steps  $\sim$  100 CPU hours /model

Minimum grid number can be taken

But, hopefully, we need hypercomputers  
for well-resolved simulations.

(e.g. Earth simulator  $\sim$  10TBytes,  $\sim$  40TFlops)

# Summary of current status

- Einstein evolution equations solver OK
- Gauge conditions (coordinate conditions) OK
- GR Hydrodynamic equations solver OK
- Powerful supercomputer ~OK

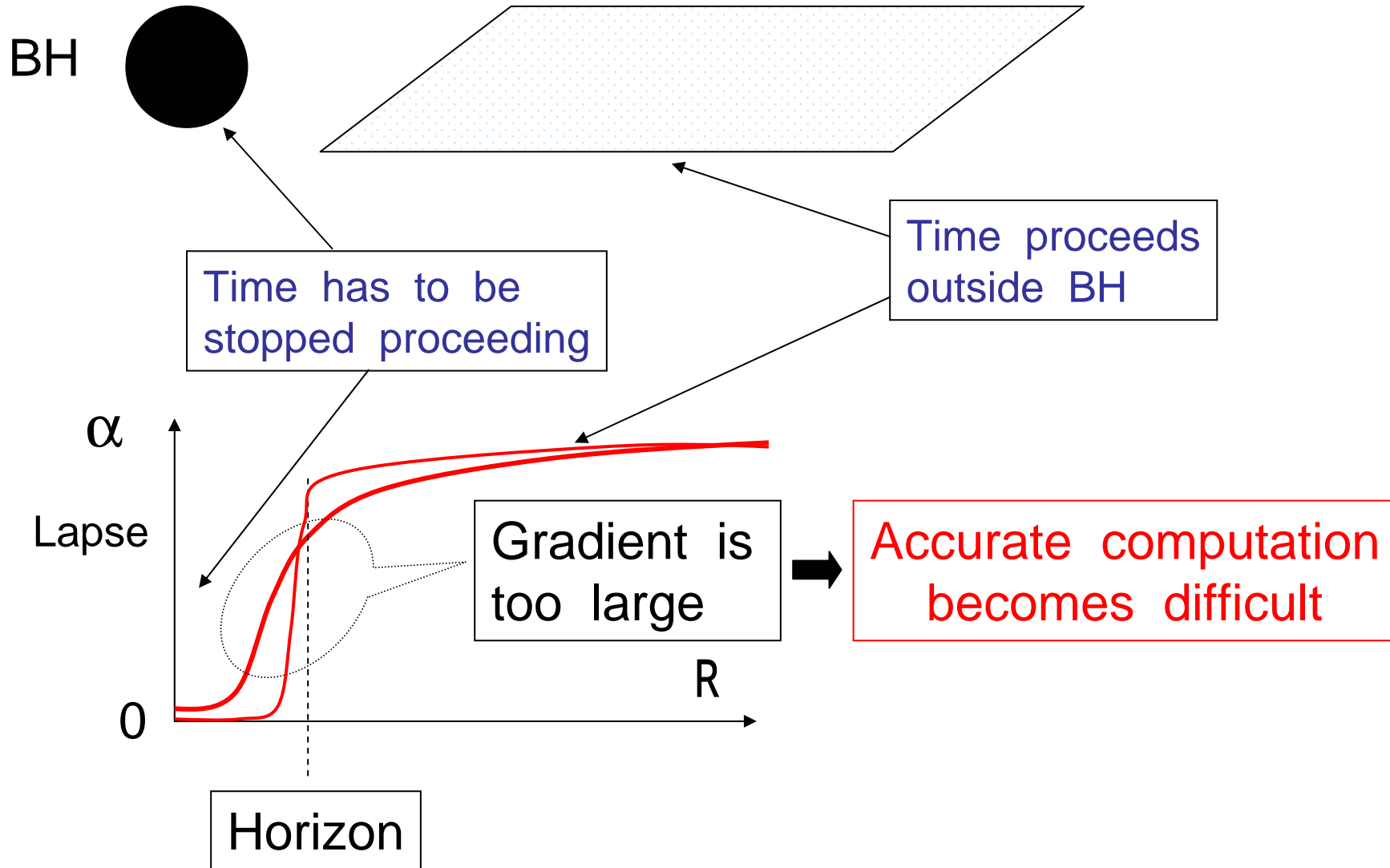
but hopefully need hypercomputers

Long-term GR simulations are feasible  
(*in the absence of BHs*)

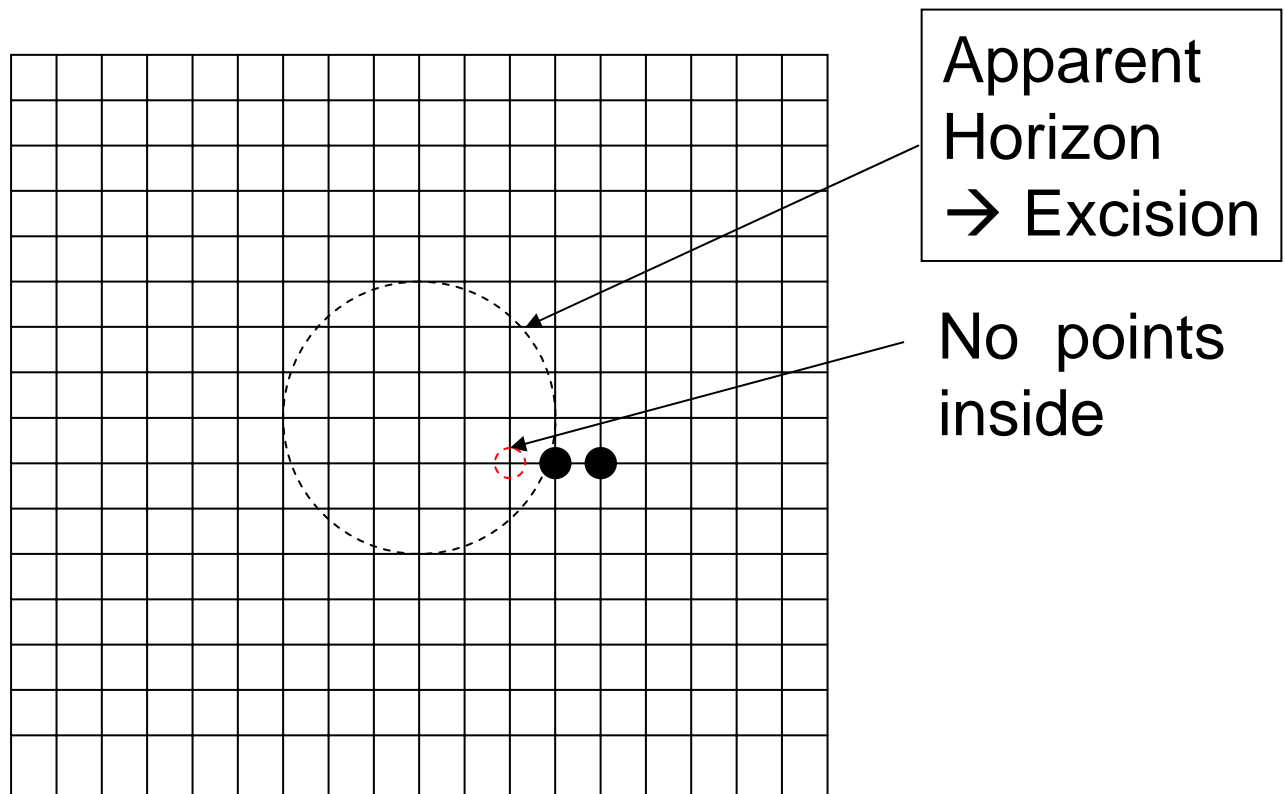
In the past 5 yrs, computations have been done for

- NS-NS merger (Shibata-Uryu, Miller, ...)
- Stellar core collapse (Font, Papadopoulos, Mueller, Shibata)
- Collapse of supermassive star (Shibata-Shapiro)
- Bar-instabilities of NSs (Shibata-Baumgarte-Shapiro)
- Oscillation of NSs (Shibata, Font-Stergioulas, ....)

# Unsolved Issue : Handling BHs



# A solution = Excision (U n r u h)



What are appropriate formulation, gauge,  
boundary conditions .... ?

-- 1BH → OK (Cornell, Potsdam, Illinois...)

-- 2BH → No success for a longterm simulation

(But see gr-qc/0312112, Bruegmann et al. for one orbit)

# 4. Our latest numerical results:

## Current implementation in our group

1. **GR** : BSSN (or Nakamura-Shibata). But modified year by year; e.g., latest version = Shibata et al. 2003 has improved accuracy significantly
2. **Gauge** : Maximal slicing ( $K=0$ ) + **Dynamical gauge**
3. **Hydro** : **High-resolution shock-capturing scheme**  
(Roe-type method with 3<sup>rd</sup>-order PPM interpolation)

# Latest results for merger of 2NS

EOS: Initial;  $P = K \rho^\Gamma$ ,  $\Gamma = 2$ ;  $K = 1.535e5$  cgs

$$M = 1.40 M_{\text{solar}} \rightarrow R = 14.8 \text{ km}$$

$$1.60 M_{\text{solar}} \rightarrow R = 13.3 \text{ km}$$

(Maximum mass for the spherical case =  $1.68 M_{\text{solar}}$ )

During the evolution:  $P = (\Gamma - 1)\rho\varepsilon$

I here show animations for merger of 2NS

(a) 1.40 – 1.40  $M_{\text{solar}}$ ,

(b) 1.33 – 1.46  $M_{\text{solar}}$ ,

(c) 1.52 – 1.52  $M_{\text{solar}}$ ,

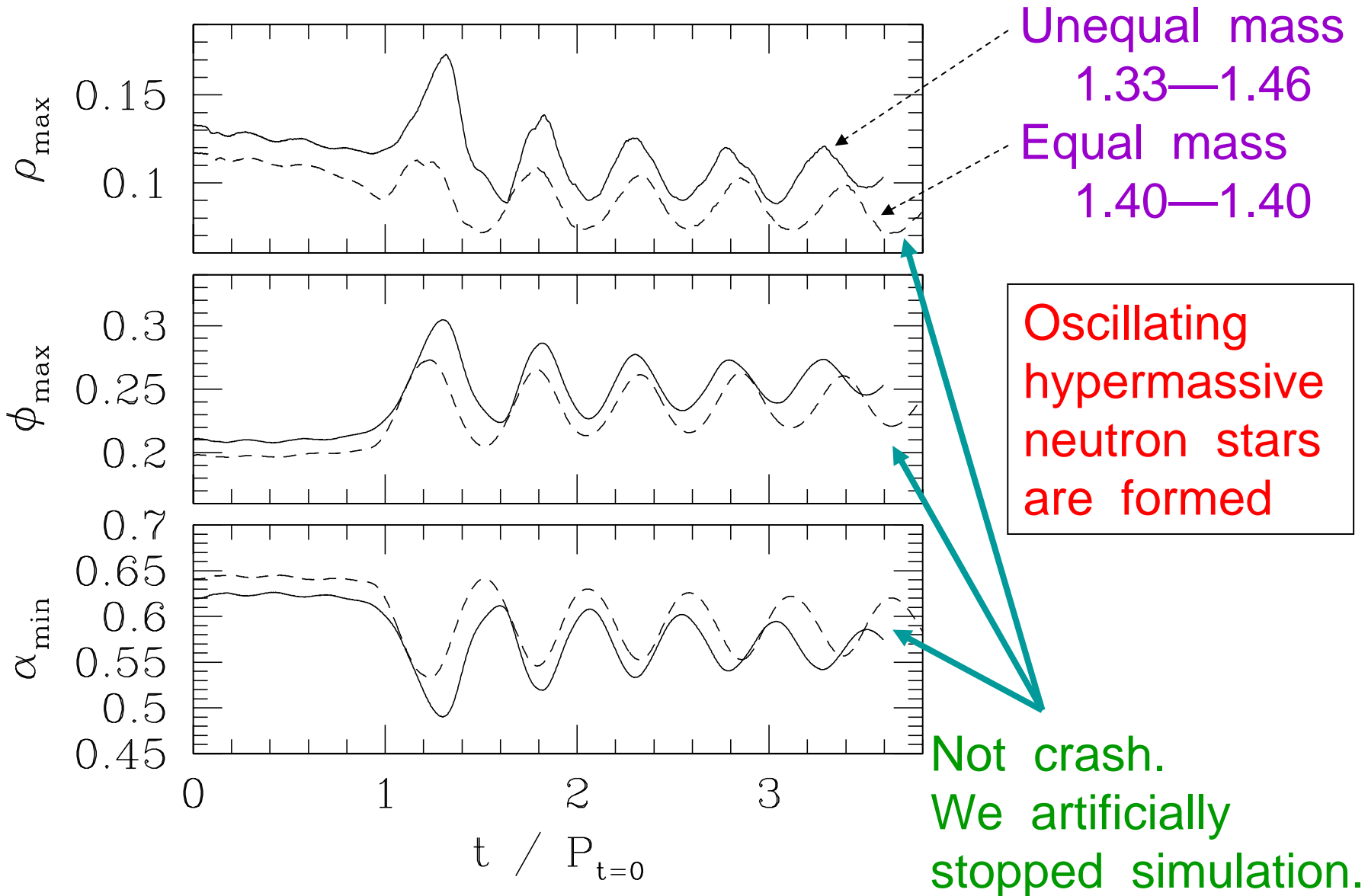
(d) 1.40 – 1.60  $M_{\text{solar}}$

(See, Shibata et al. PRD 68, 084020, 2003)

Typical grid size : 633 \* 633 \* 317

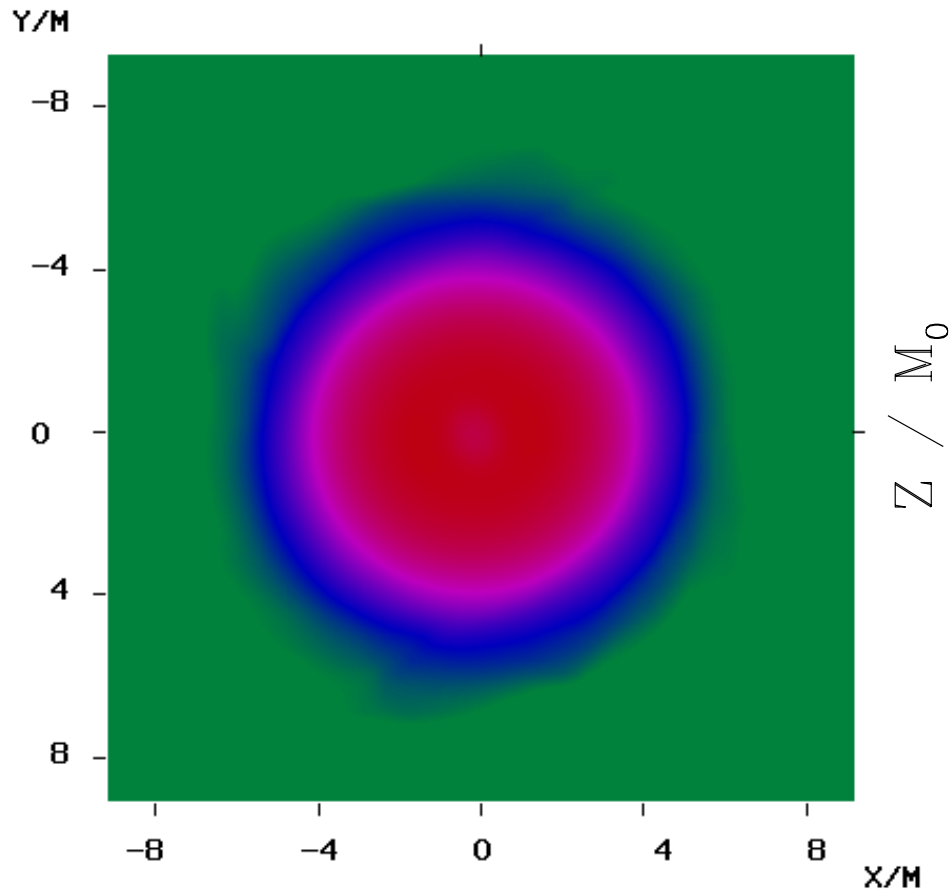
(max we have taken, 761 \* 761 \* 381)

# Evolution of maximum density in NS formation

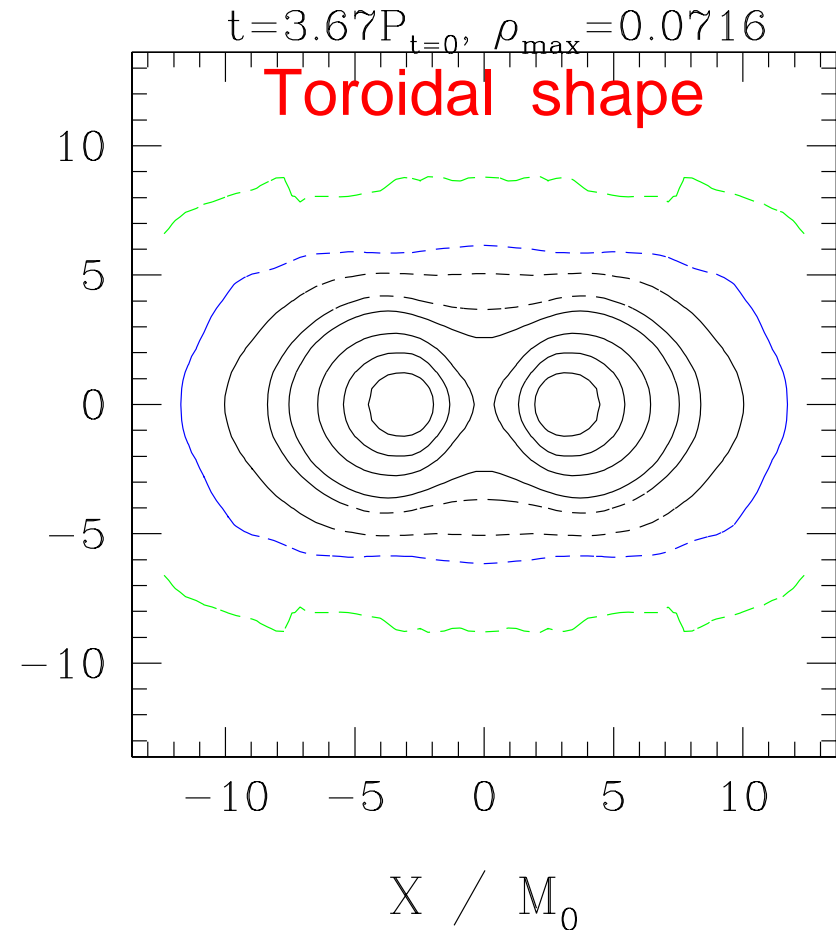


1.40 – 1.40  $M_{\text{solar}}$  case : final snapshot

Massive toroidal neutron star is formed  
(slightly elliptical)

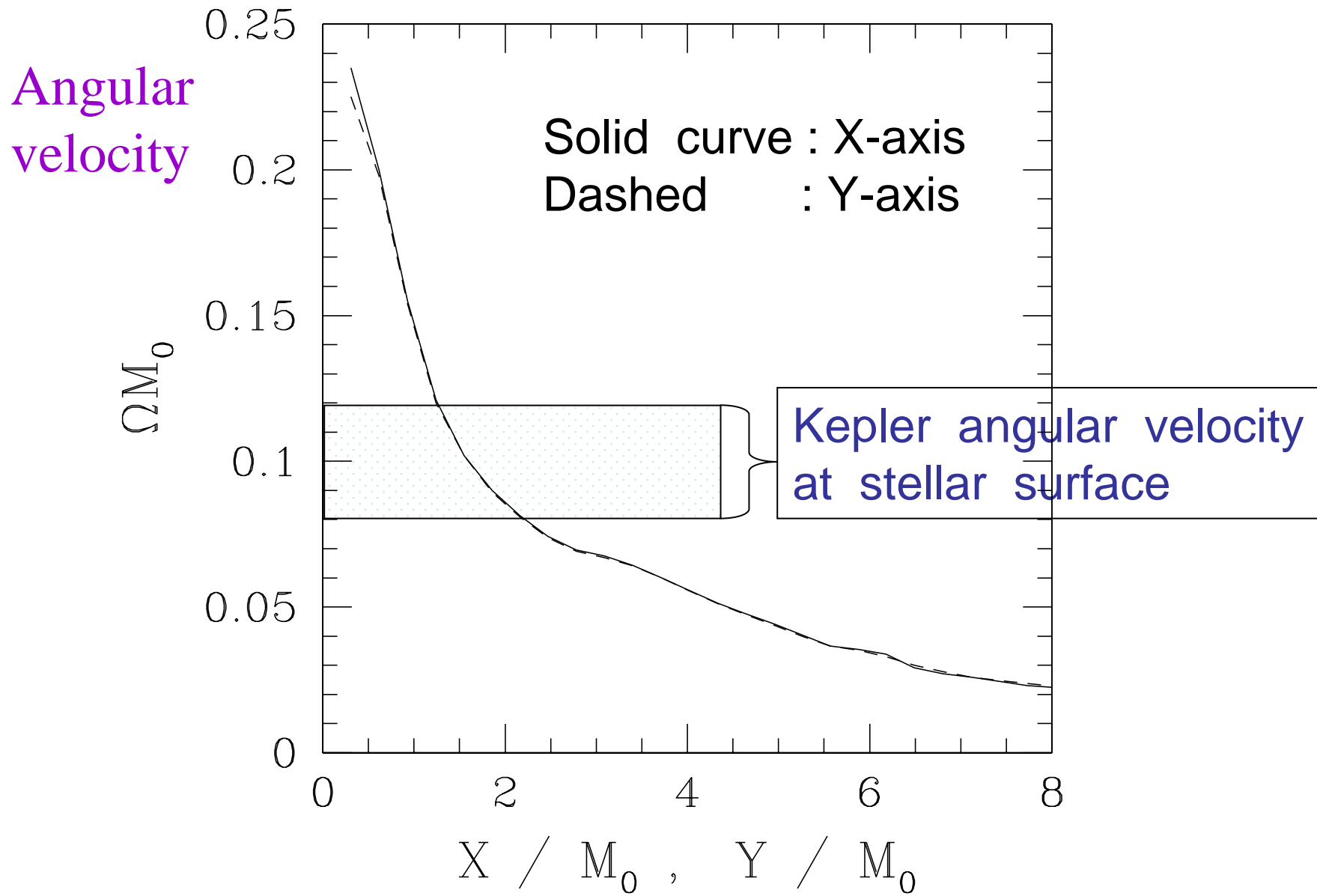


X – Y contour plot



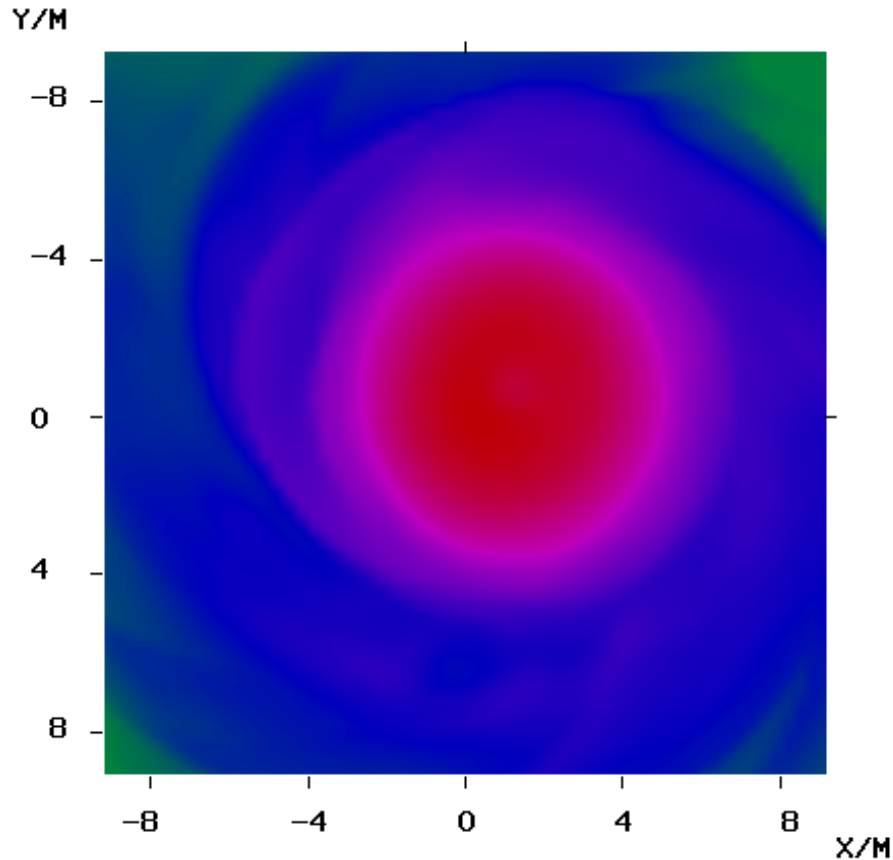
X – Z contour plot

# Formed Massive toroidal NS is differentially and rapidly rotating



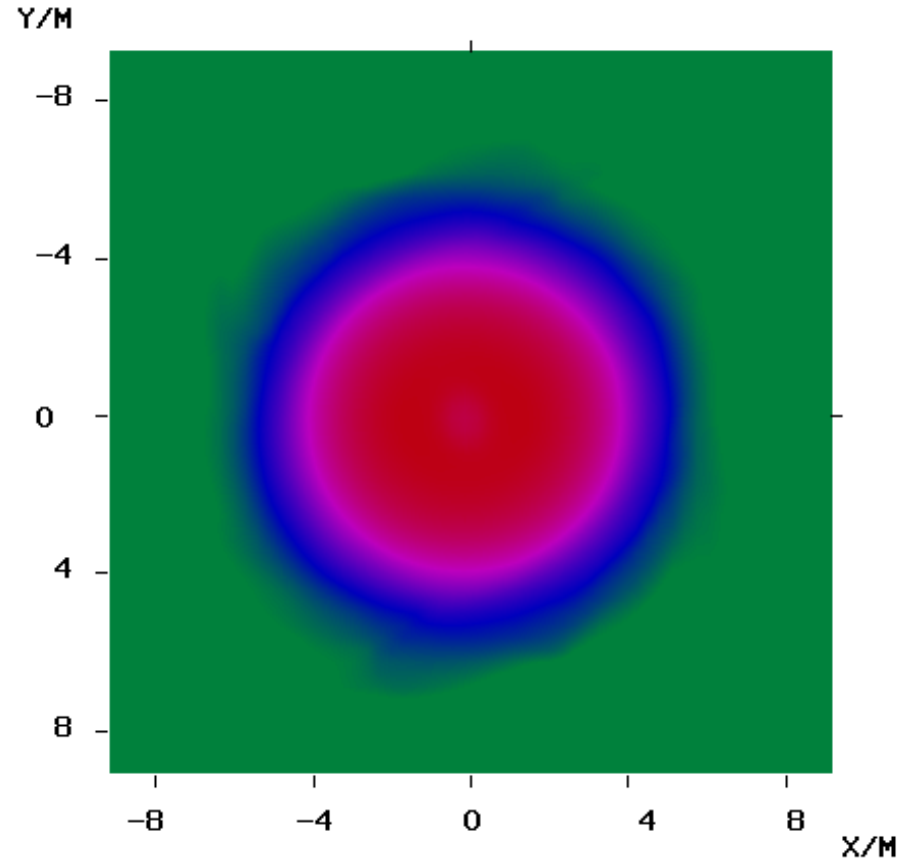
# Comparison between equal and unequal mass mergers

1.33—1.46:  
Massive NS + disk



Unequal-mass case  
Mass ratio  $\sim 0.90$

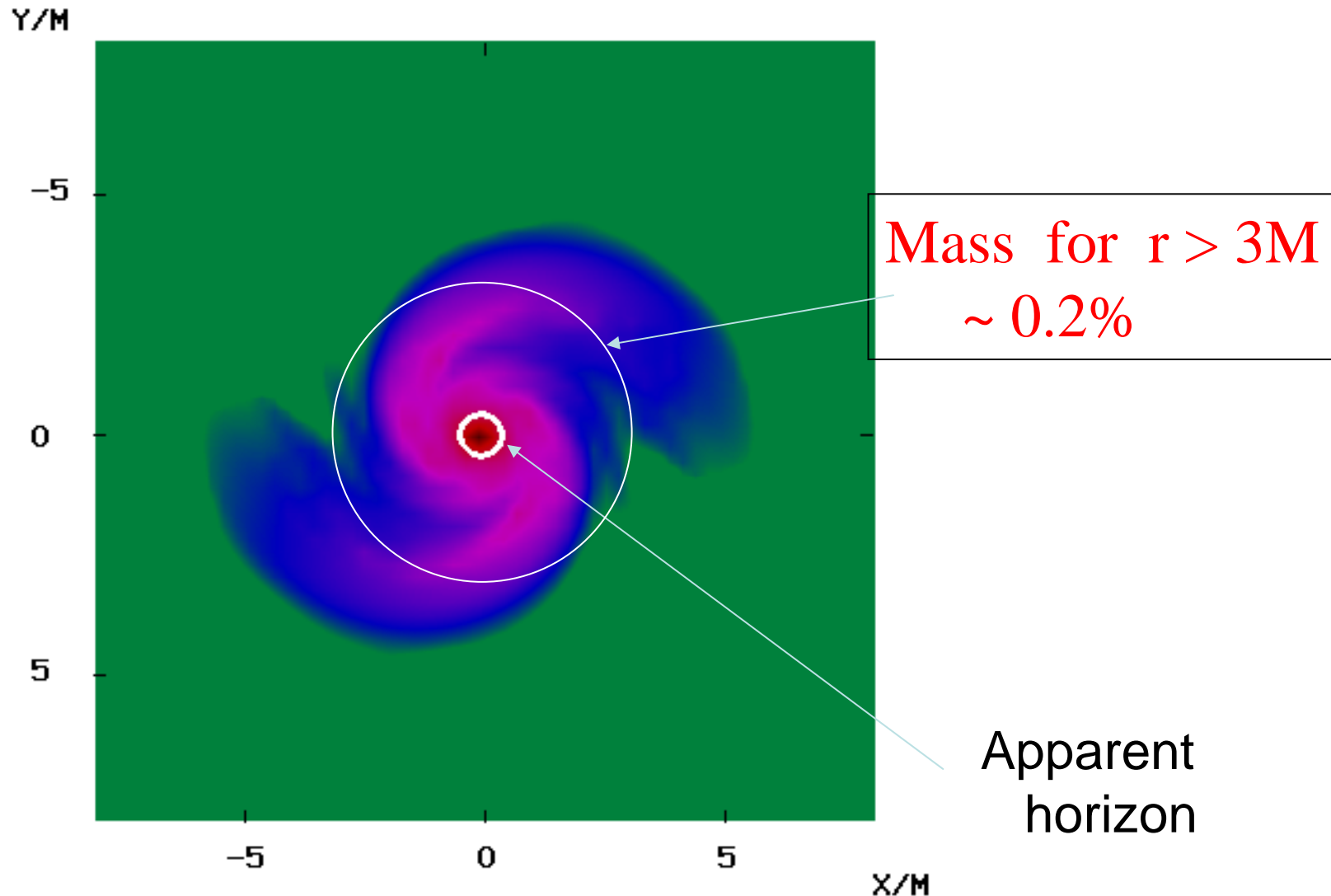
1.40—1.40:  
Massive NS



Equal-mass case

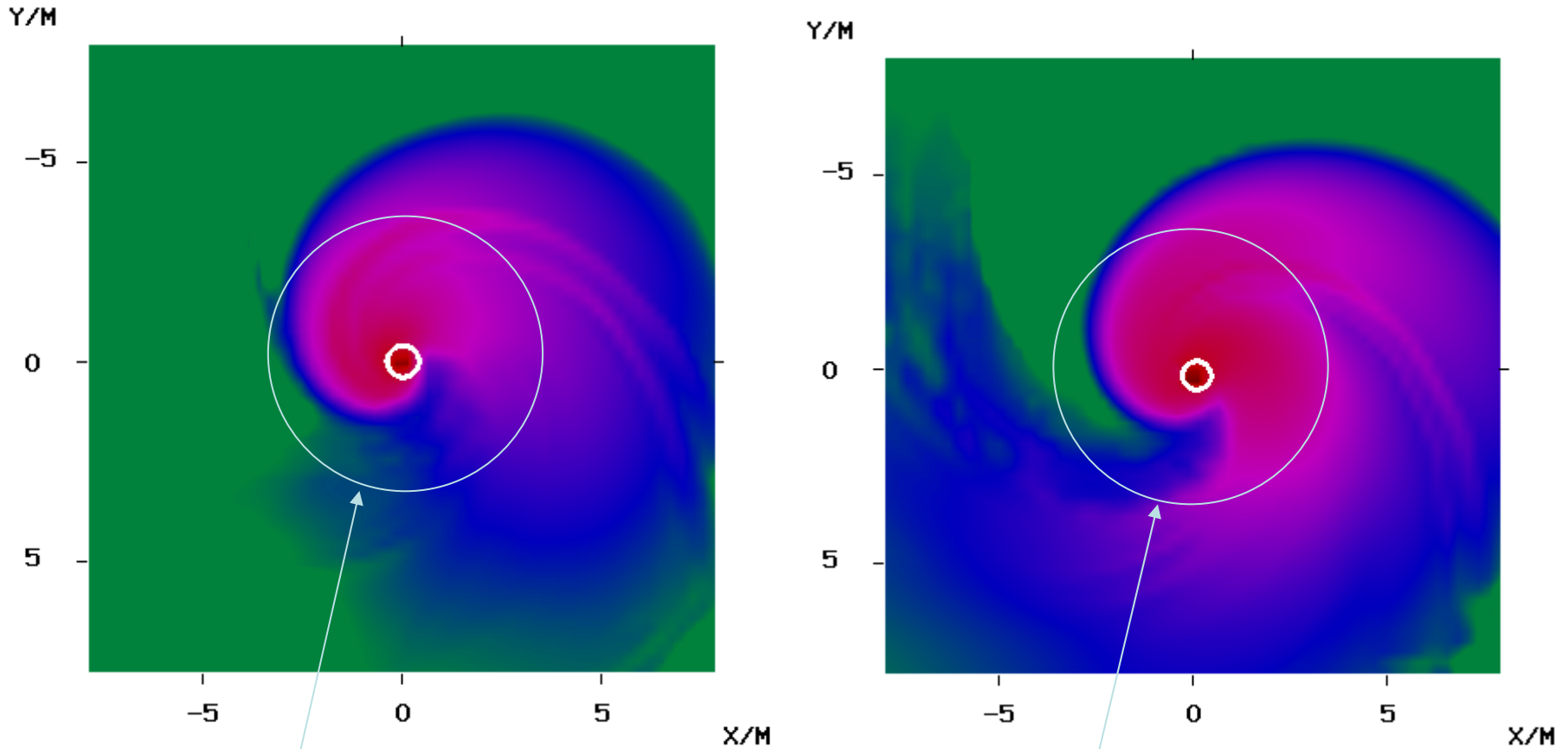
# Black hole formation case: 1.52-1.52

Equal-mass case



# Disk mass for unequal-mass merger

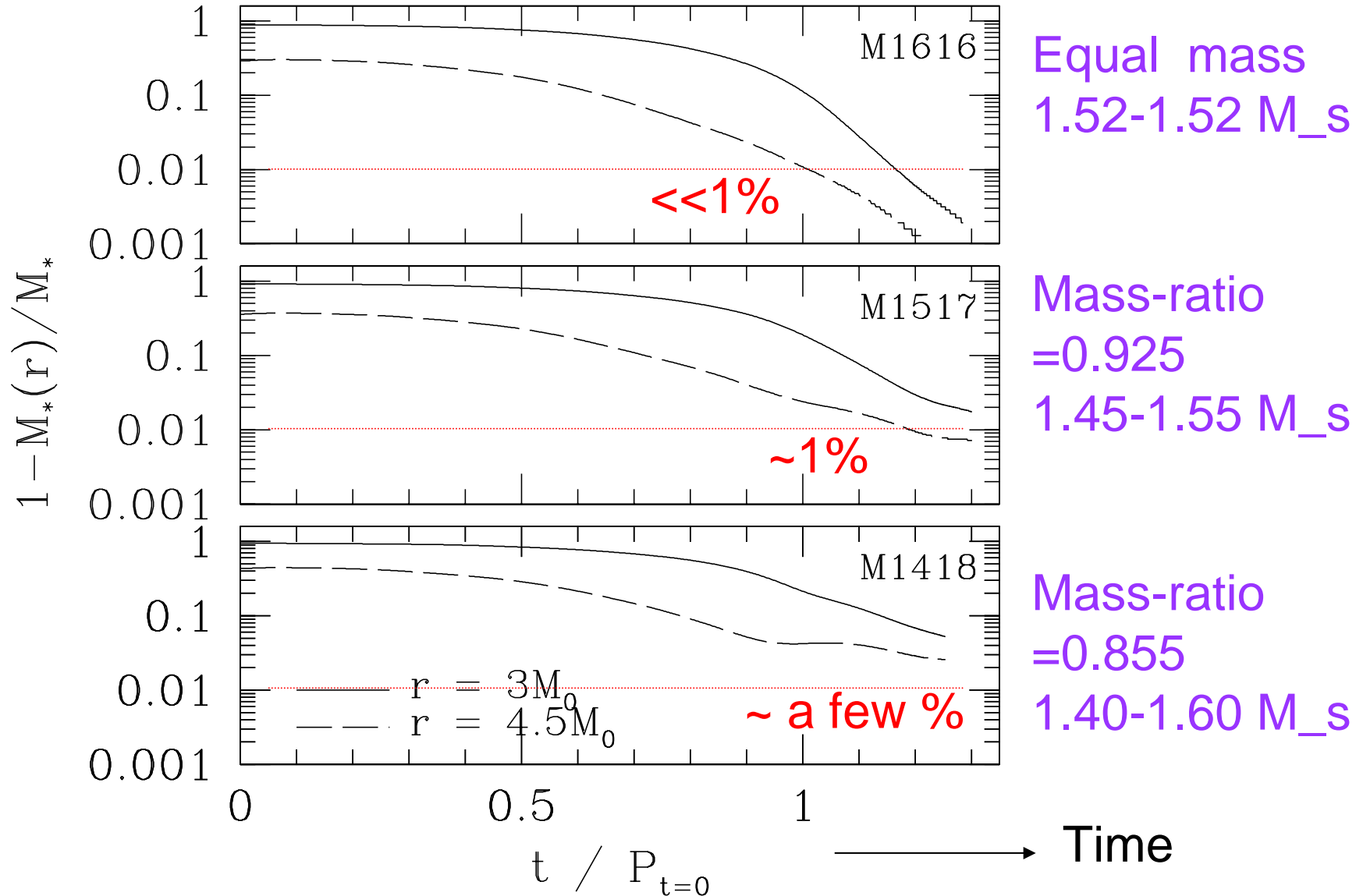
1.45—1.55, Mass ratio 0.925    1.40—1.60, Mass ratio 0.855



Mass for  $r > 3M$   
~ 2%

Mass for  $r > 3M$   
~ 4%

# Mass fraction outside a sphere for BH formation case



# Products of mergers for $\Gamma = 2$

## Equal – mass cases

- Low mass cases
  - Hypermassive neutron stars
  - of nonaxisymmetric & quasiradial oscillations.
- High mass cases
  - Direct formation of Black holes
  - with very small disk mass

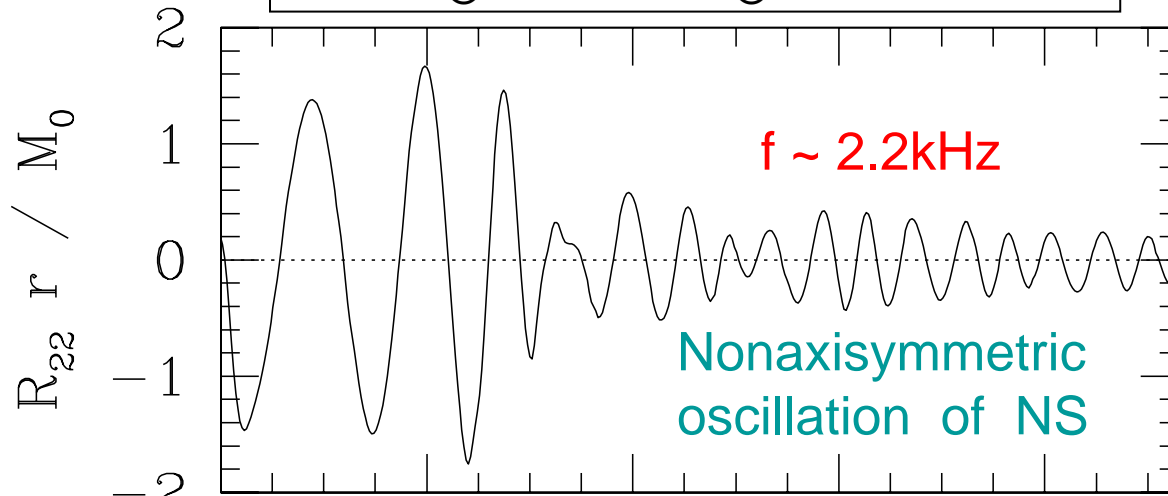
## Unequal – mass cases (mass ratio $\sim 0.9$ )

- Likely to form disks of mass
  - $\sim$  several percents of total mass
  - BH(NS) + Disk ( $\sim 0.1 M_{\text{solar}}$ )
  - Maybe a candidate for short GRB

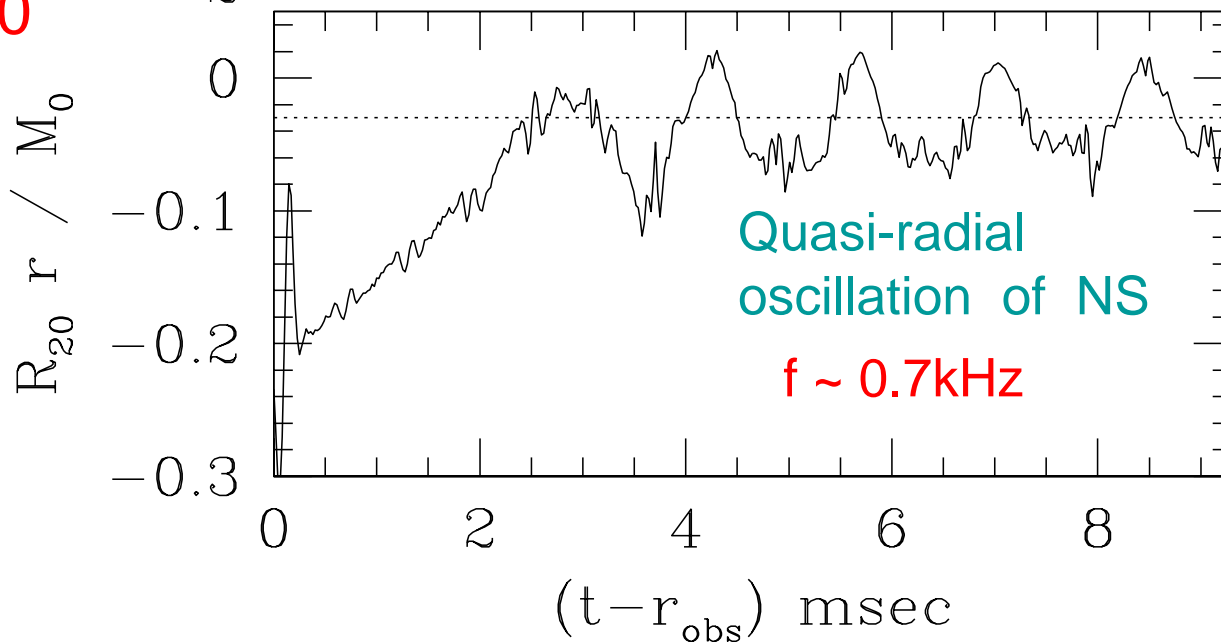
# Gravitational waves for NS formation

$1.4M_{\odot} - 1.4M_{\odot}, R = 15\text{km}$

$l=m=2$   
mode



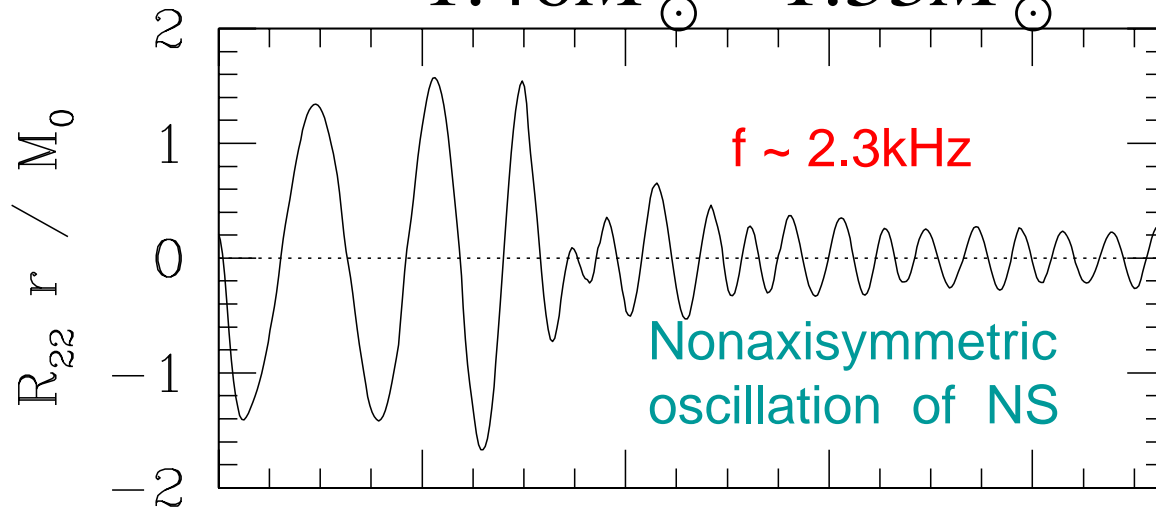
$l=2, m=0$   
mode



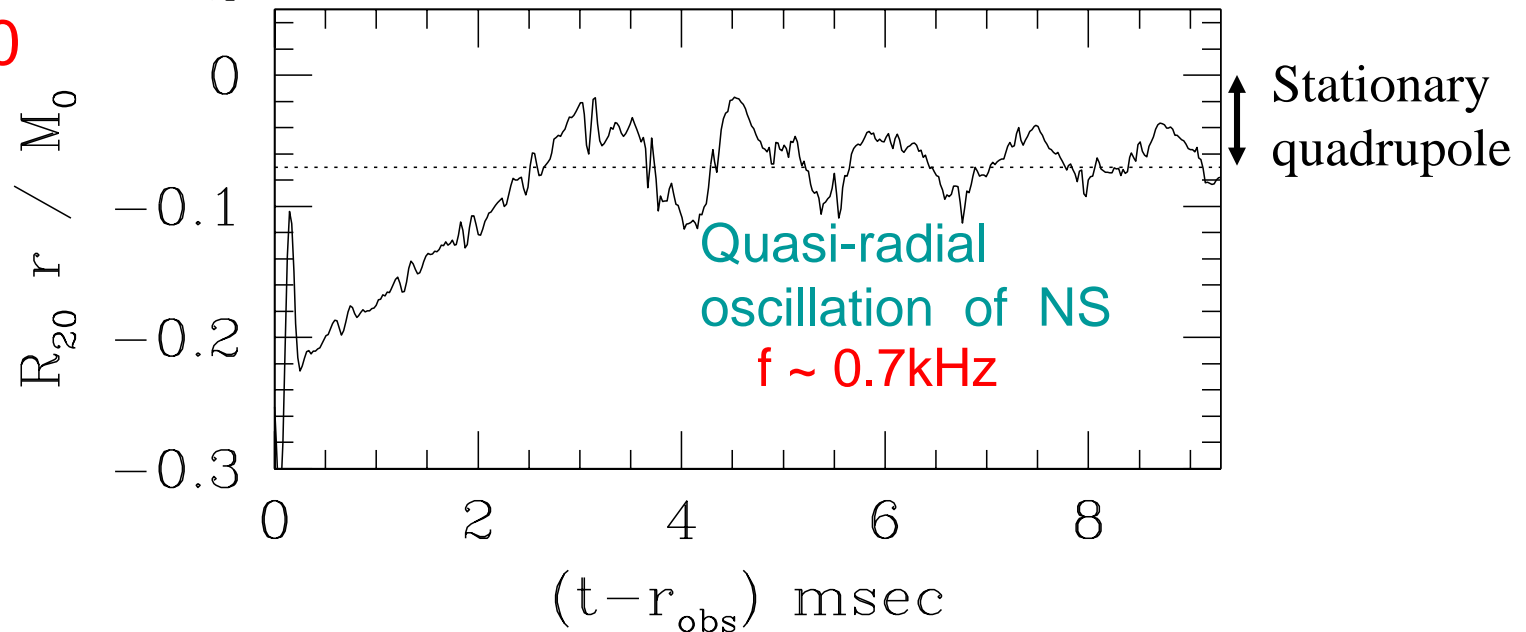
# Gravitational waves from unequal-mass merger to NS formation

$1.46M_{\odot} - 1.33M_{\odot}$

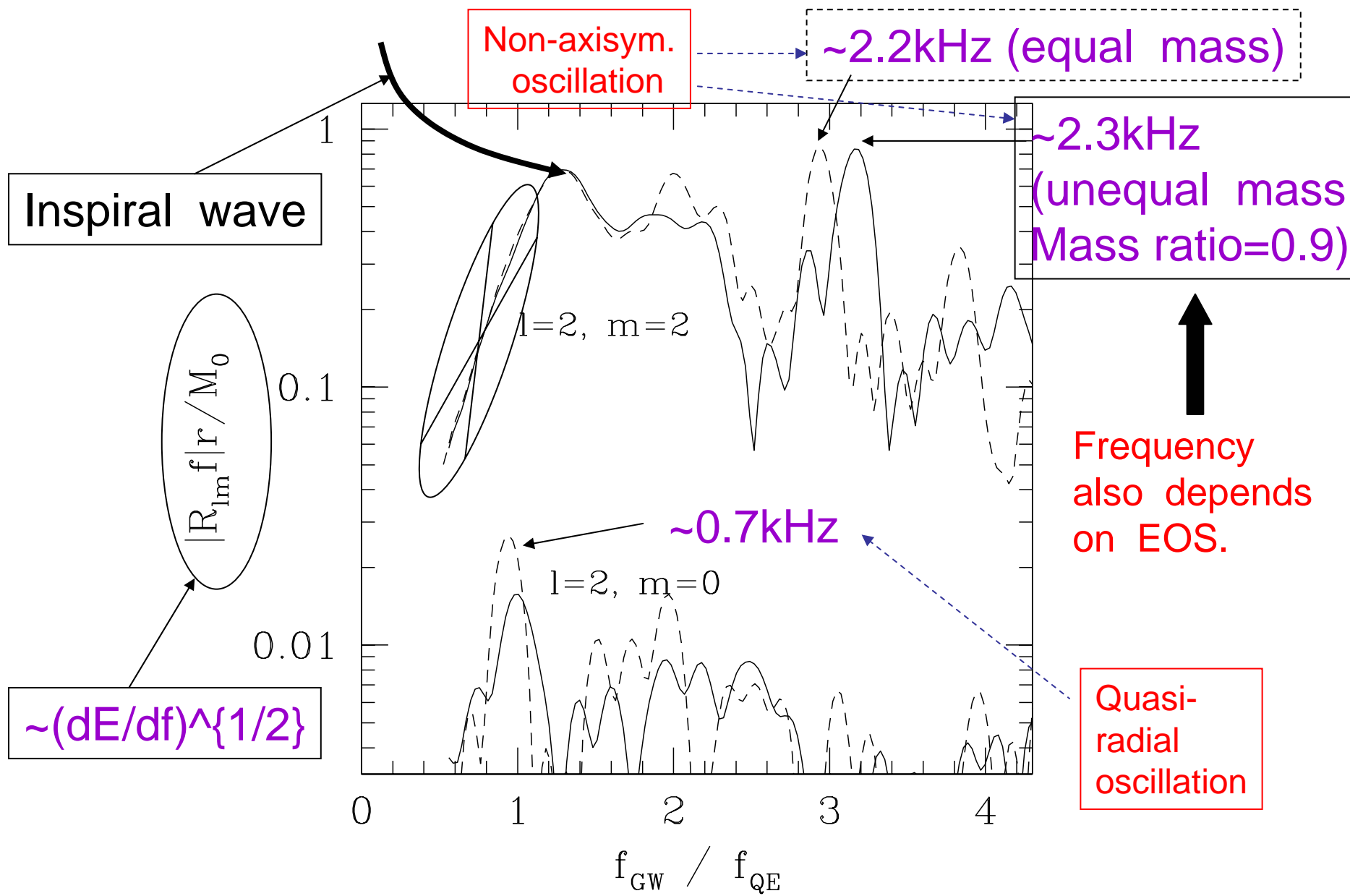
$l=m=2$   
mode



$l=2, m=0$   
mode

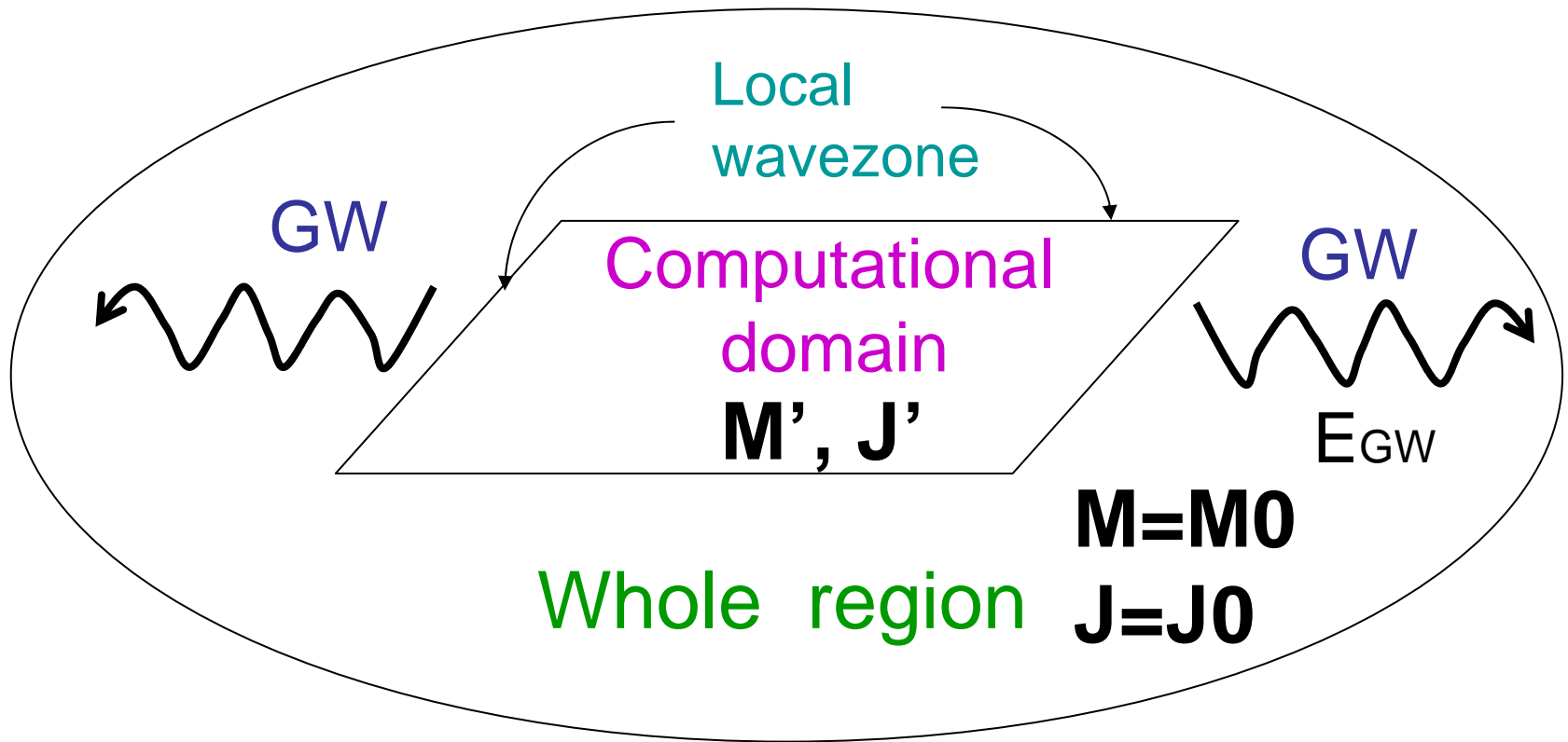


# Fourier spectrum in NS formation



# Computation of mass and angular momentum

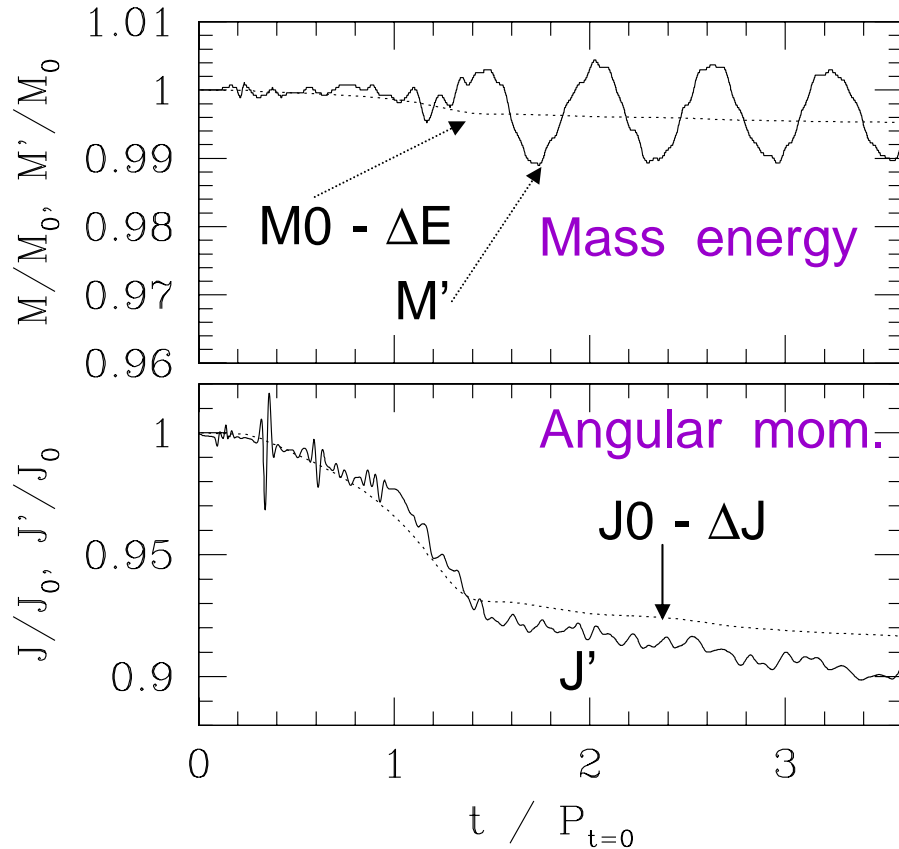
-- Check of the conservation --



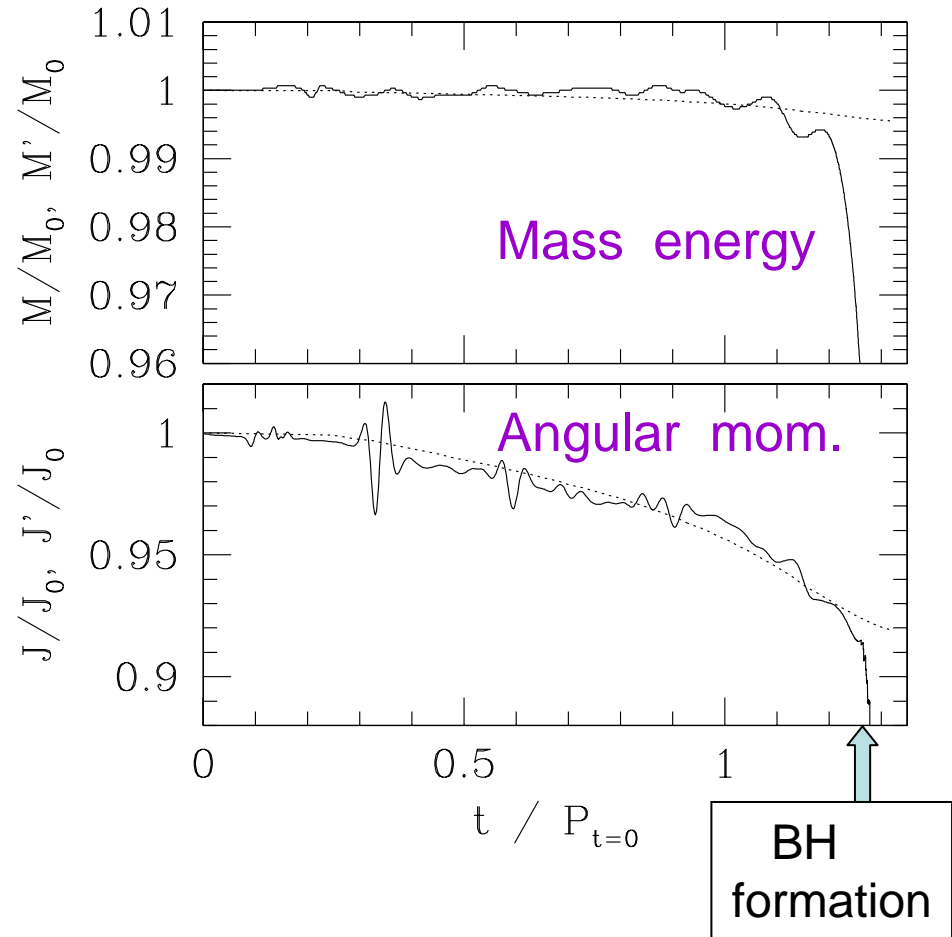
**$M_0 - E_{GW} = M'$  &  $J_0 - J_{GW} = J'$   
should be satisfied**

# Radiation reaction : OK within $\sim 1\%$

## NS formation: equal mass



## BH formation: unequal mass



Solid curves : computed from data sets in finite domain.  
Dotted curves: computed from fluxes of gravitational waves

# 5 Summary

- 1 Rapid progress in particular in the past 5 yrs
- 2 Scientific (quantitative) runs are feasible now.
- 3 (Astrophysically) Accurate and longterm simulations are feasible for many phenomena in the absence of BHs : **NS-NS merger, Stellar collapse, Bar-instabilities of NSs ....**
- 4 (I think) numerical implementations for fundamental parts have been almost established (for the BH-absent spacetimes)

# Issues for the near future

## 1 Several (technical) Issues still remain :

- Grid numbers are still not large enough in 3D  
→ We would need hypercomputer (~10TBytes, ~10TFlops)

Probably becomes available in a couple of yrs.

- Computation crashed due to grid stretching around BH horizon  
→ We need to develop excision techniques.
- How to achieve a very high accuracy for making GW templates ?

## 2 Incorporate more realistic physics in hydro simulation

More realistic EOS, Neutrino cooling, Magnetic fields

# Where are we ?

- 1: Make a code which runs anyhow stably (do not care accuracy)
- 2: Improve the code which can provide a **qualitatively correct result**; care accuracy somewhat (say we admit an error of ~10%)
- 3: Improve the code gradually getting **qualitatively new results** which can be obtained only by an improved code
- ★ 4: Goal: Make a code which provides a **quantitatively accurate result**.



We are here.

Similar to construction of detectors in some sense

# Animations

- <http://esa.c.u-tokyo.ac.jp/~shibata/anim.html>