Shock Propagation in Loosely Packed Granular Media

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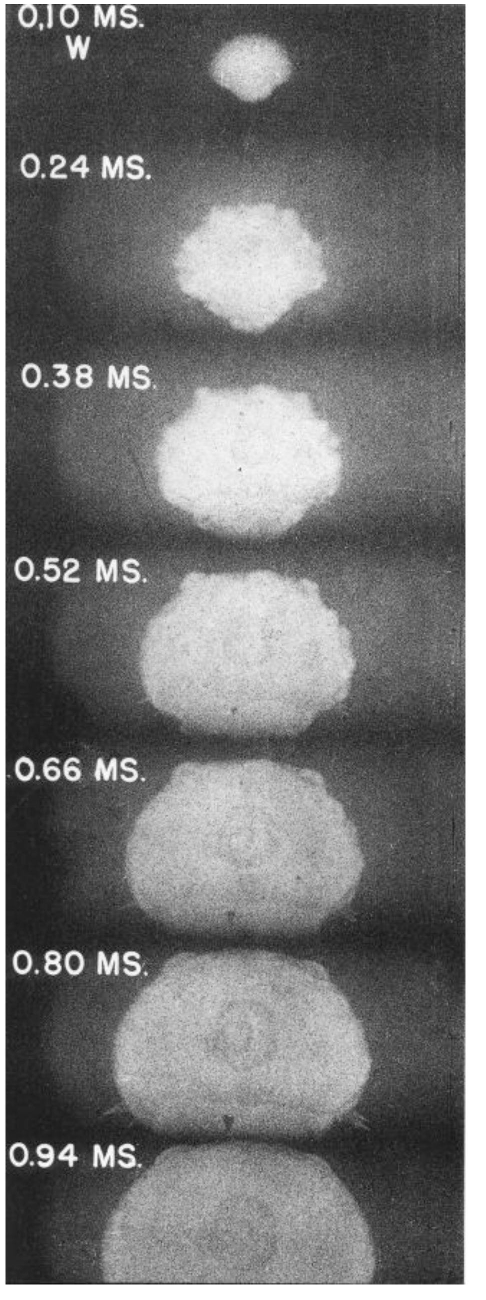
Outline of the talk

- The problem
  - ⋆ Nuclear Explosion
  - ⋆ Granular Explosion
- Motivation
- Analysis
- Comparison with experiments
- Modified models
- Conclusions

Nuclear Explosion



How does the radius increase with time?

Dimensional Analysis

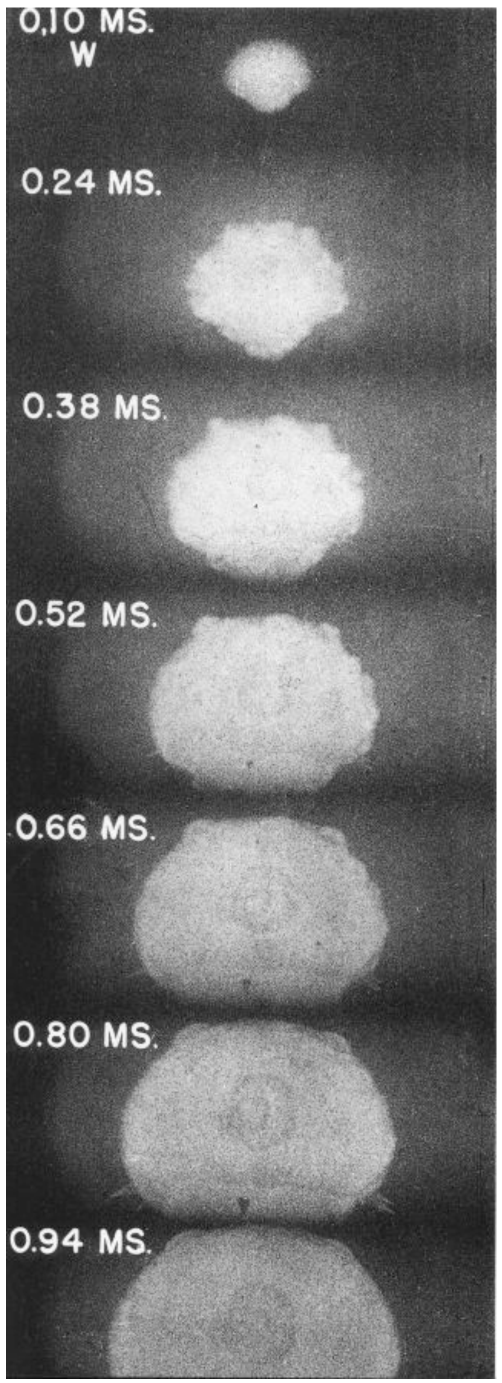
$$R(t) = f(E_0, t, \rho, \cancel{T_0})$$

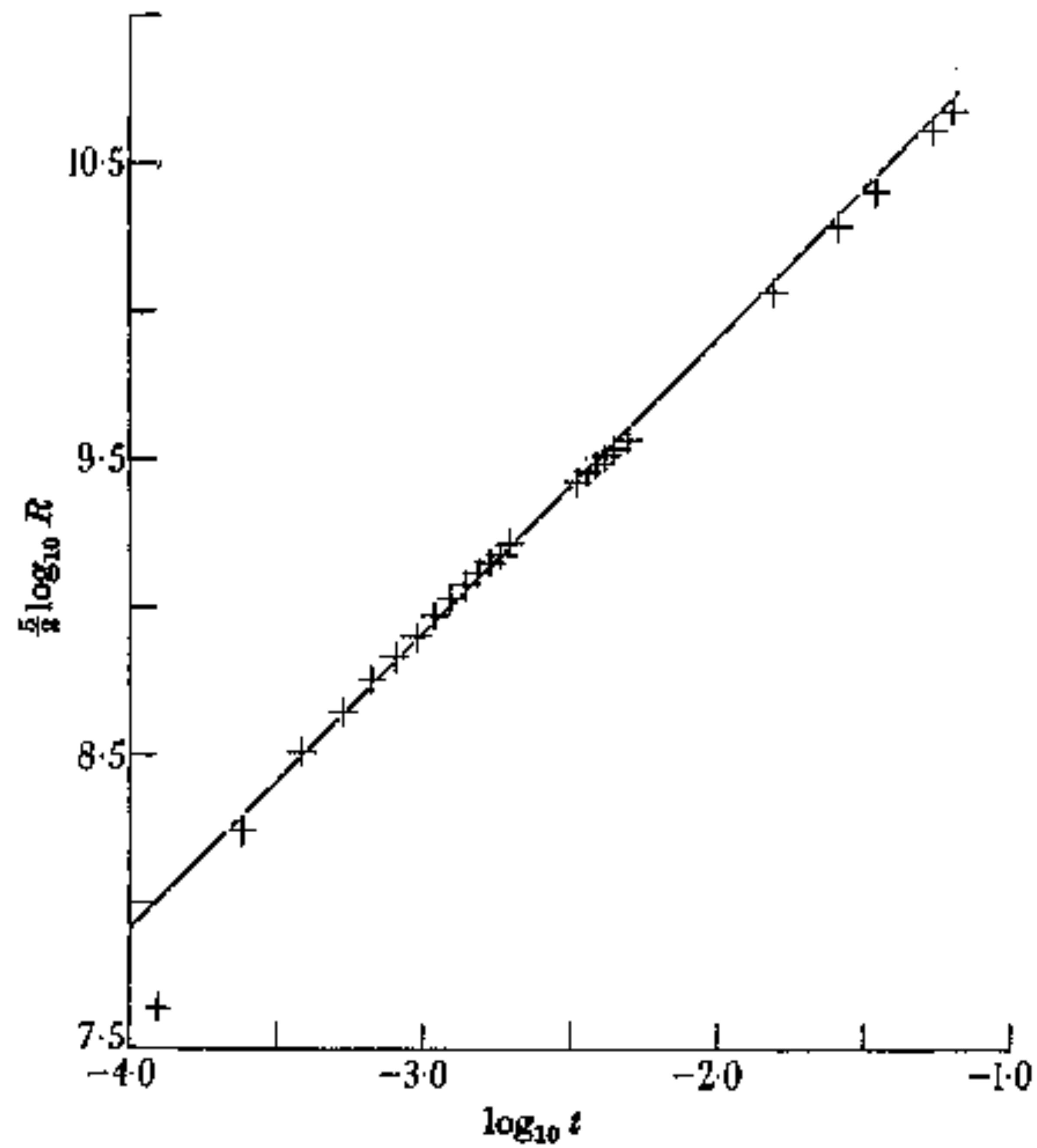
$$\begin{aligned} [E_0] &= ML^2T^{-2} \\ [\rho] &= ML^{-d} \\ [t] &= T \end{aligned}$$

$$\boxed{R(t) = c\left(\frac{E_0 t^2}{\rho}\right)^{\frac{1}{d+2}}}$$

$$d = 3 \implies R(t) \propto t^{2/5}$$

Comparison with data

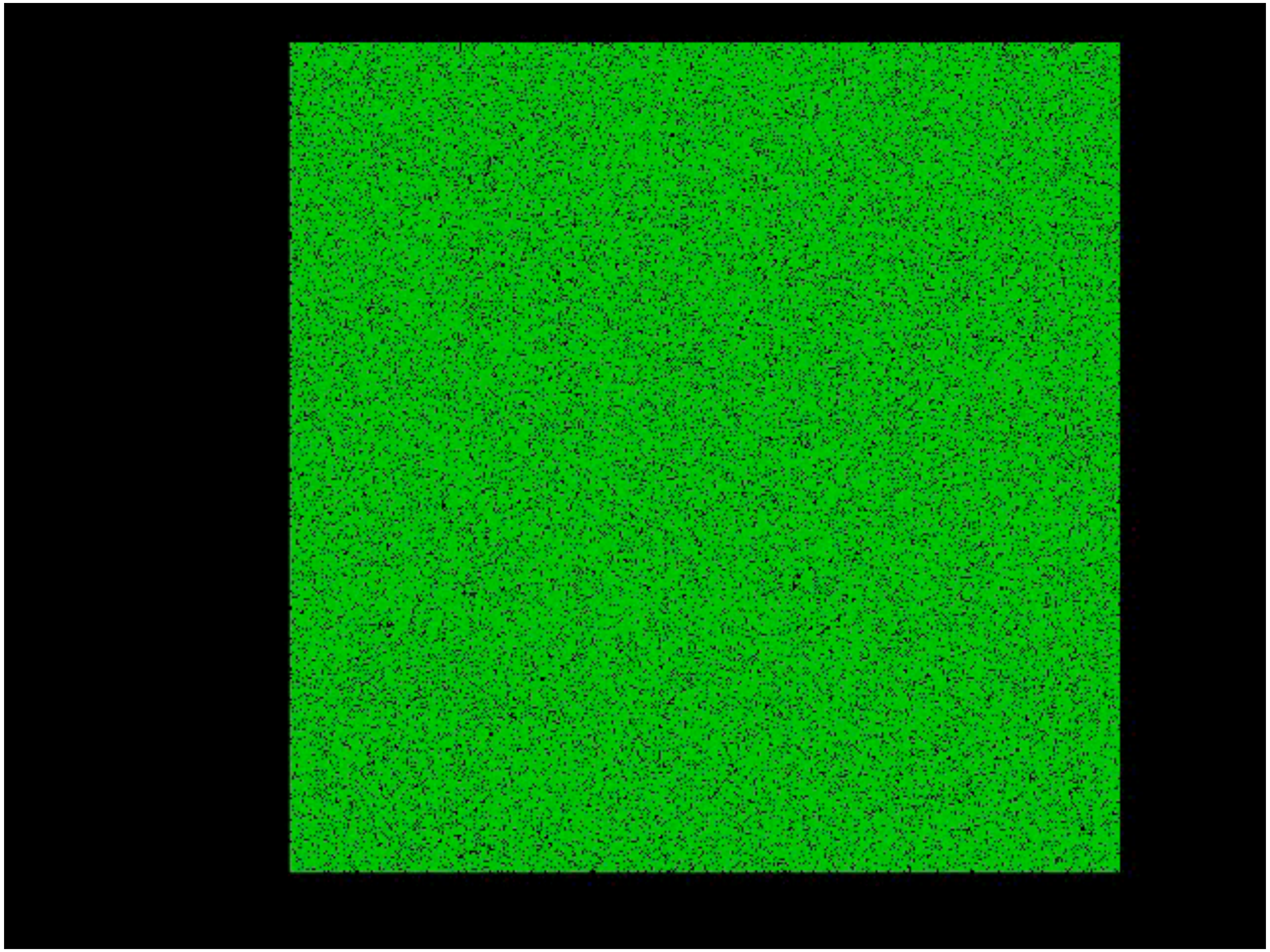




A computer model

- Particles at rest
- One particle given an impulse
- Interaction only on contact
  - ⋆ Energy conserving
  - ⋆ Momentum conserving

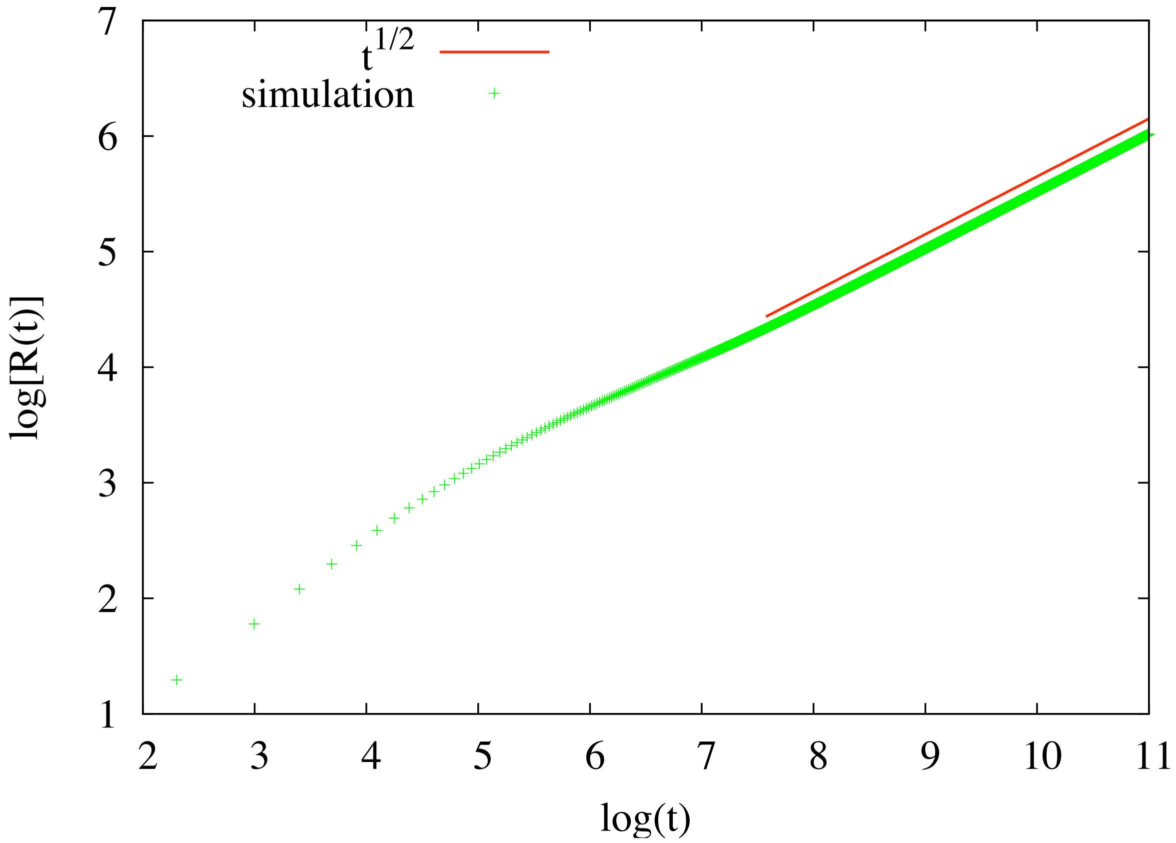
A computer model



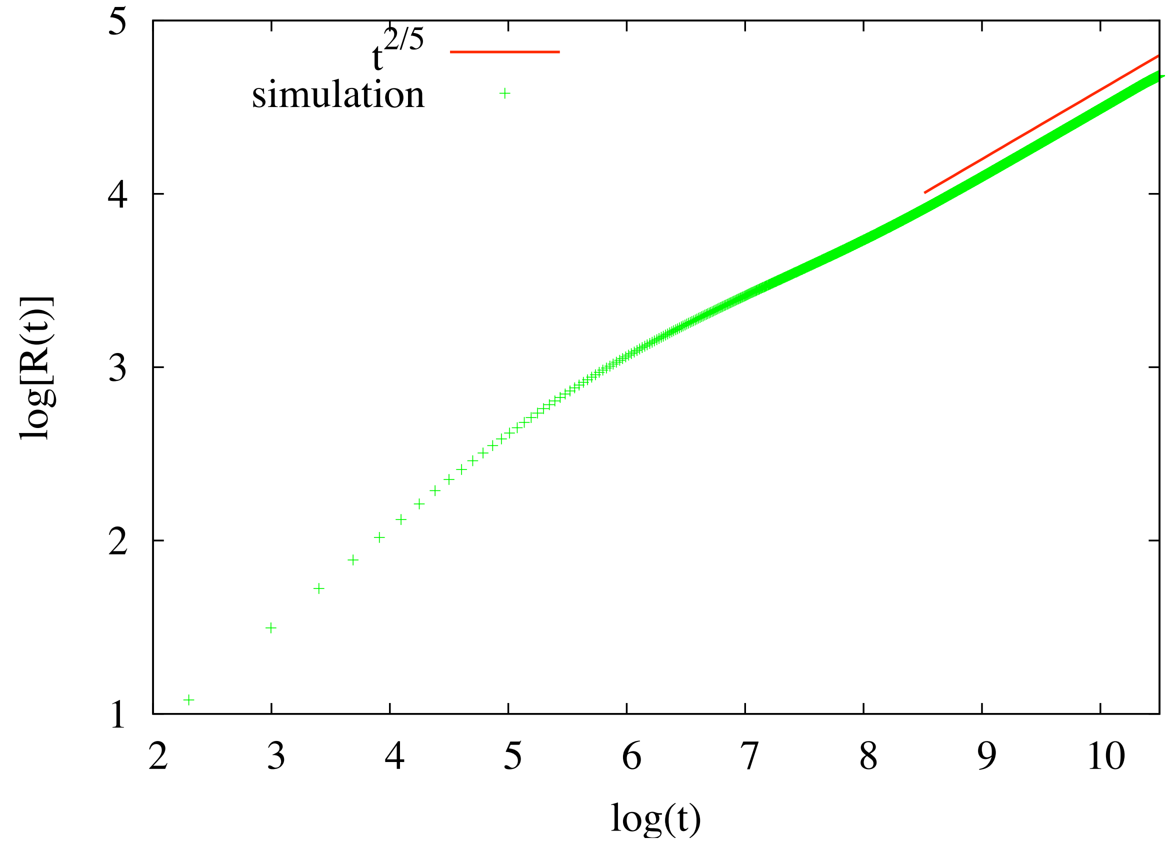
Radius vs time

$$R(t) = c\left(\frac{E_0 t^2}{\rho}\right)^{\frac{1}{d+2}}$$

2 dimensions



3 dimensions



Question

Take the above model and make the collisions inelastic.

Do the results change?

Granular systems

Sand, steel balls, talcum powder

$$\text{Size} \sim 1\mu m \text{ to } 1mm$$

$$\text{Mass} \sim 1\ mg$$

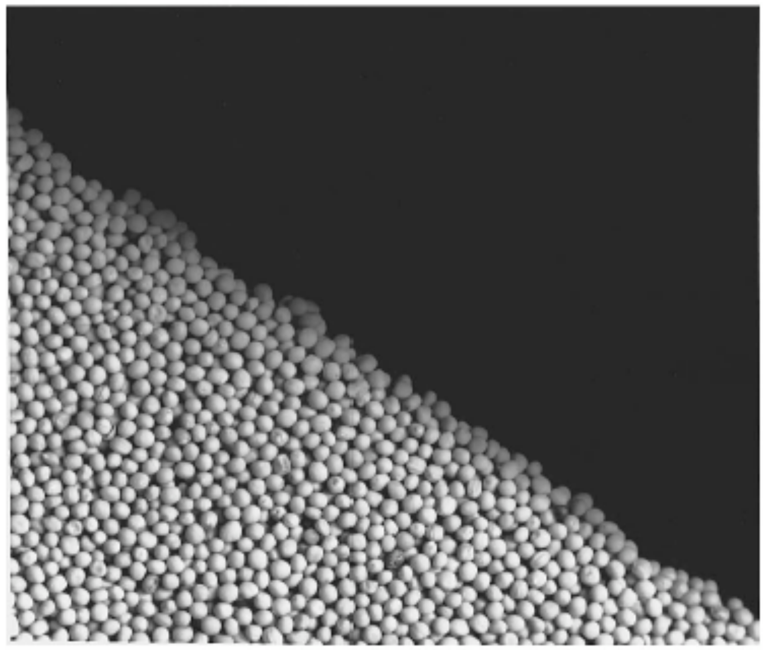
$$\text{Velocity} \sim 1cm/s$$

$$\frac{KE}{kT} = \frac{10^{-6}10^{-4}}{kT} \approx 10^{10}$$

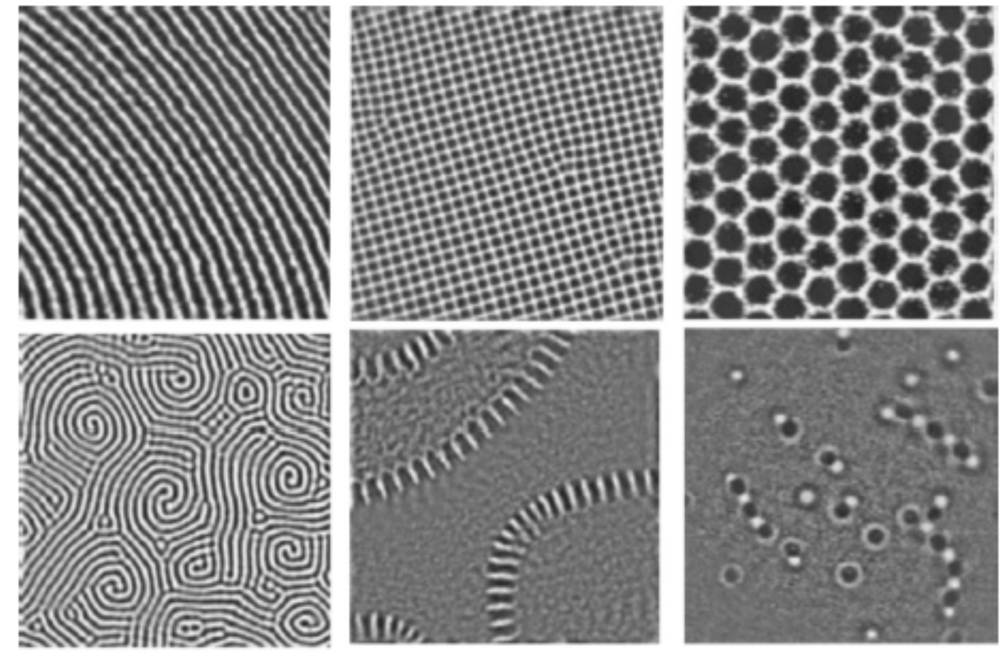
$$\frac{PE}{kT} = \frac{10^{-6}1010^{-2}}{kT} \approx 10^{13}$$

Temperature plays no role

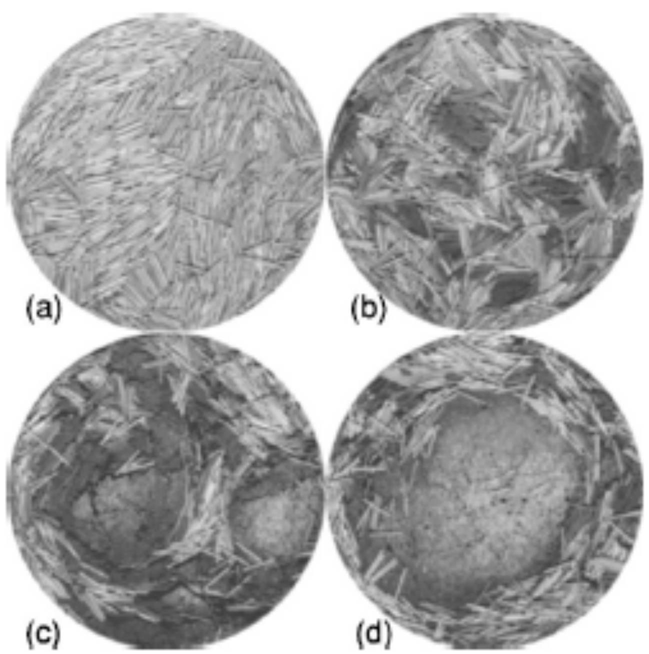
Different Cases



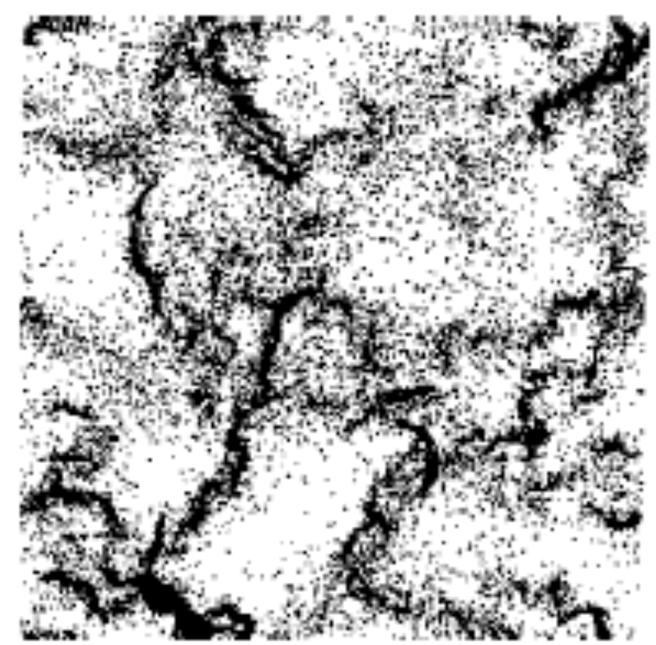
Jaeger et al, 1996



Aranson et al, 2006



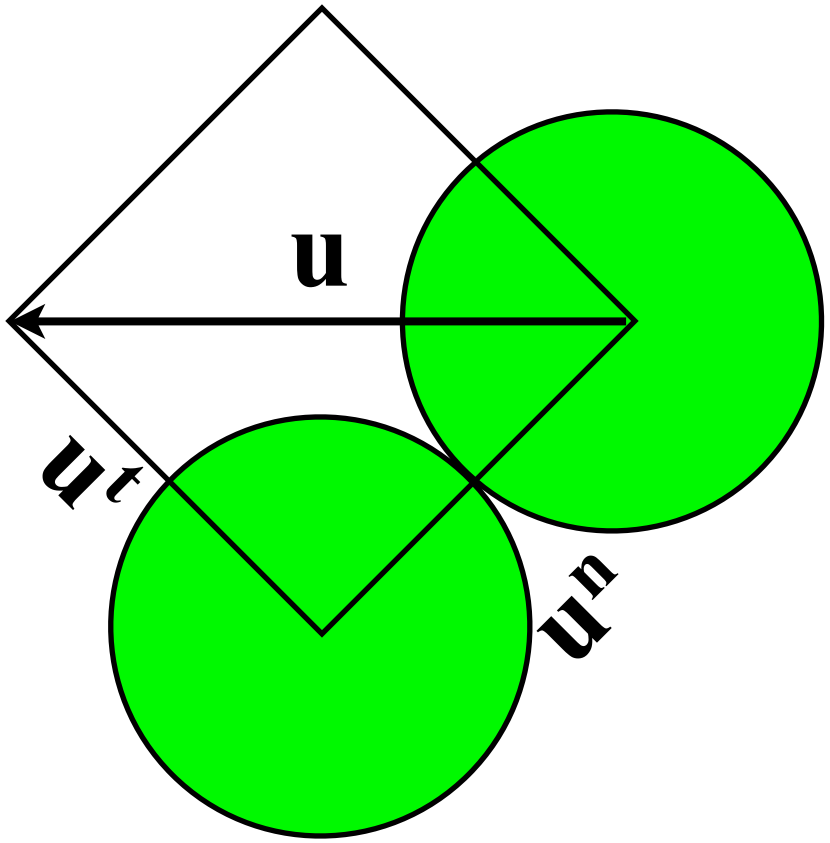
Blair et al, 2003



Goldhirsch et al, 1993

Key ingredient

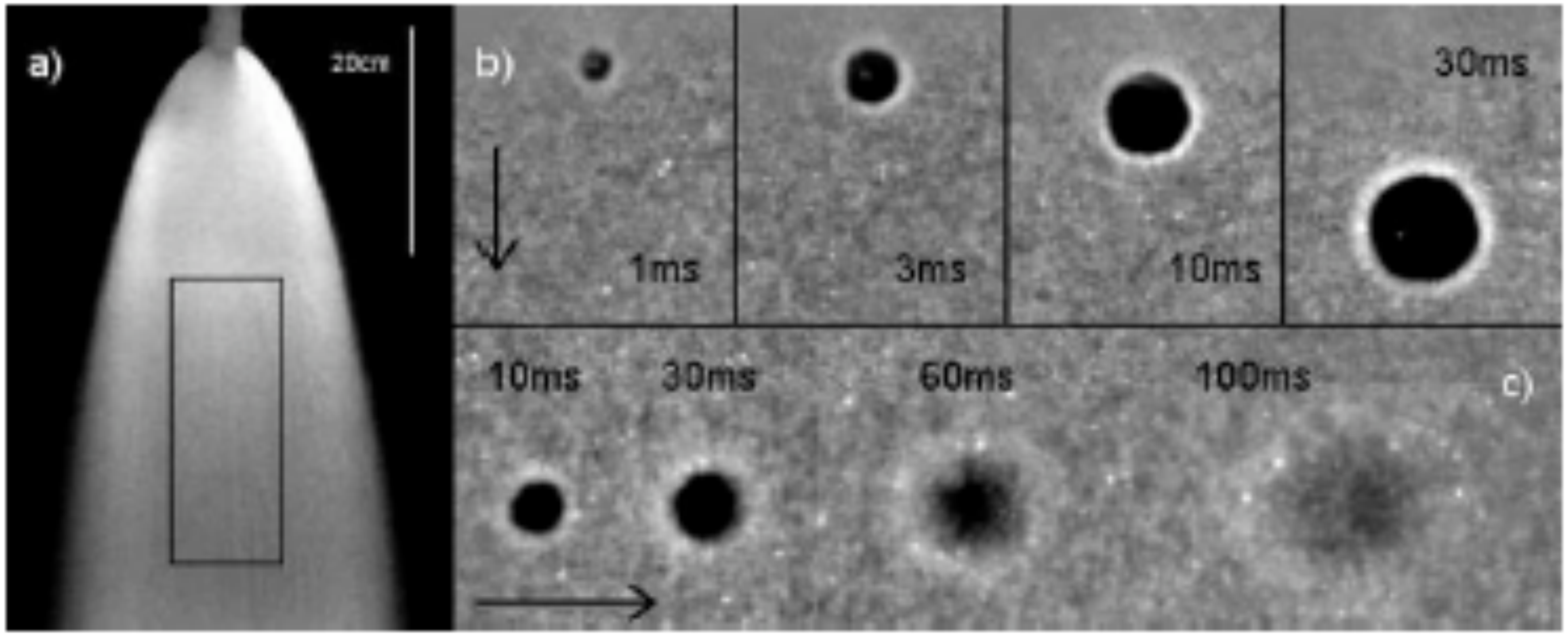
Collisions are inelastic



$$\begin{aligned} v^t &= u^t \\ v^n &= -ru^n \end{aligned}$$

$$r < 1$$

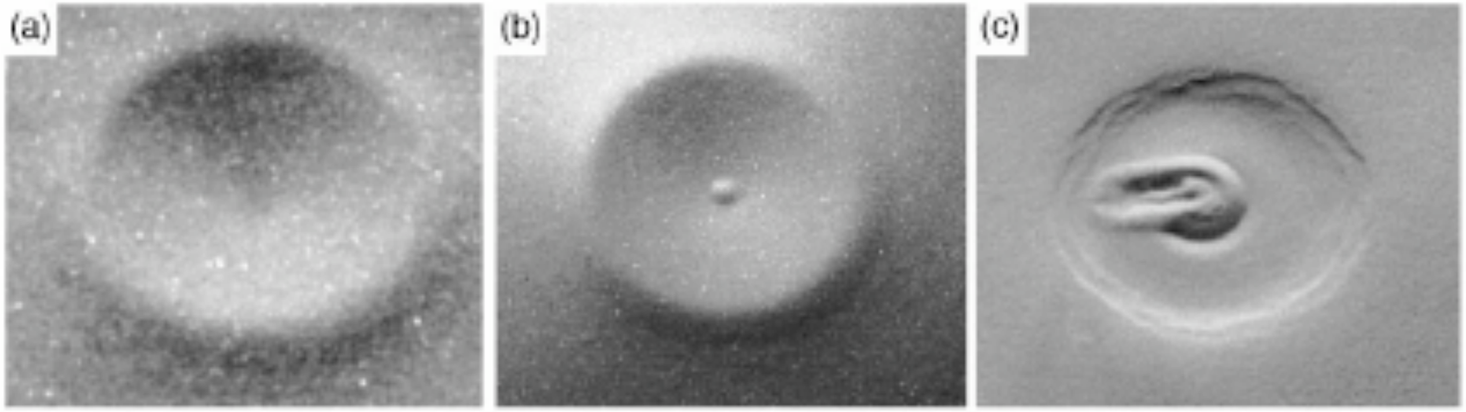
Experiments



Boudet et al, PRL 2009

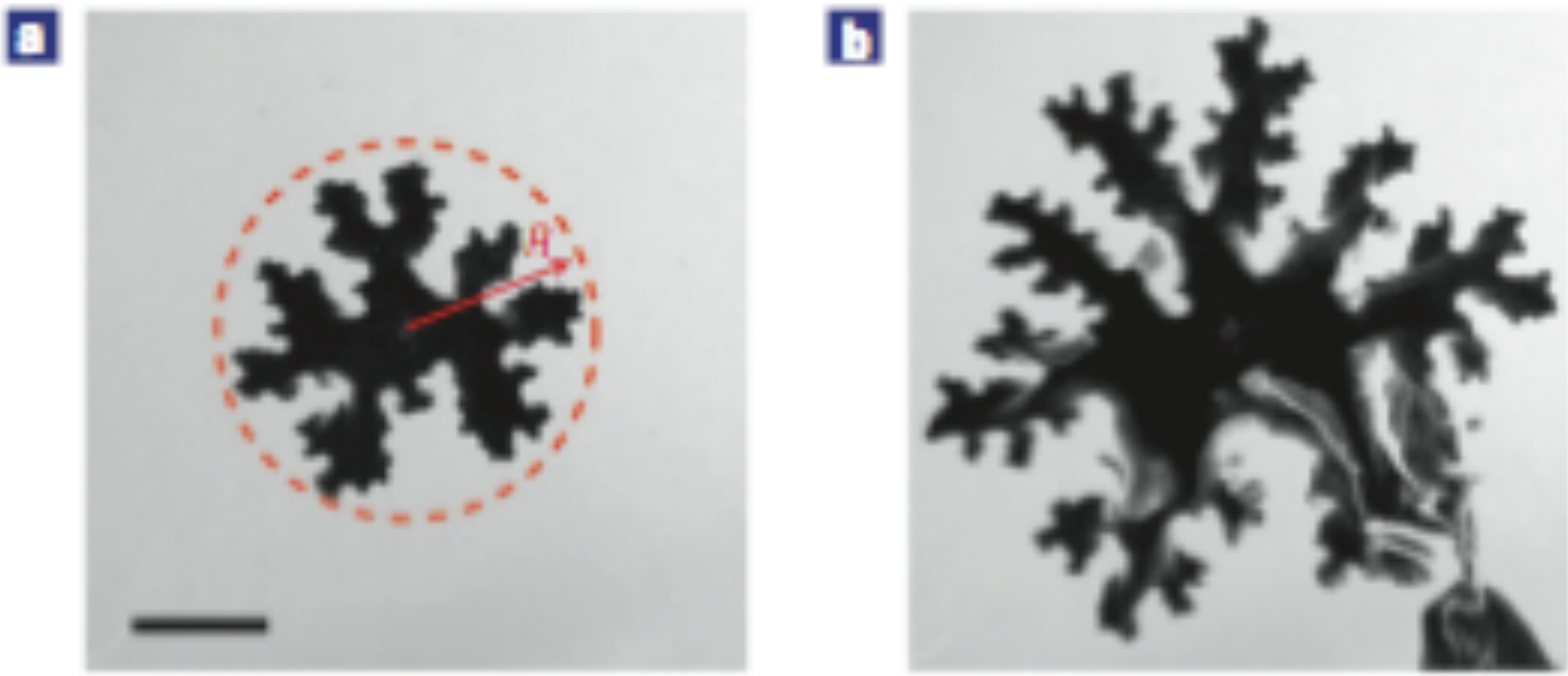
Experiments

Crater formation



Walsh et al, PRL 2003

Experiments



Cheng et al, Nature Phys, 2008

Freely cooling granular gas

- Give initial energy to particles
- Isolate system
- Energy loss through collisions
- Why study?
  - ★ Isolates effects of inelastic collisions
  - ★ Direct experiments
  - ★ As parts of larger driven systems
  - ★ Interacting particle systems

Homogeneous Cooling

$$\frac{dE}{dt} = -\frac{\Delta E}{\tau}$$

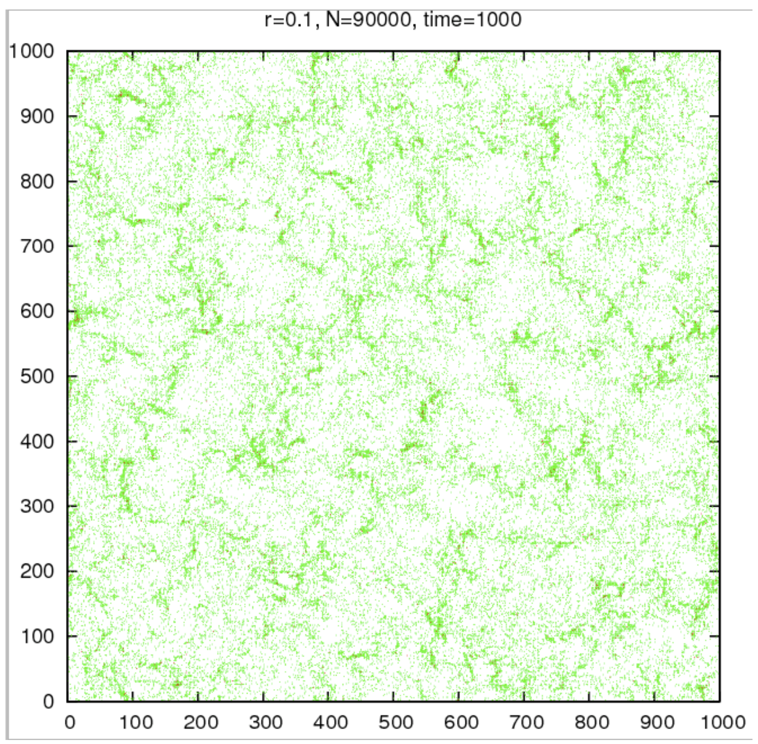
$$\frac{dE}{dt} \sim \frac{(1-r^2)E}{a/\sqrt{E}}$$

$$E \sim \frac{1}{(1-r^2)t^2 + c^2}$$

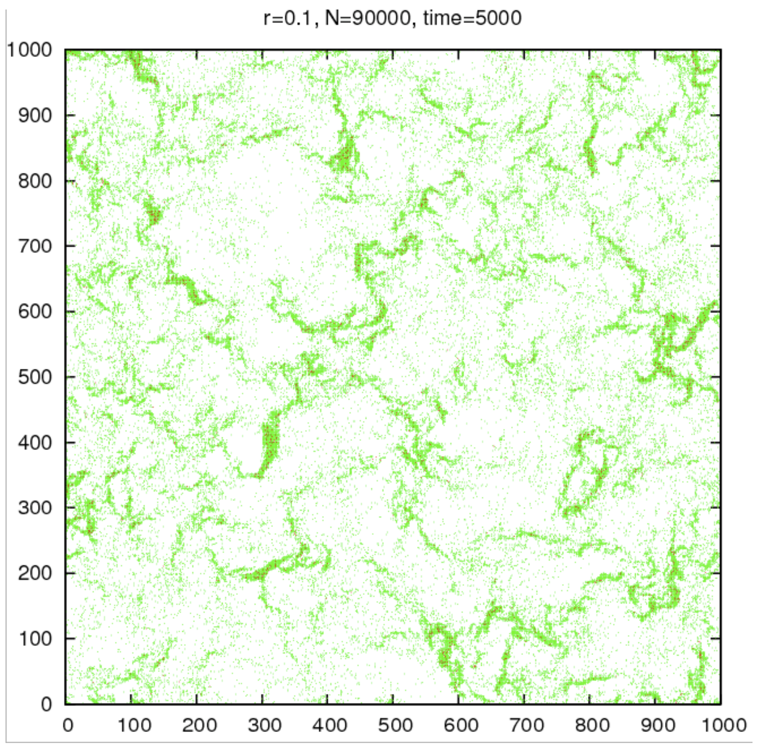
Haff's law   Haff, 1982

Assumption: particles are homogeneously distributed

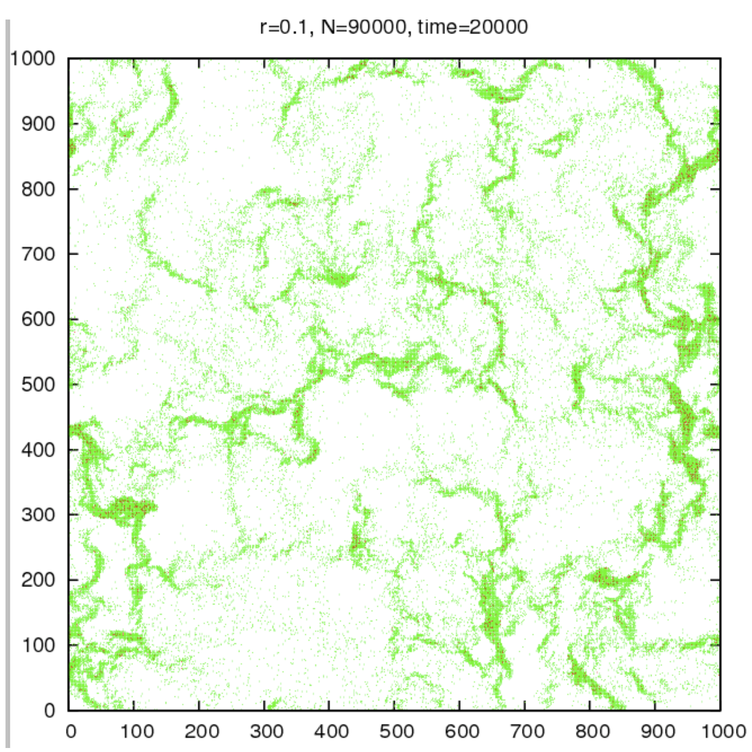
Clustering



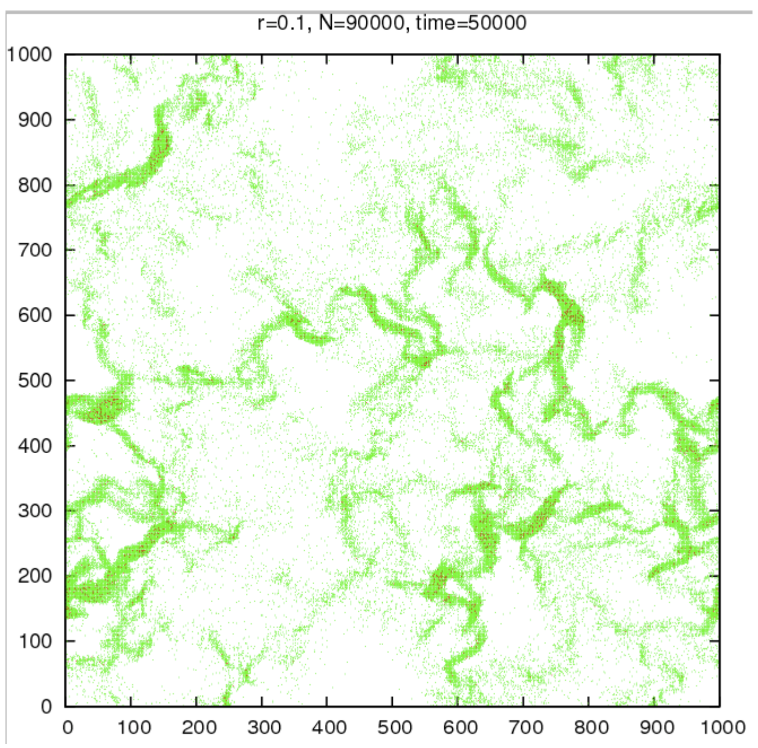
r=0.1, N=90000, time=1000



r=0.1, N=90000, time=5000



r=0.1, N=90000, time=20000



r=0.1, N=90000, time=50000

Clustering

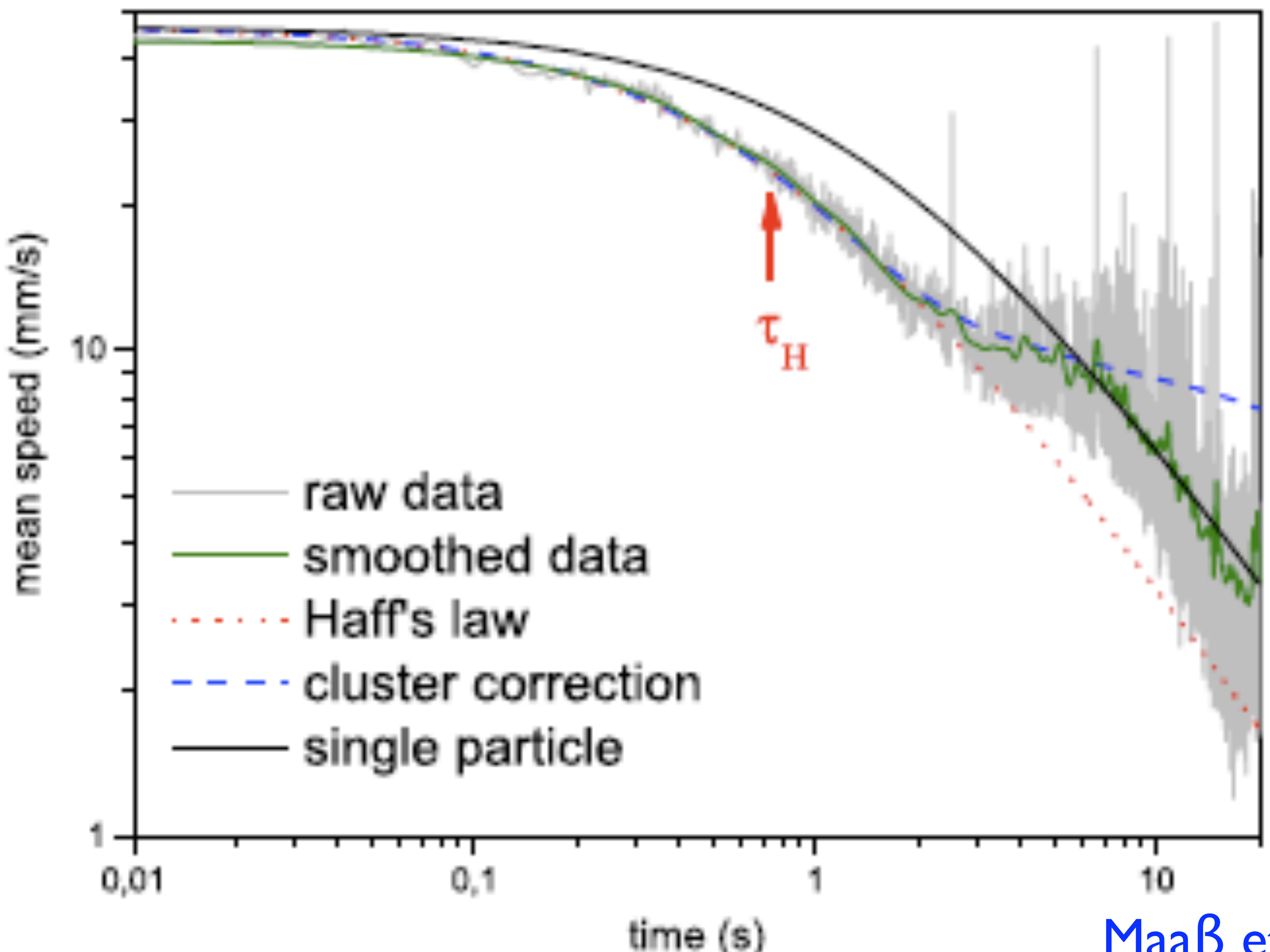
- Breakdown of Haff’s law (kinetic theory)
- New regime: inhomogeneous clustered regime

Experiments

- friction
- boundary effects

Experiments

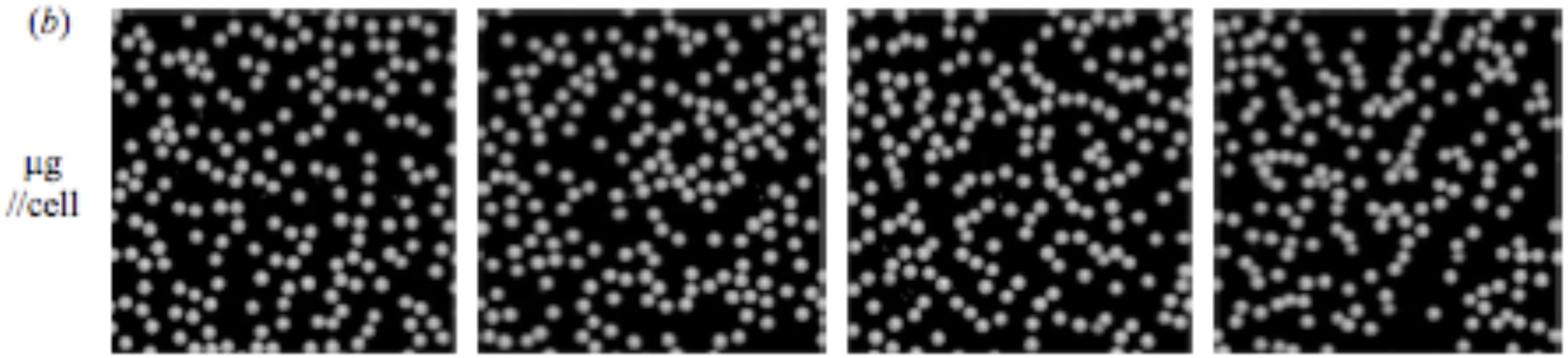
Levitation

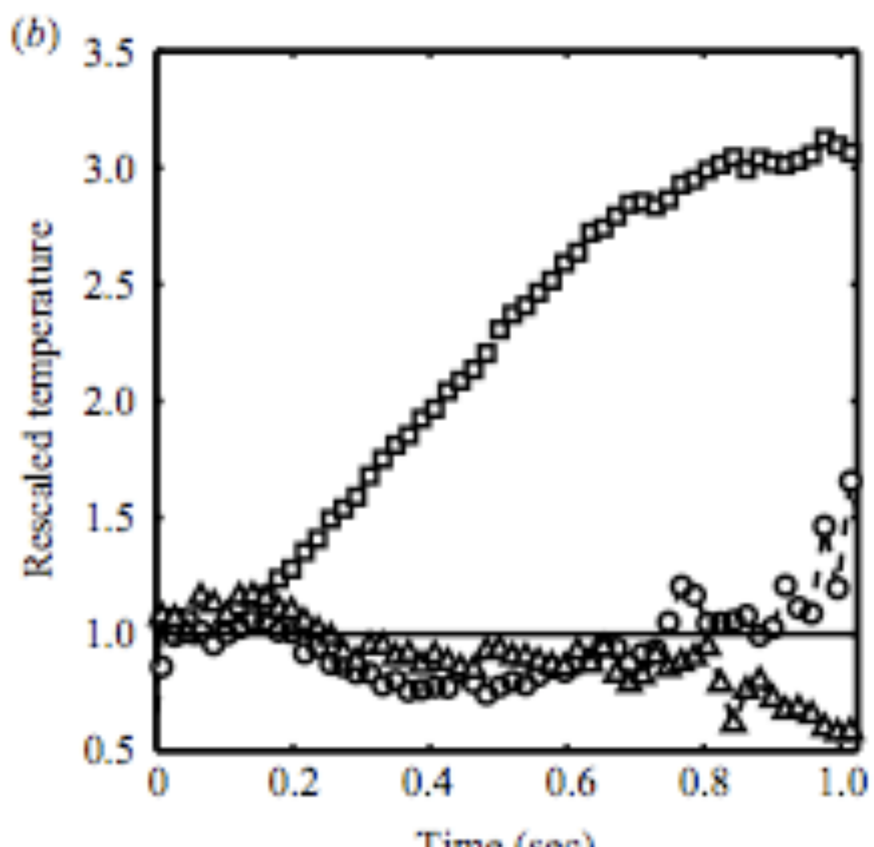


Maaβ et al, 2008

Experiments

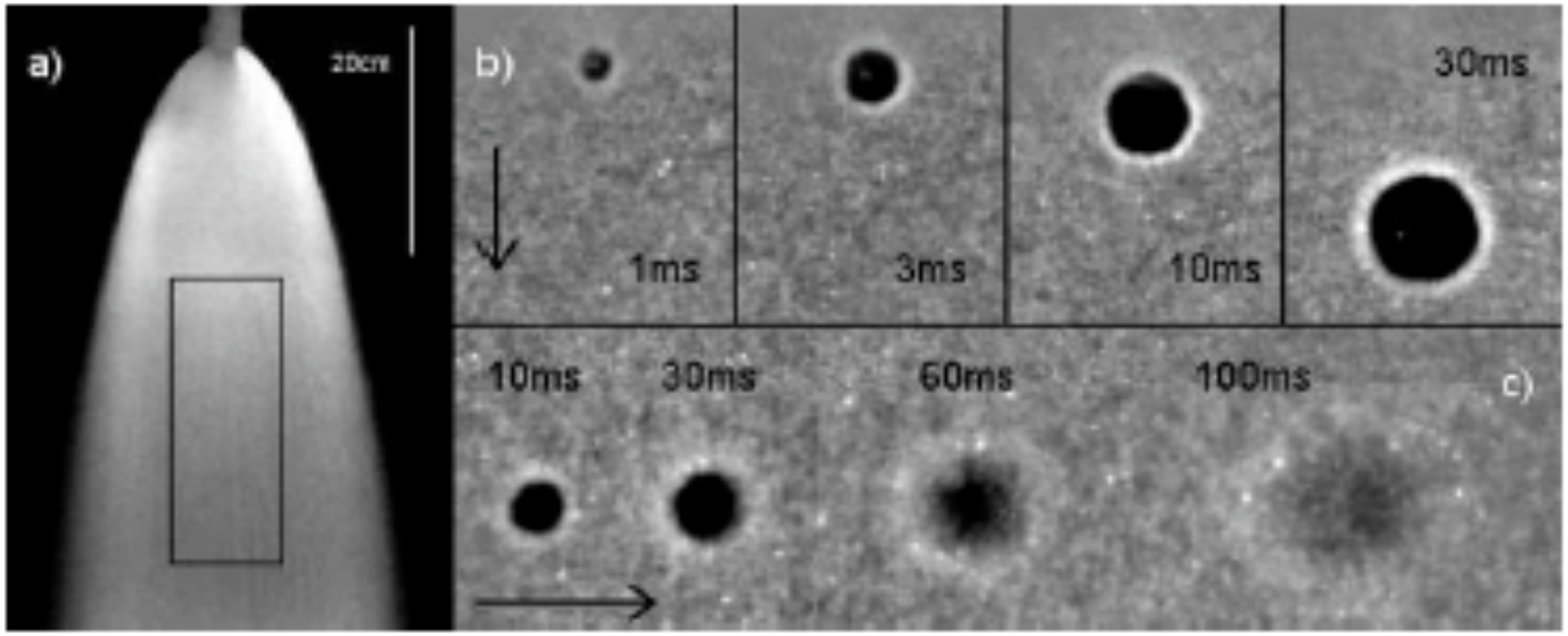
Microgravity





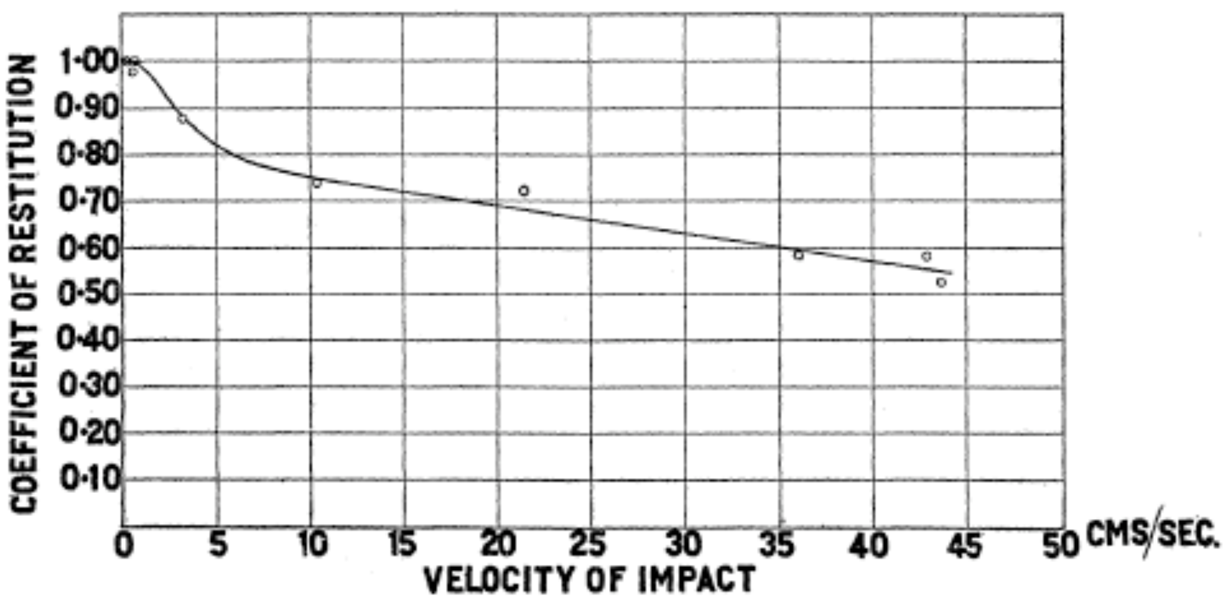
Tatsumi et al, 2009

Clustering

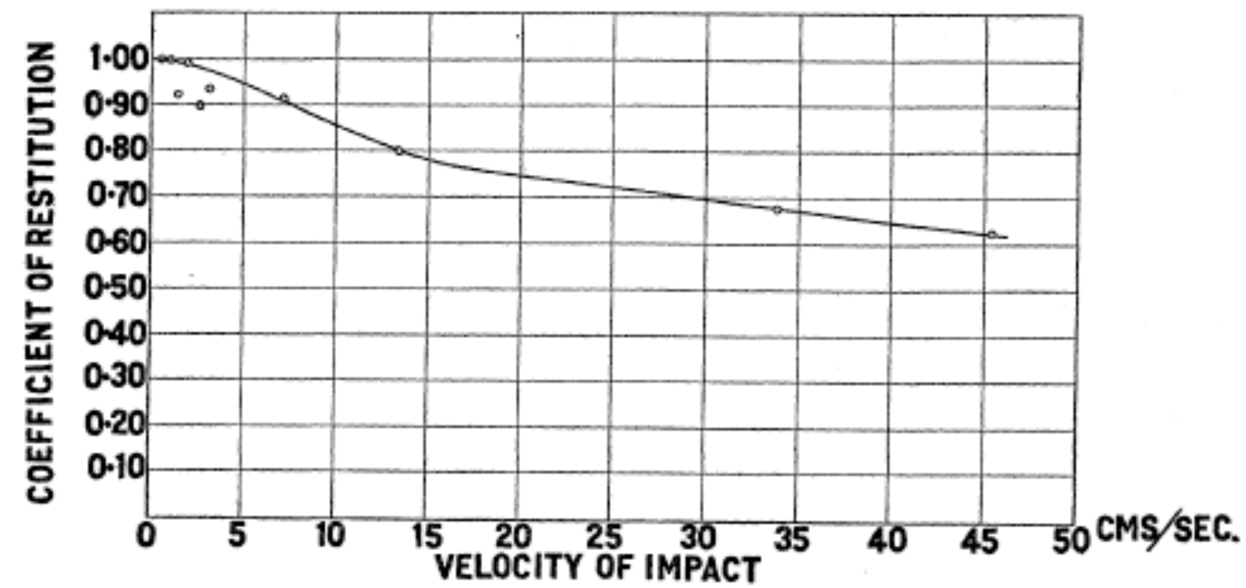


Boudet et al, PRL 2009

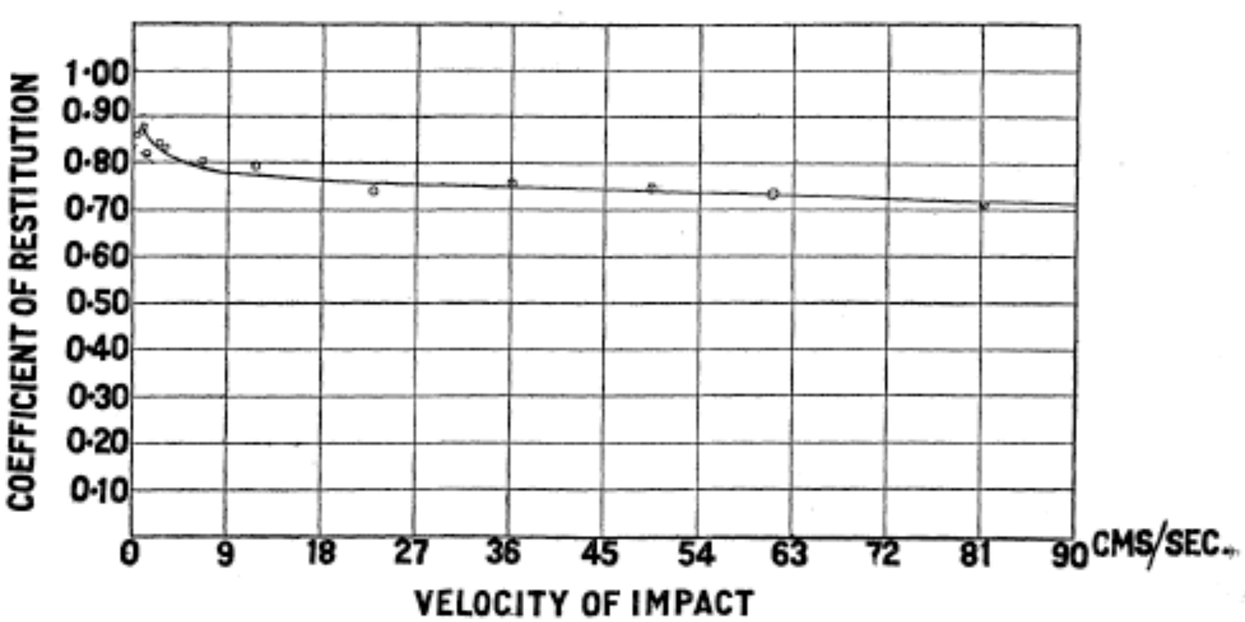
A constant coefficient of restitution?



Brass



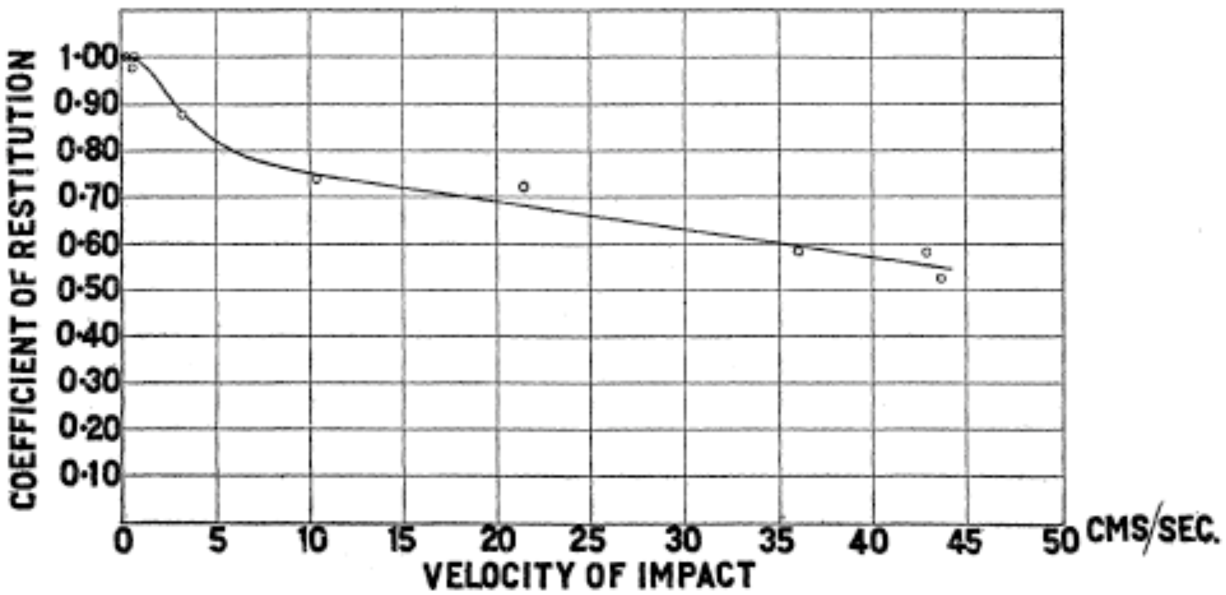
Aluminium



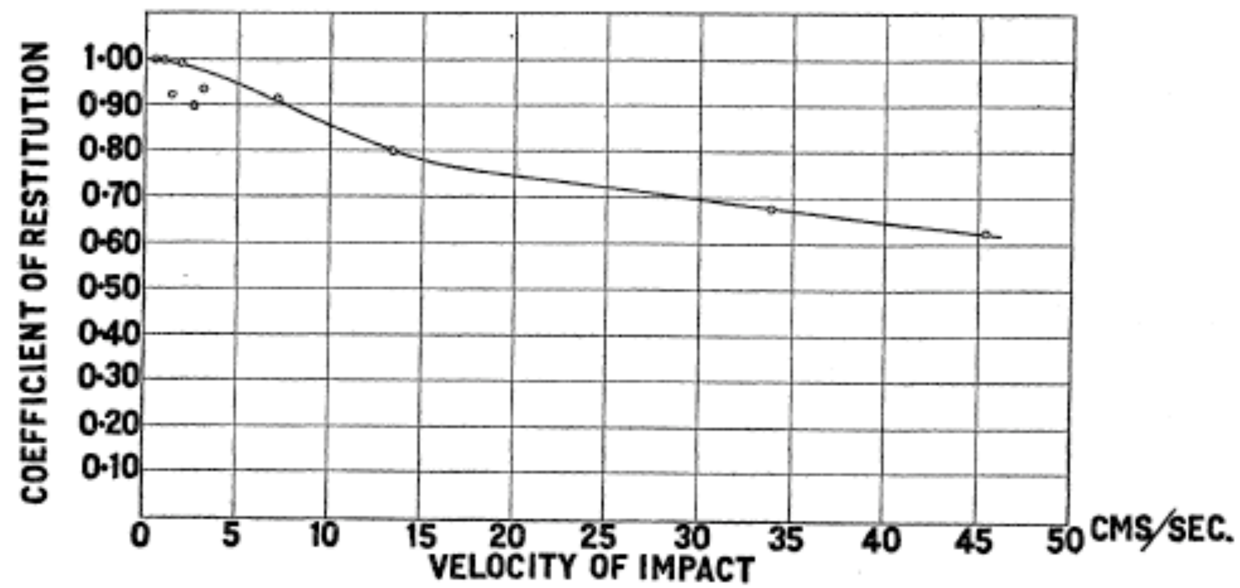
Marble

C.V. Raman, 1918

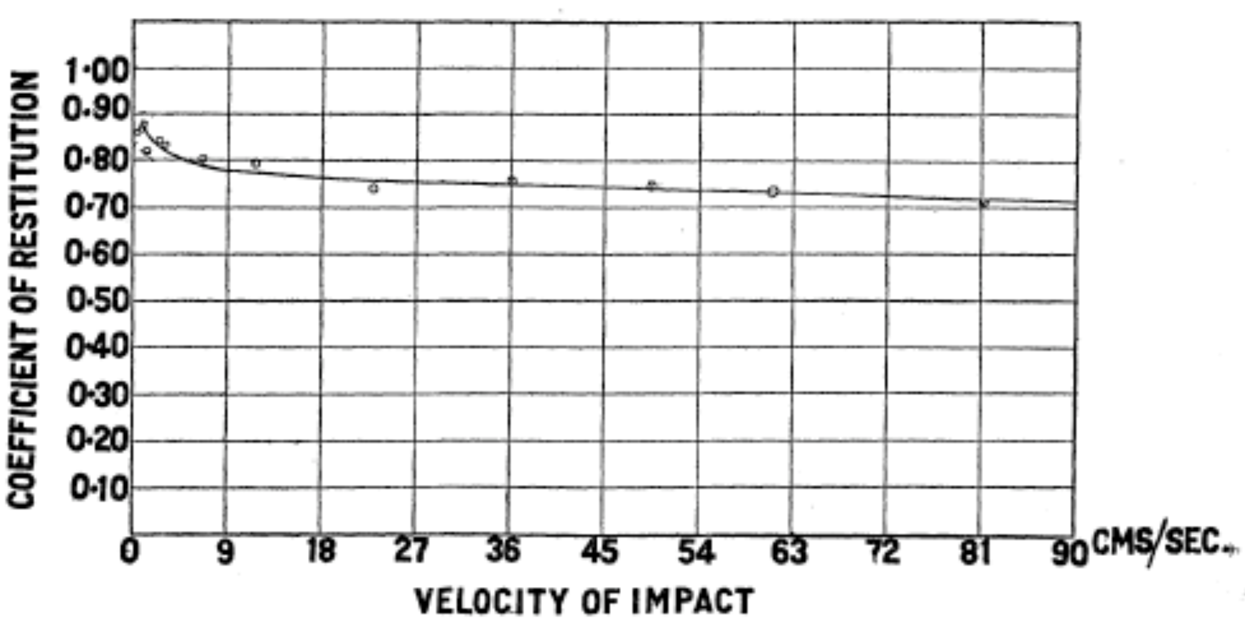
A constant coefficient of restitution?



Brass



Aluminium



Marble

$$r \longrightarrow 1 \text{ when } v \longrightarrow 0$$

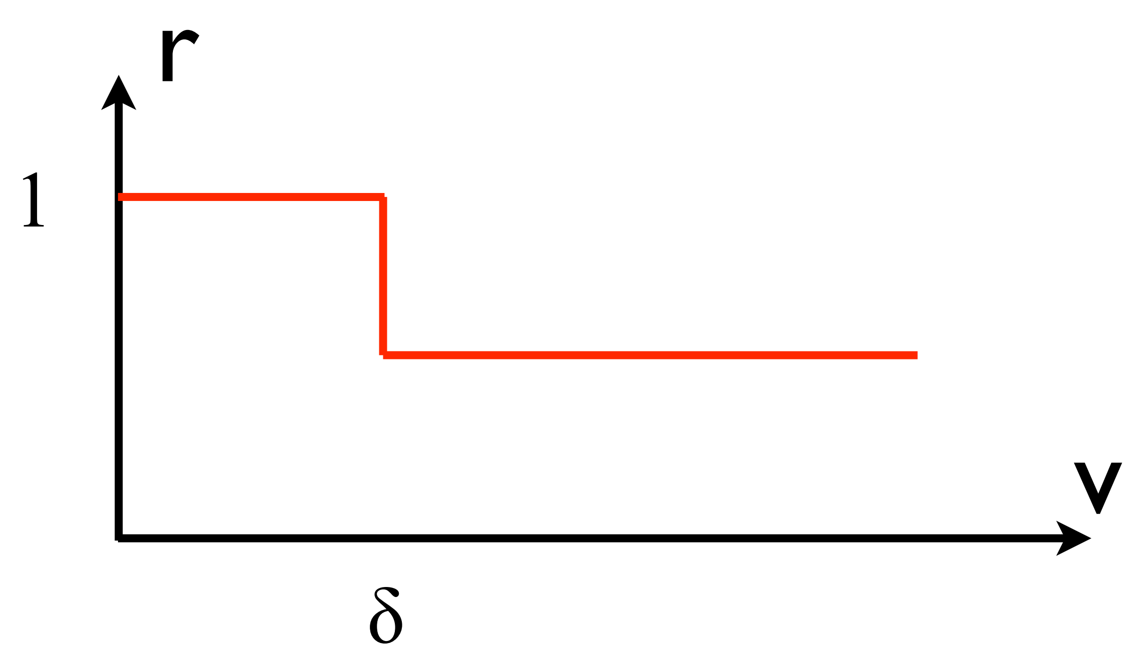
$$r \longrightarrow r_0 \text{ when } v \longrightarrow \infty$$

C.V. Raman, 1918

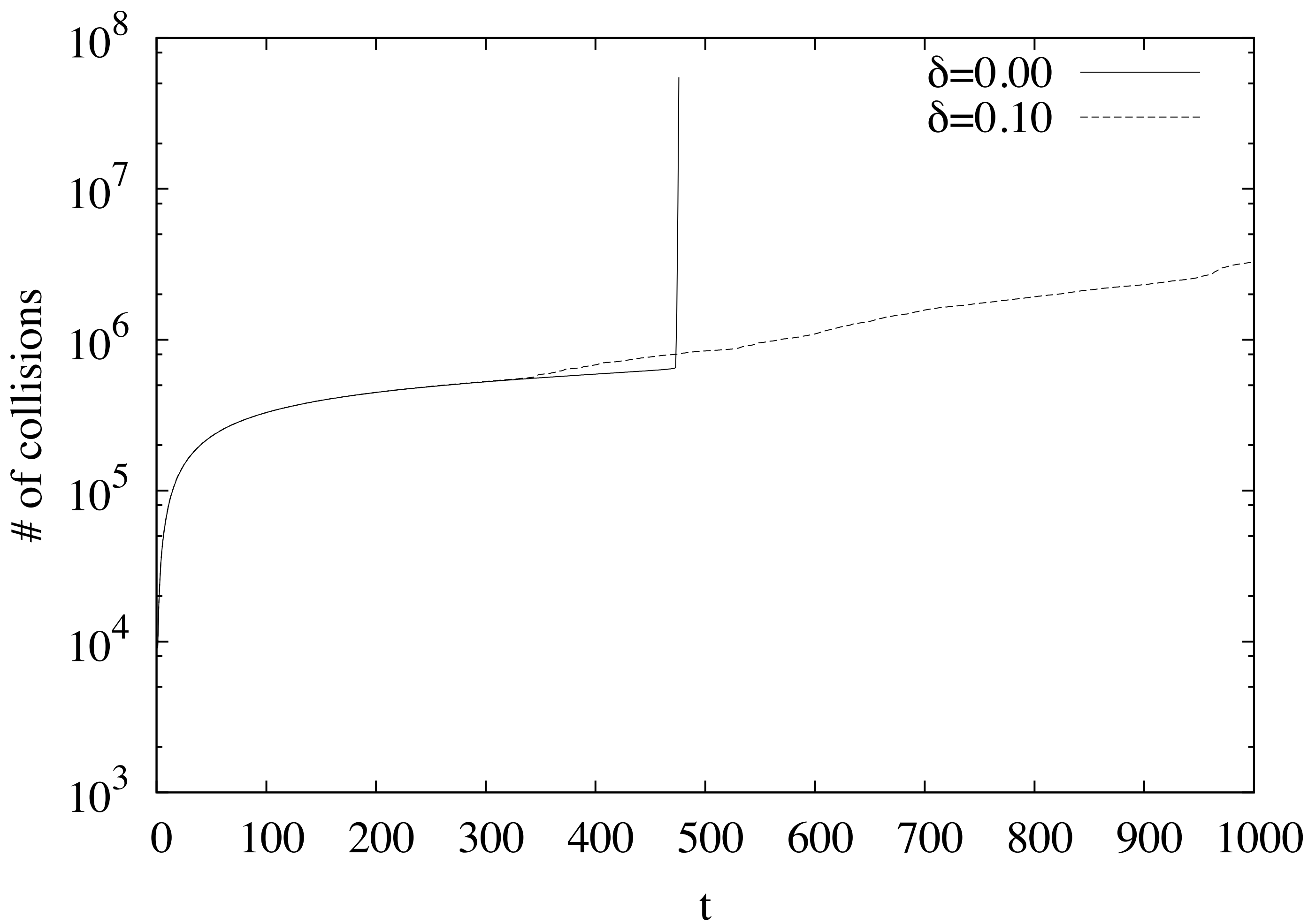
Coefficient of Restitution

$$\text{r} \rightarrow 1 \text{ when } v \rightarrow 0$$

$$\text{r} \rightarrow r_0 \text{ when } v \rightarrow \infty$$

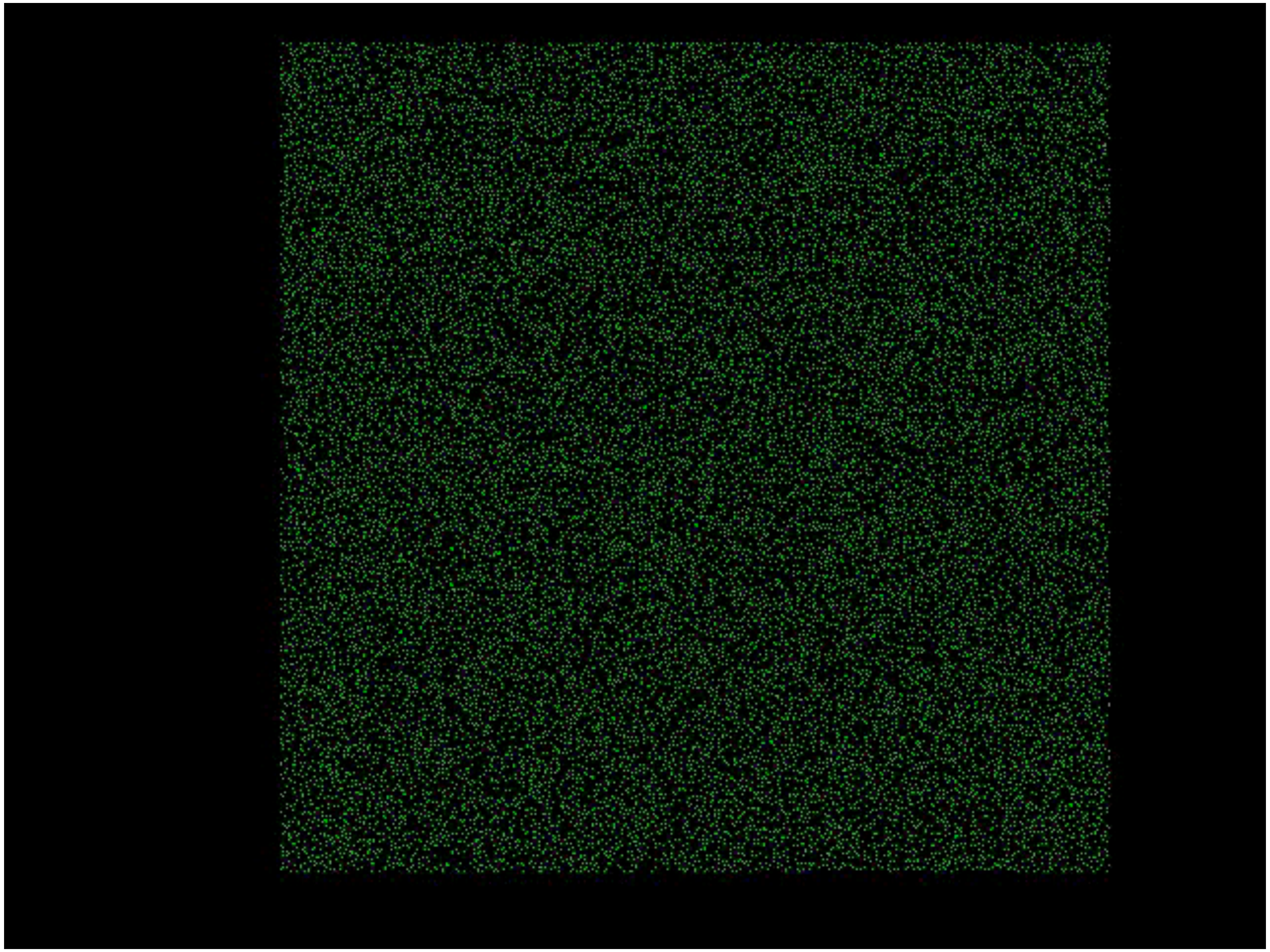


Do we need δ?

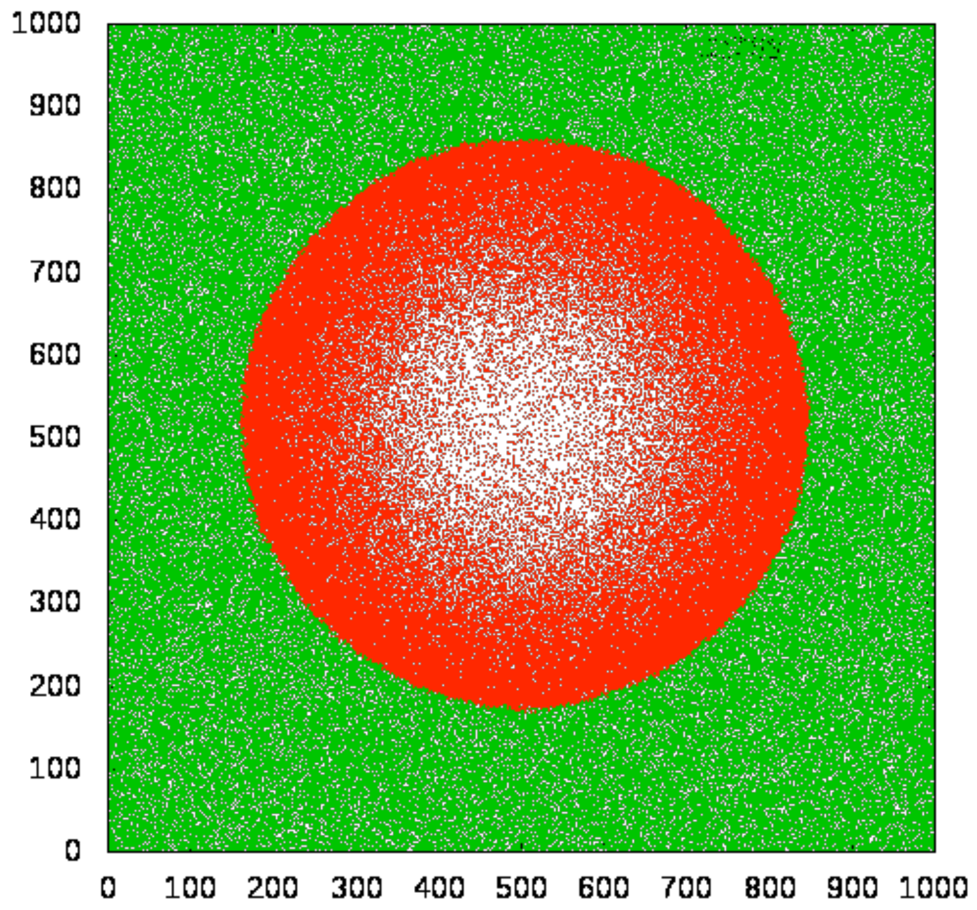


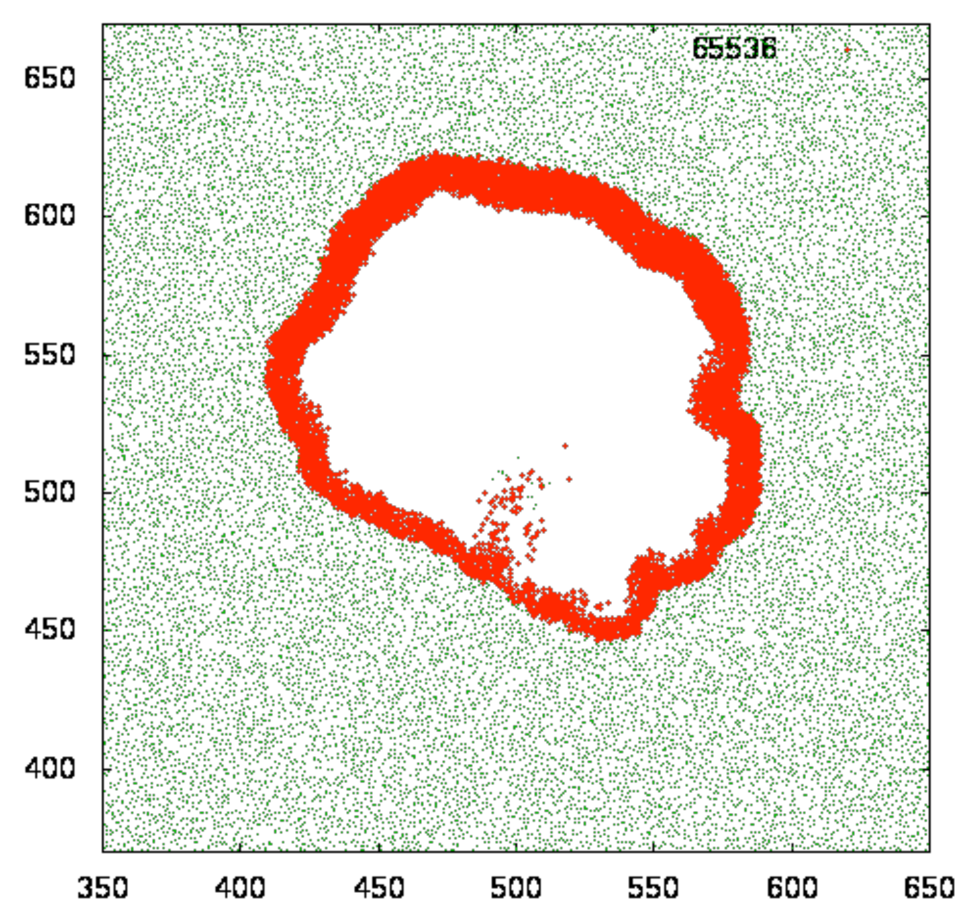
Inelastic collapse

Computer simulation



Elastic vs Inelastic

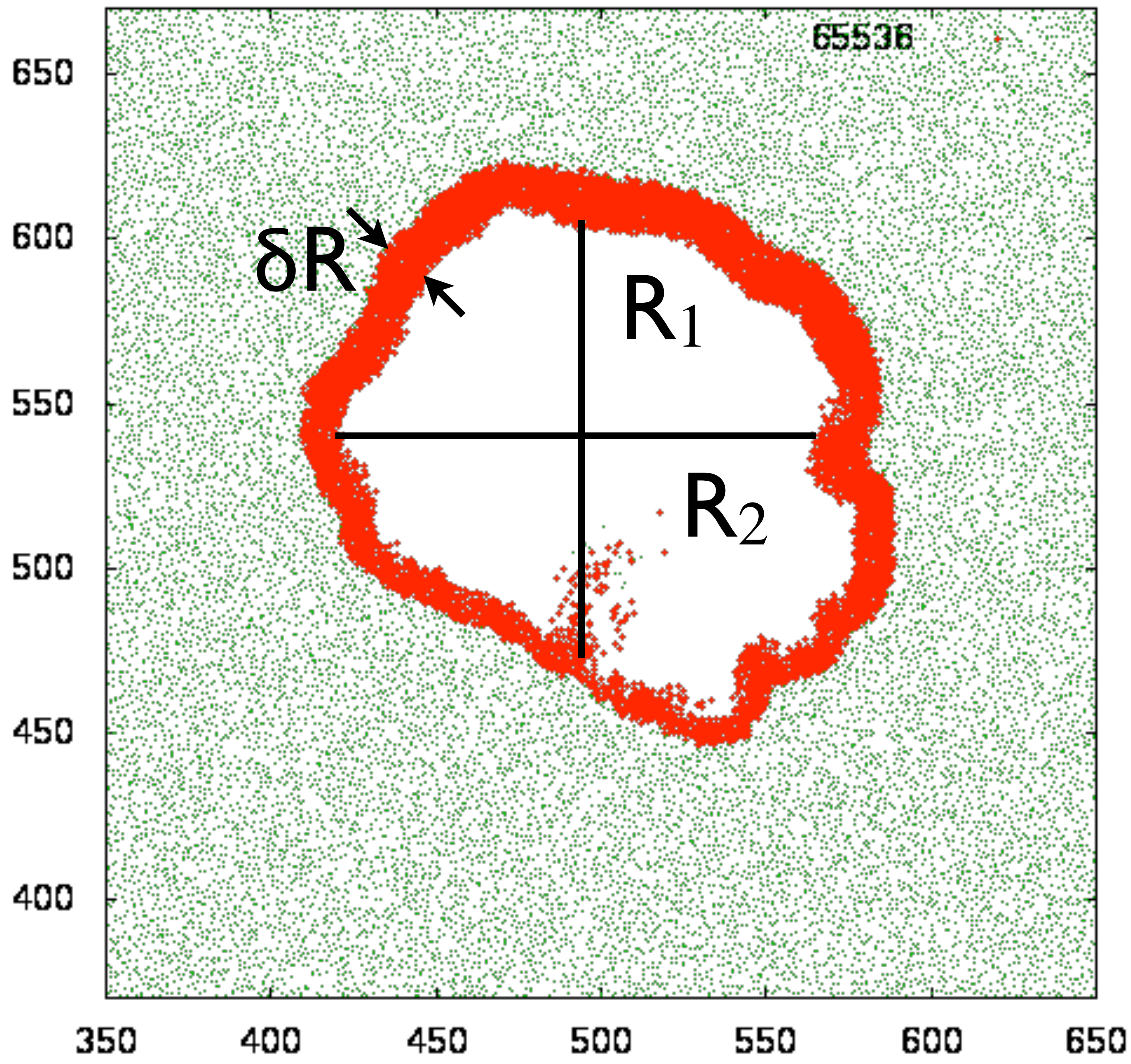




Scaling analysis

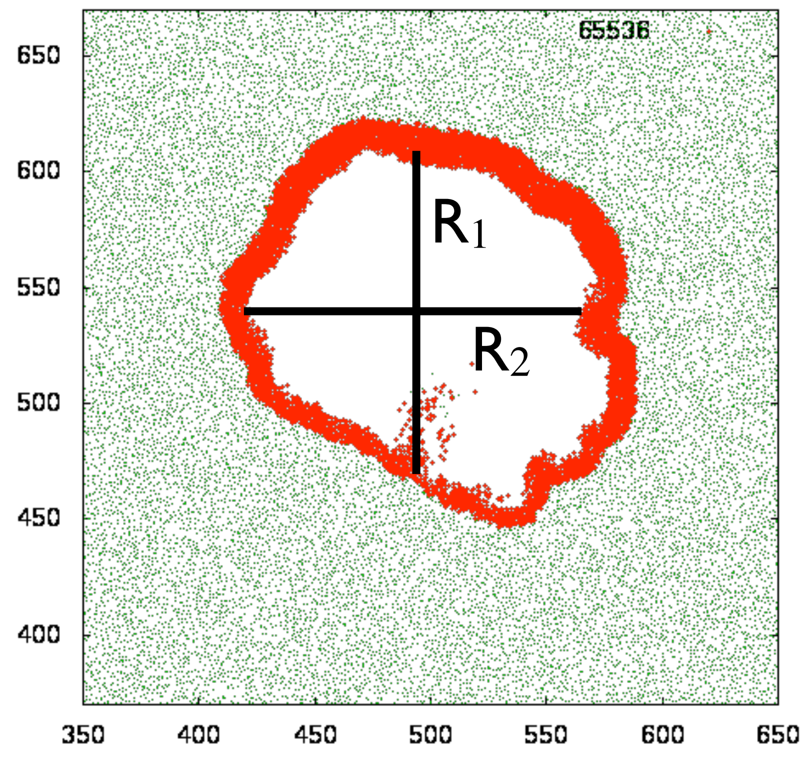
Let $R_t \sim t^{\alpha}$

Length scales



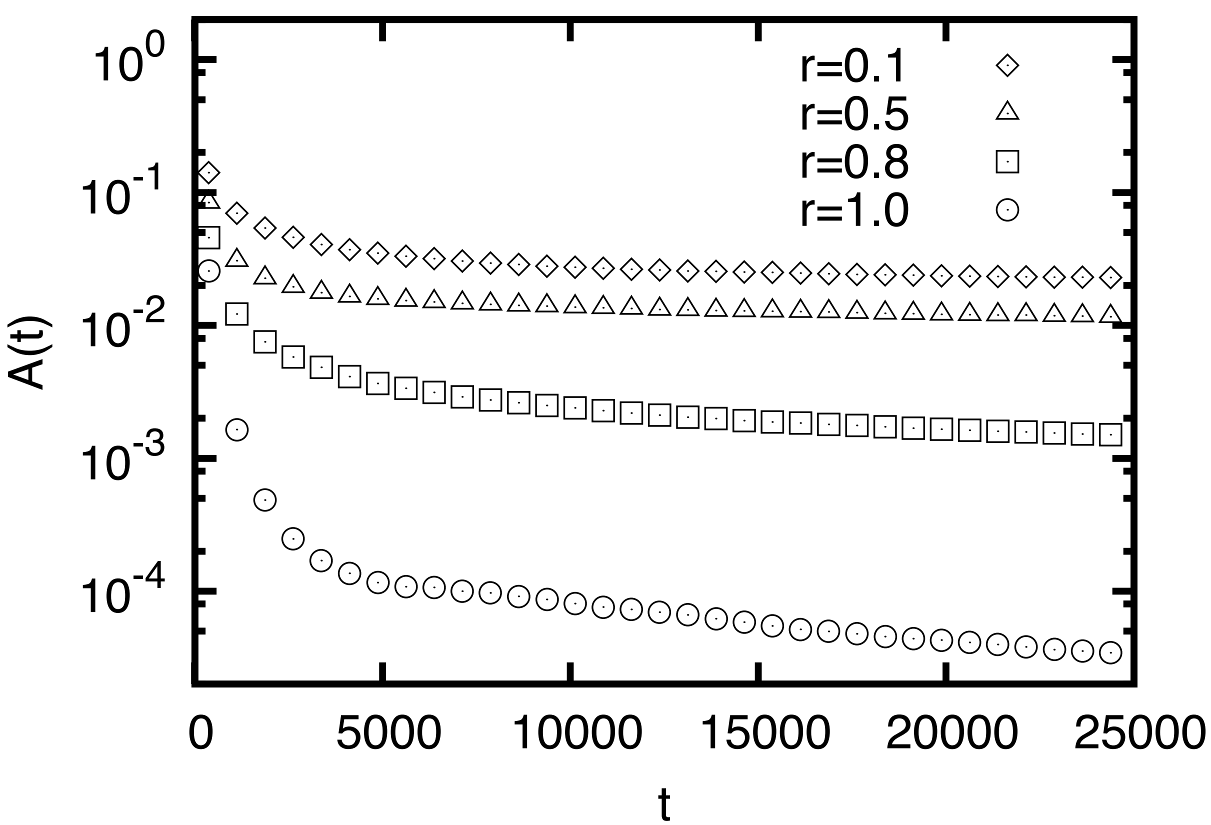
Do all these lengths scale as t^{α} ?

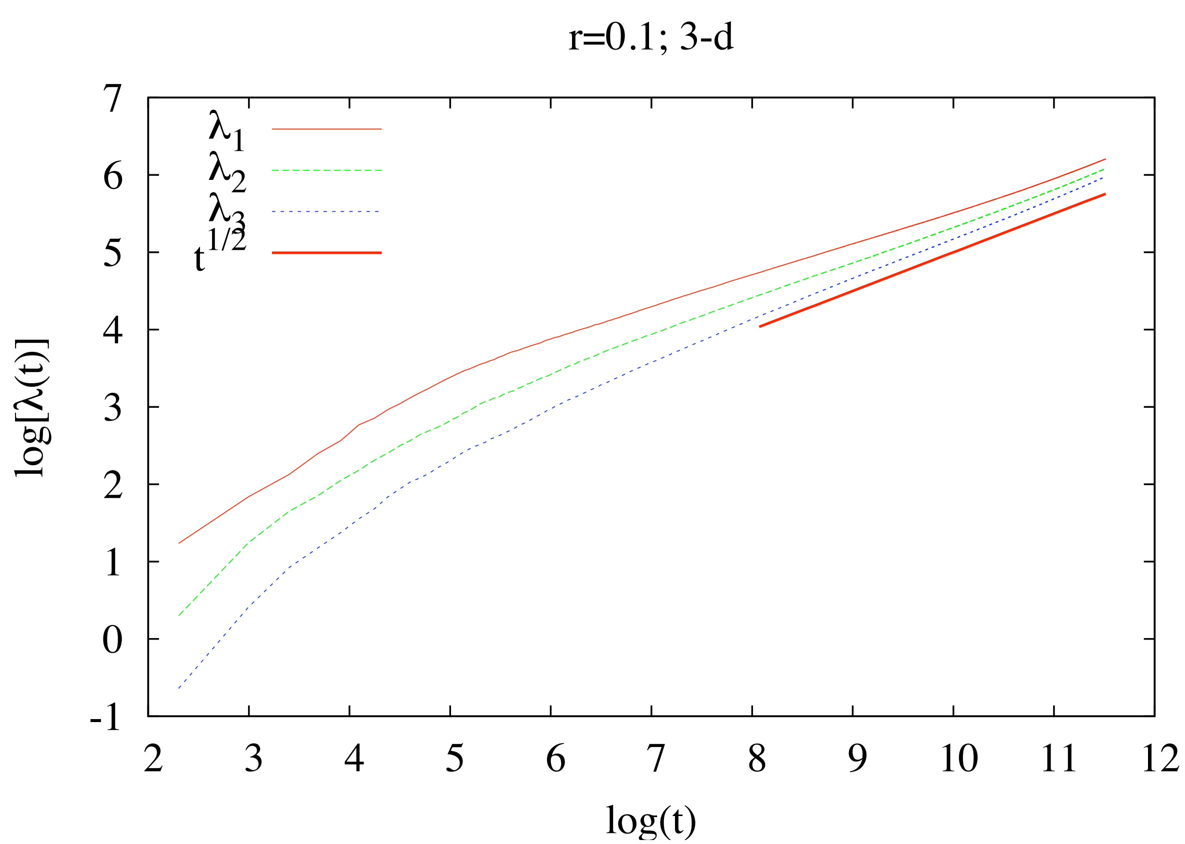
Length scales



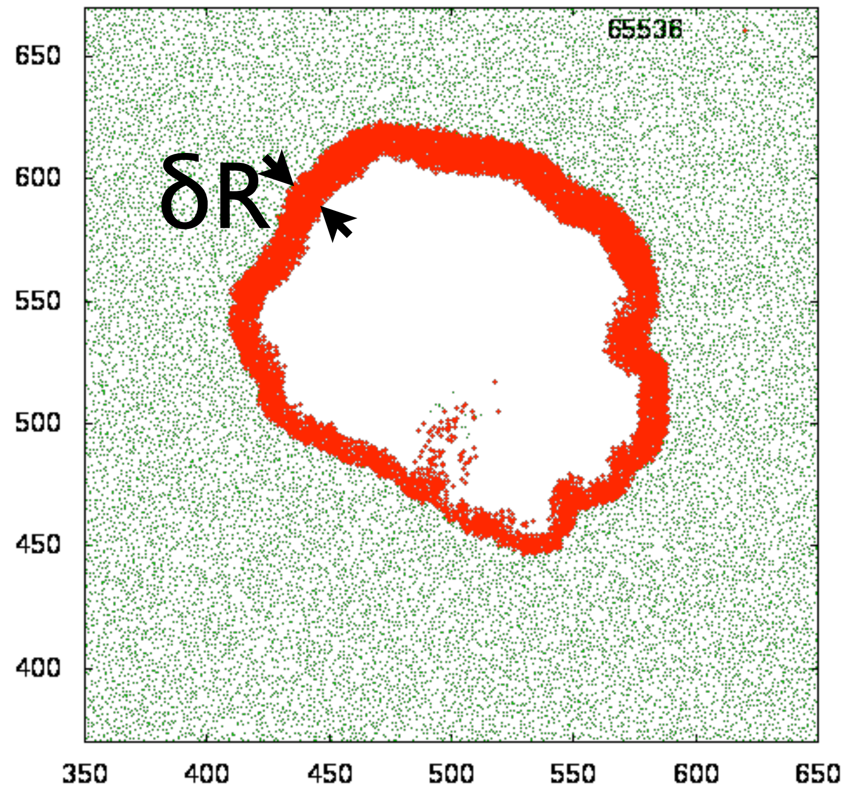
$$\begin{pmatrix} \langle R_x^2 \rangle & \langle R_x R_y \rangle \\ \langle R_x R_y \rangle & \langle R_y^2 \rangle \end{pmatrix}$$

$$A = \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \right)^2$$





Length scales

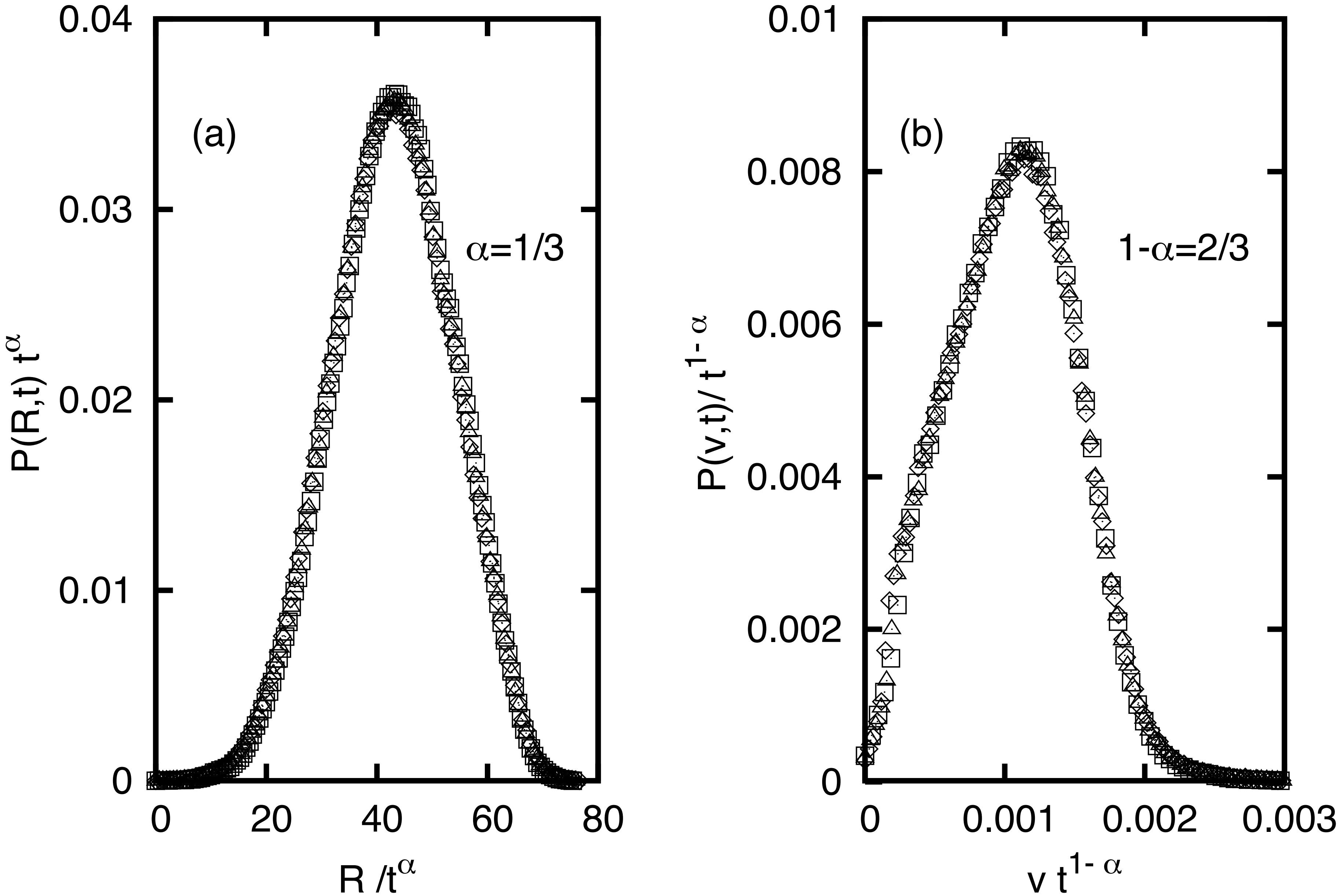


$$P(R,t) = f(R, \delta R, t)$$

$$P(R,t) = f(R,t)$$

$$P(R,t) = \frac{1}{R} g\left(\frac{R}{t^{\alpha}}\right)$$

Prob dist (2-D)



(a)
α=1/3
P(R,t) t^α
R /t^α
0.04
0.03
0.02
0.01
0
0
20
40
60
80
(b)
1-α=2/3
P(v,t)/ t^1- α
v t^1- α
0.01
0.008
0.006
0.004
0.002
0
0
0.001
0.002
0.003

Scaling analysis

$$\text{Let } R_t \sim t^\alpha$$

$$v_t = \frac{dR_t}{dt} \sim t^{\alpha-1}$$

$$N_t \sim R_t^d \sim t^{\alpha d}$$

$$E_t \sim N_t v_t^2 \sim t^{\alpha d + 2\alpha - 2}$$

Scaling (elastic limit)

$$E_t \sim N_t v_t^2 \sim t^{\alpha d + 2\alpha - 2}$$

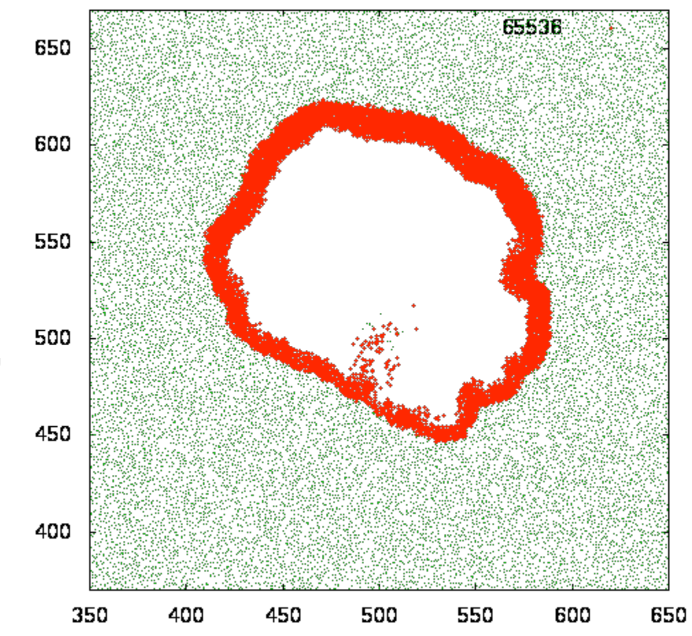
But energy is a constant

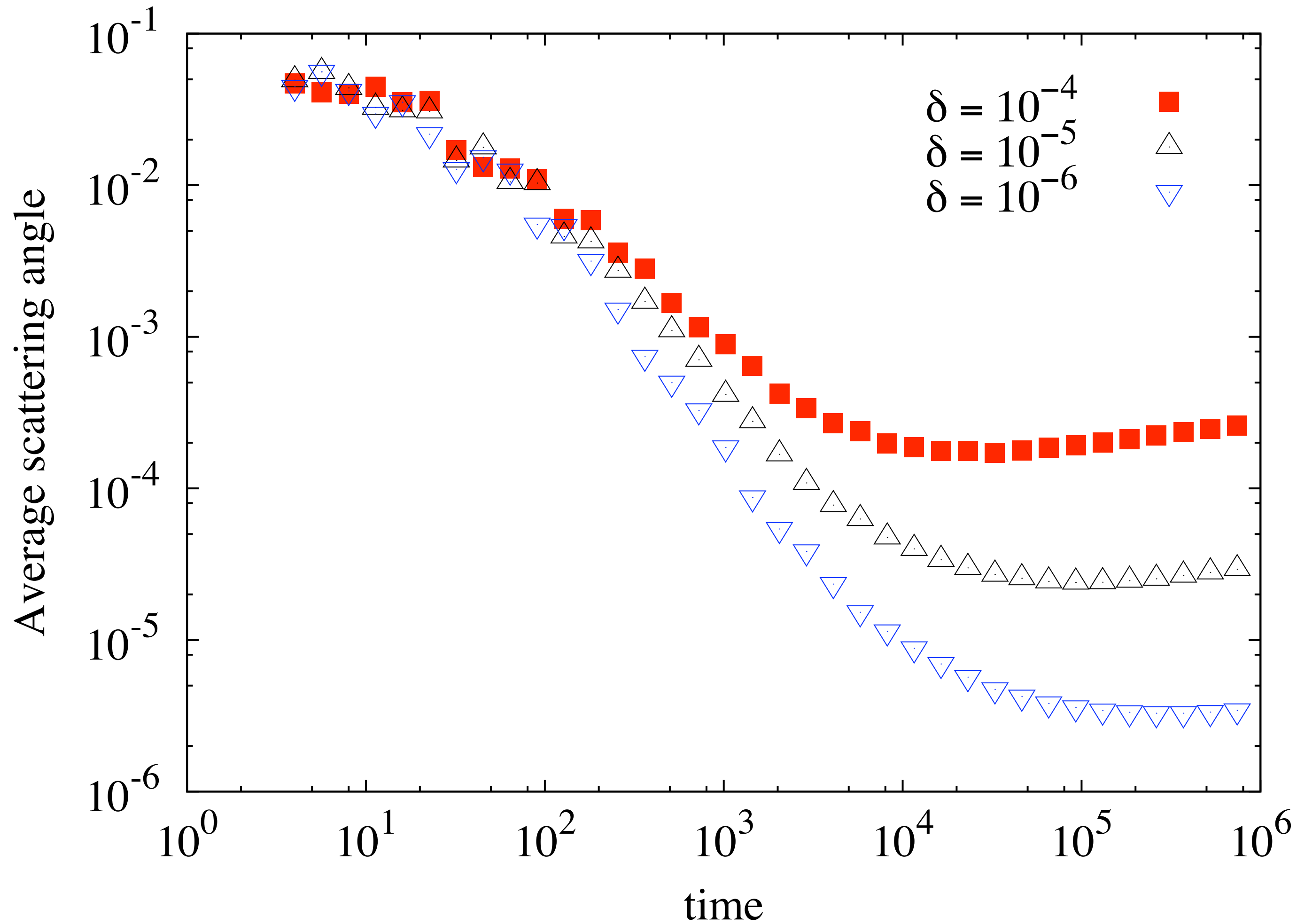
$$\alpha d + 2\alpha - 2 = 0$$

$$\alpha = \frac{2}{d+2}$$

Scaling (inelastic limit)

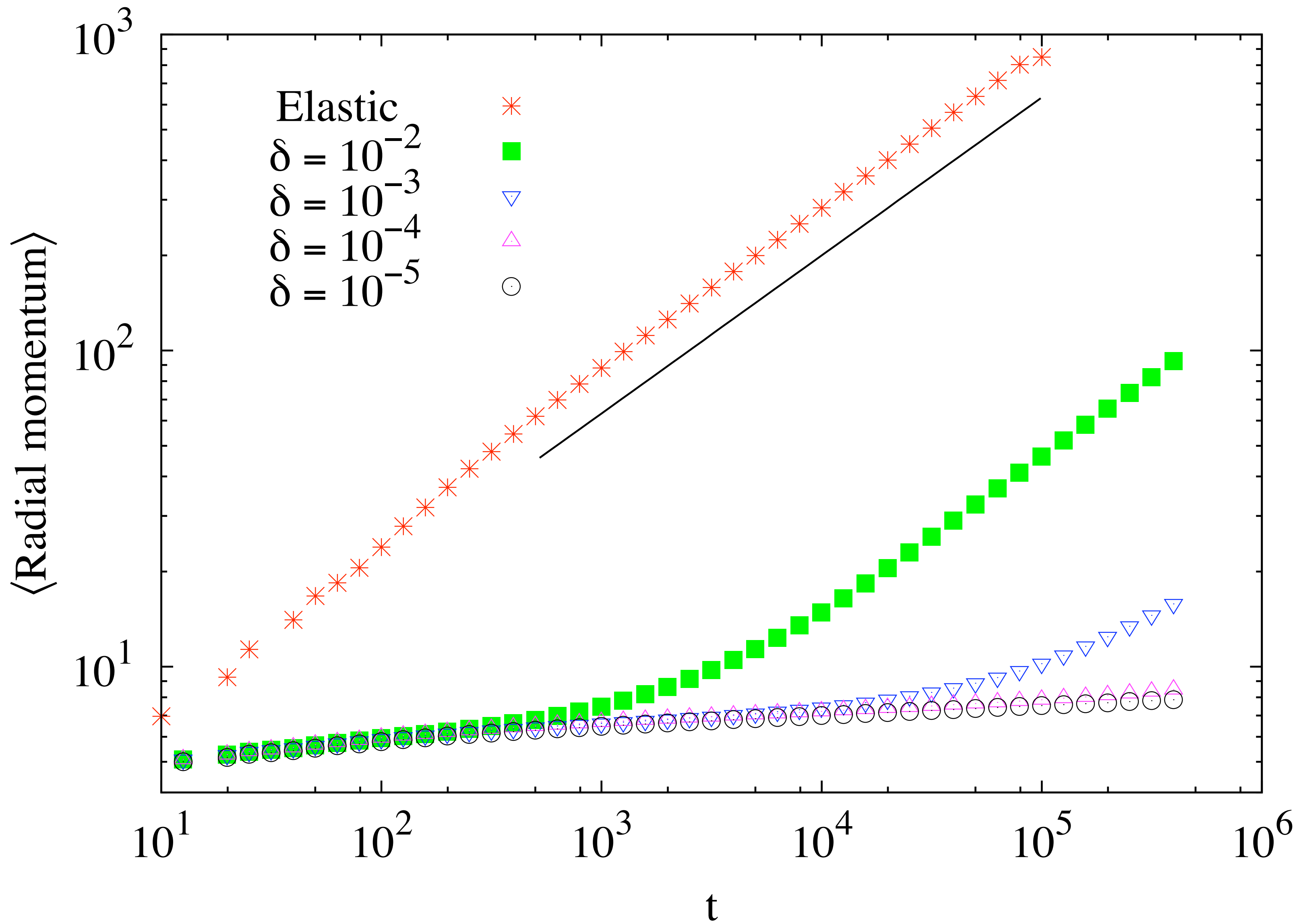
- clustering for all $r < 1$
- Particle direction remains constant





Radial Mometum

- radial momentum is conserved



Scaling (inelastic limit)

$$N_t v_t d\Omega = \text{constant}$$

$$\alpha d + \alpha - 1 = 0$$

$$\alpha = \frac{1}{d+1}$$

$$\alpha_{el} = \frac{2}{d+2}$$

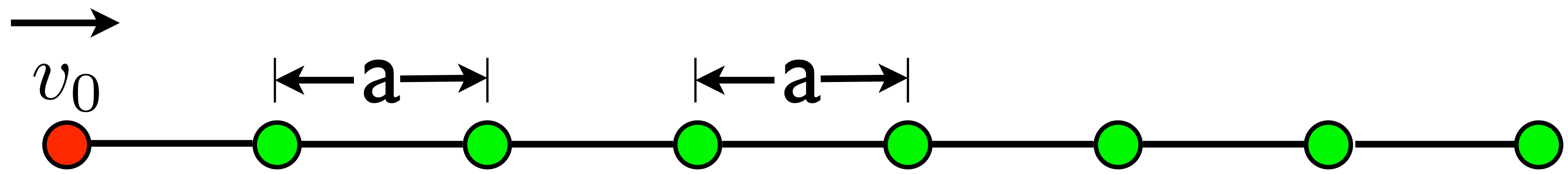
Let $R_t \sim t^{\alpha}$

$$v_t = \frac{dR_t}{dt} \sim t^{\alpha-1}$$

$$N_t \sim R_t^d \sim t^{\alpha d}$$

$$E_t \sim N_t v_t^2 \sim t^{\alpha d + 2\alpha - 2}$$

A calculation in one dimension



Special case: r=0 [sticky limit]

After $m-1$ collisions, mass of particle is m

$$\text{Momentum conservation} \implies v_{m-1} = \frac{v_0}{m}$$

$$t_m = \frac{a}{v_0} + \frac{a}{v_1} + \ldots + \frac{a}{v_{m-1}} = a \sum_{i=1}^{m} \frac{i}{v_0} \propto m^2$$

$$m \sim N_t \sim R_t \sim \sqrt{t}$$

$$\alpha = \frac{1}{2}, \quad d = 1$$

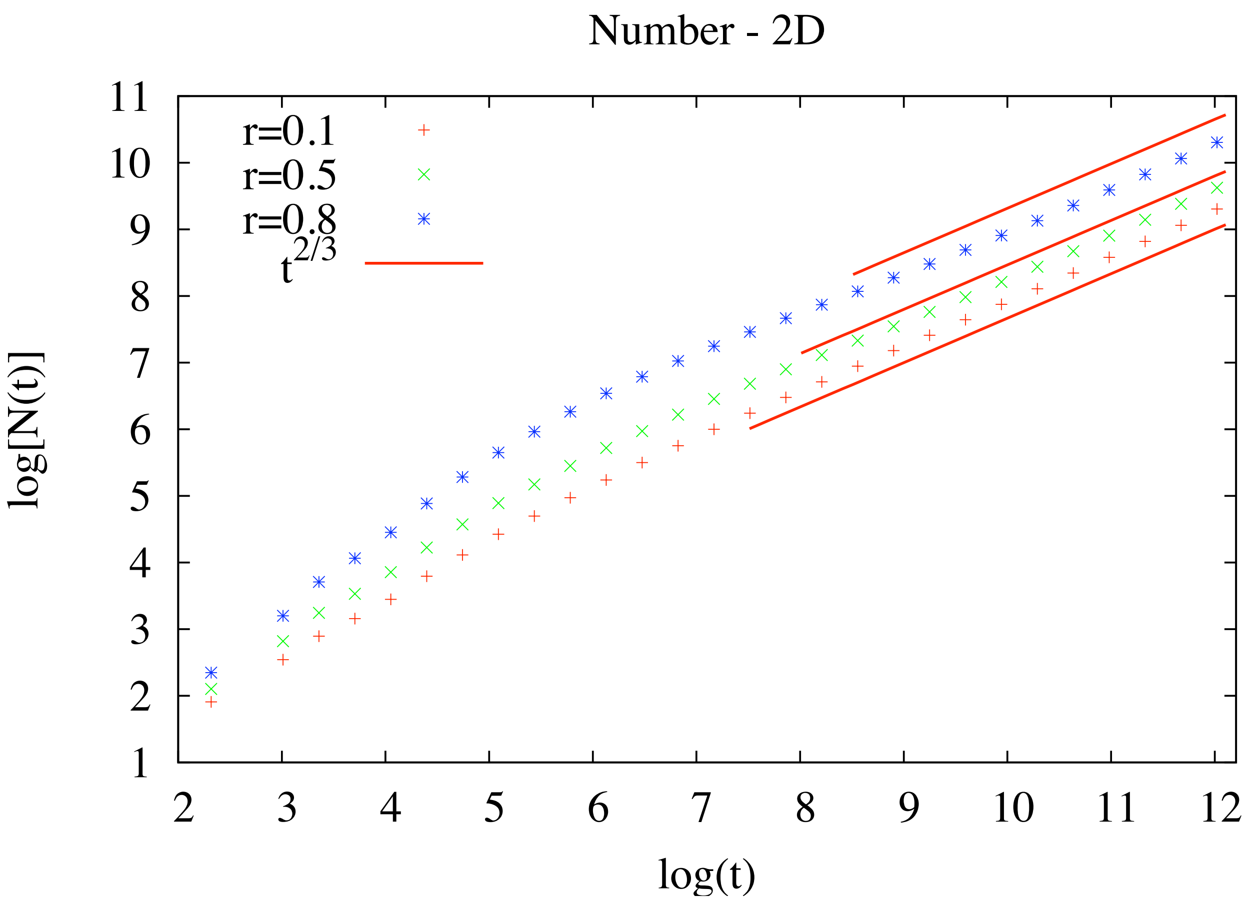
Recall

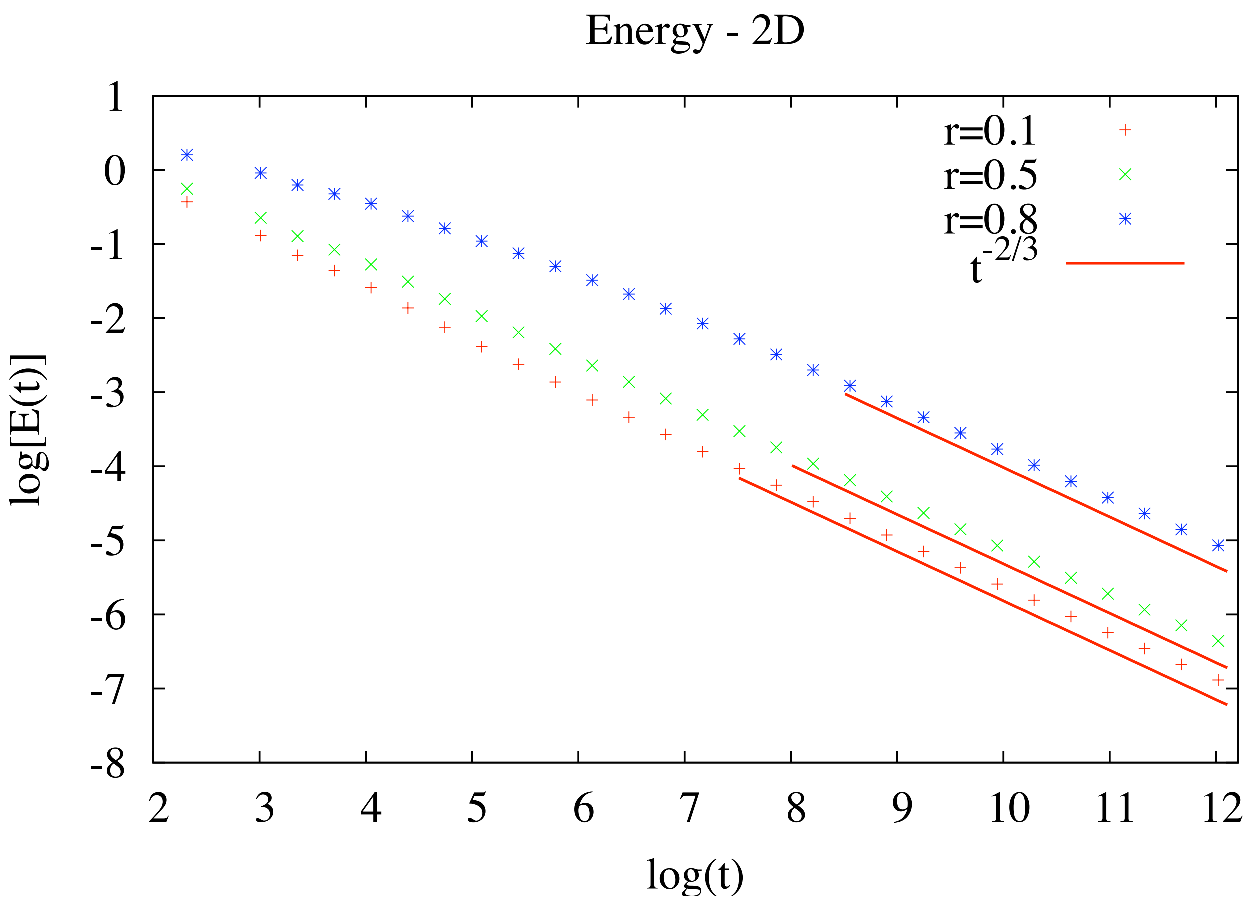
$$\alpha = \frac{1}{d+1}$$

Simulation-2d

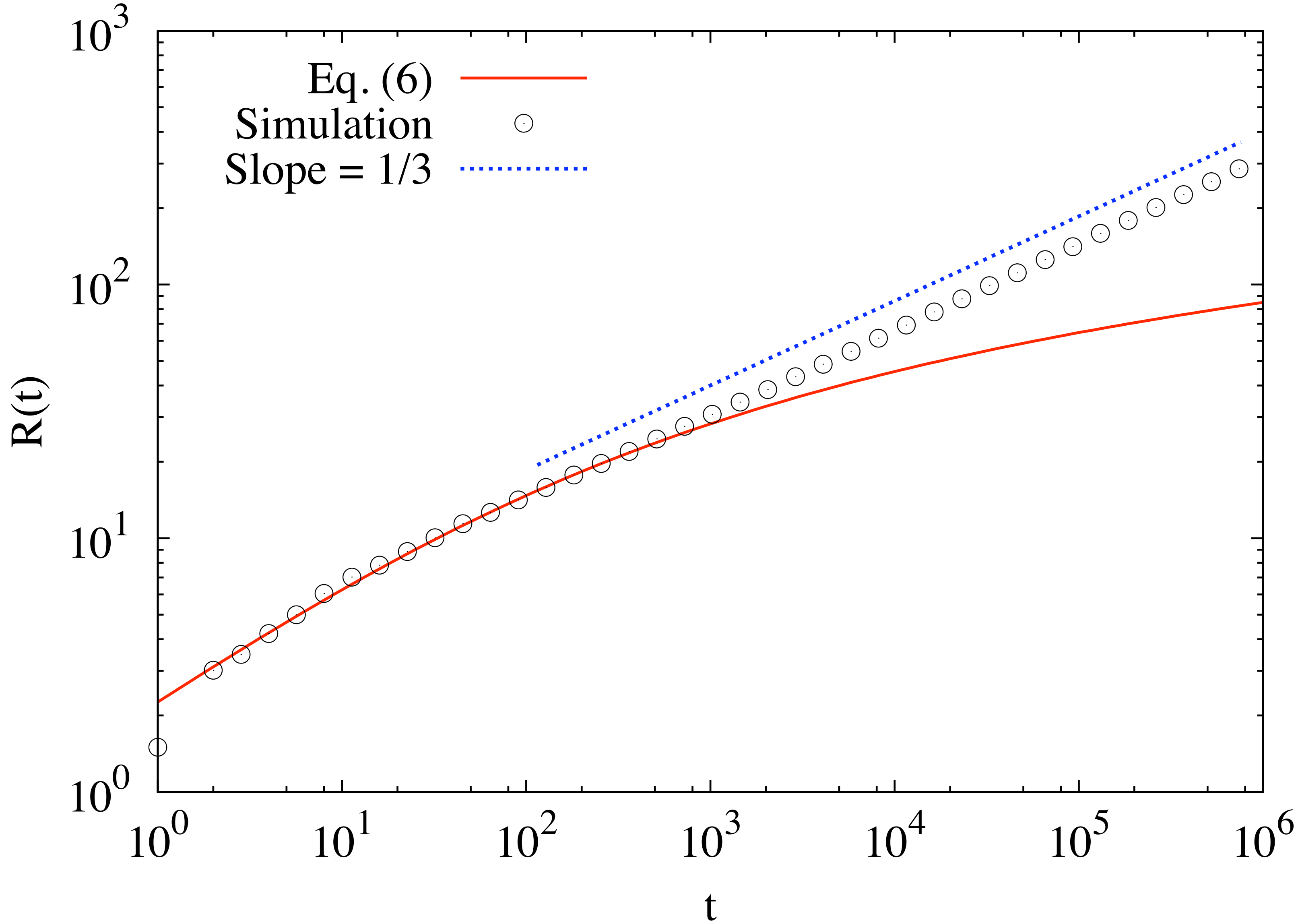
$$\langle N(t)\rangle \sim t^{2/3}$$

$$\langle E(t)\rangle \sim t^{-2/3}$$





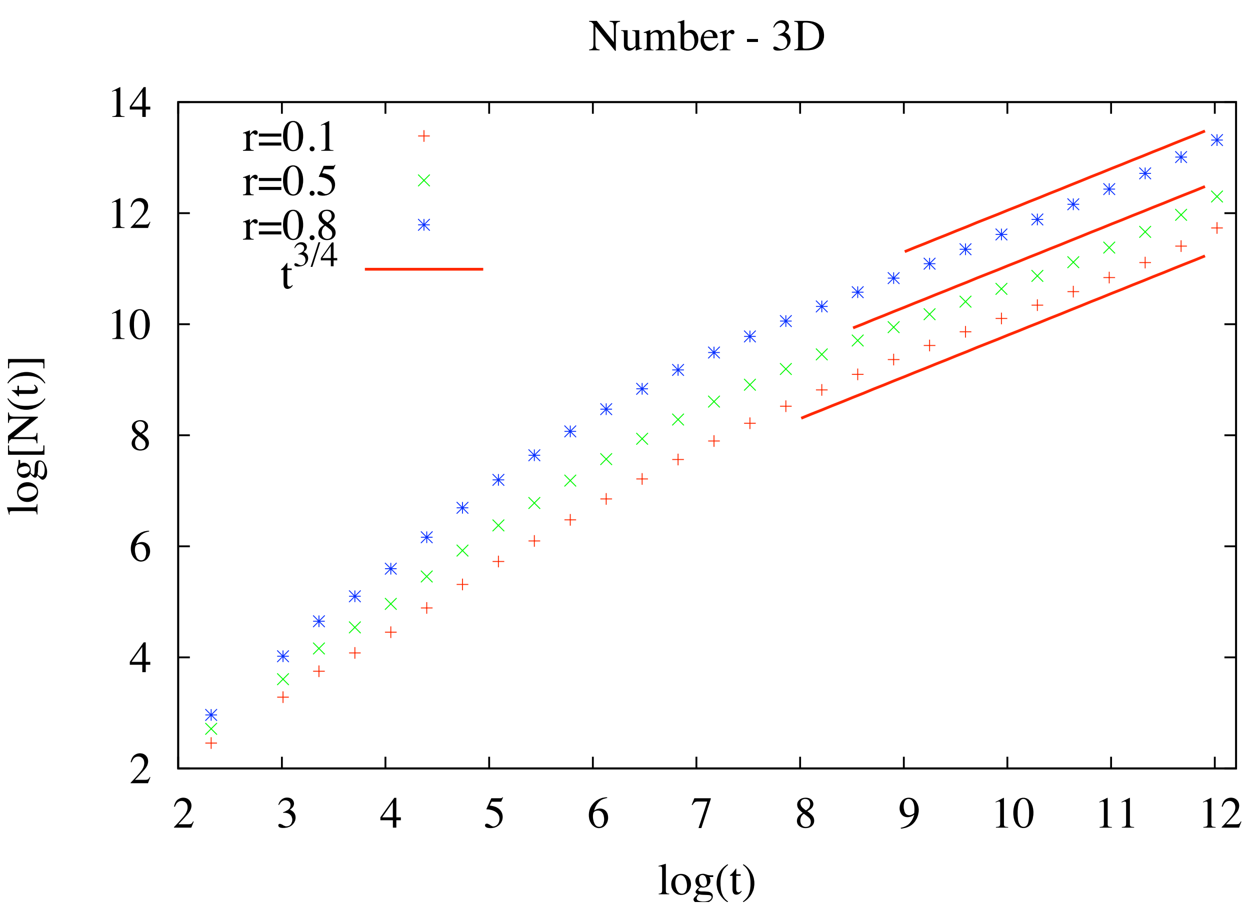
Comparison with kinetic theory

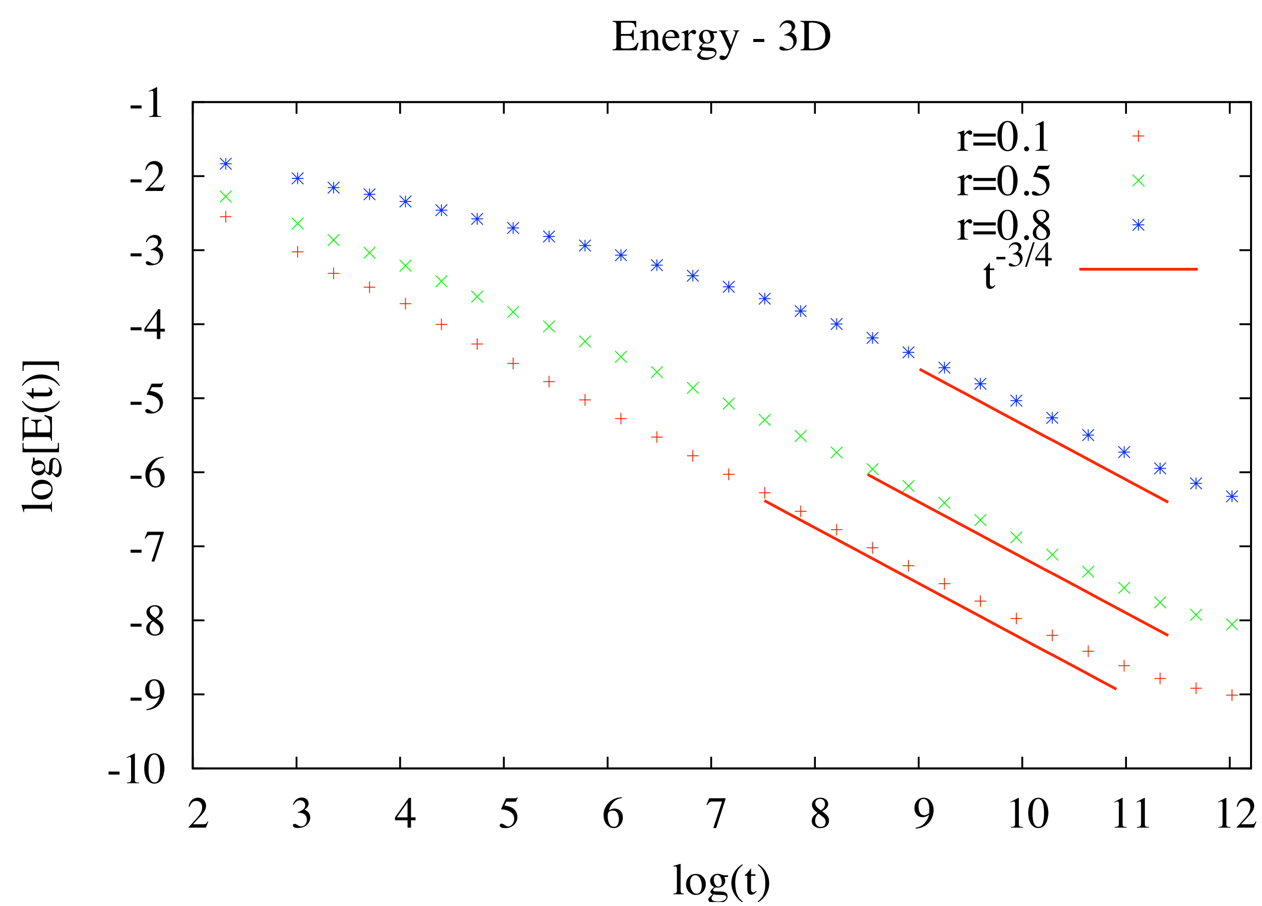


Simulation-3d

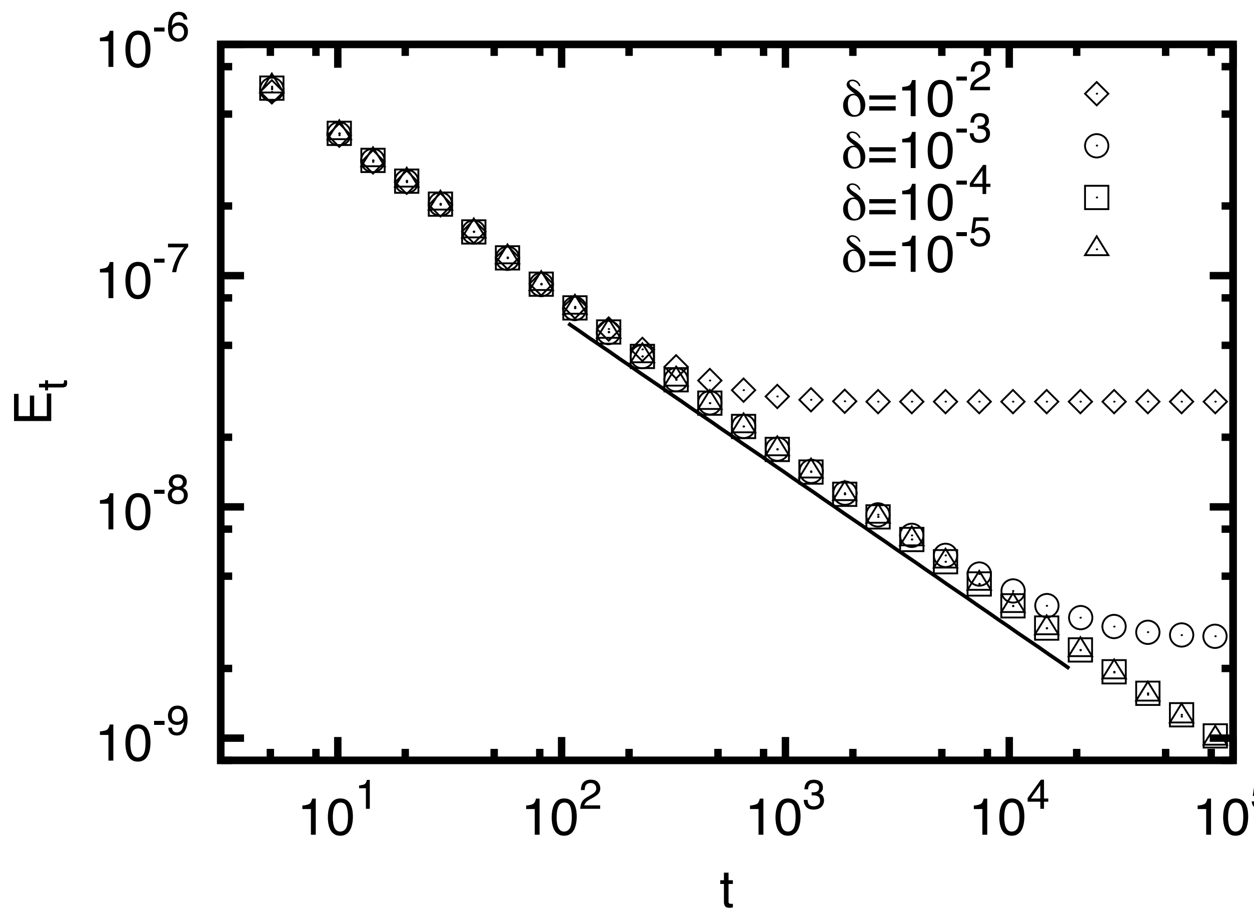
$$\langle N(t) \rangle \sim t^{3/4}$$

$$\langle E(t) \rangle \sim t^{-3/4}$$

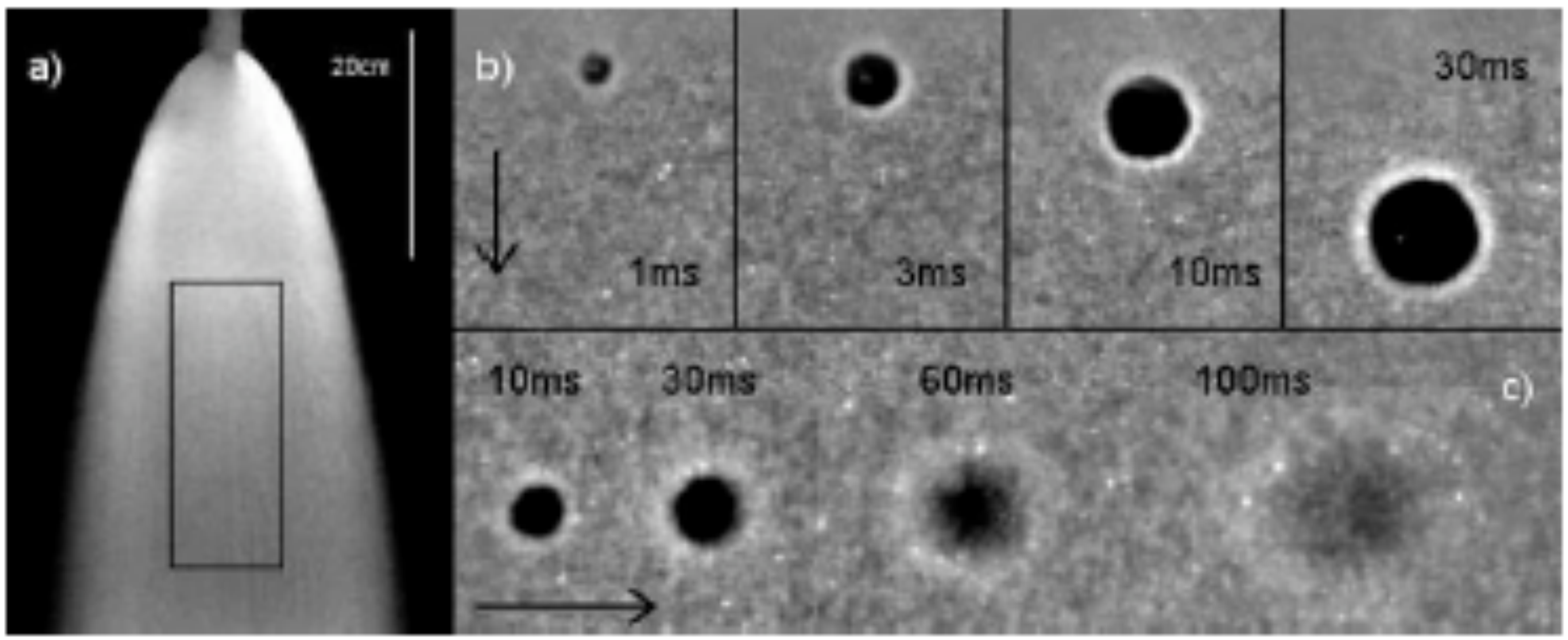




δ-dependence

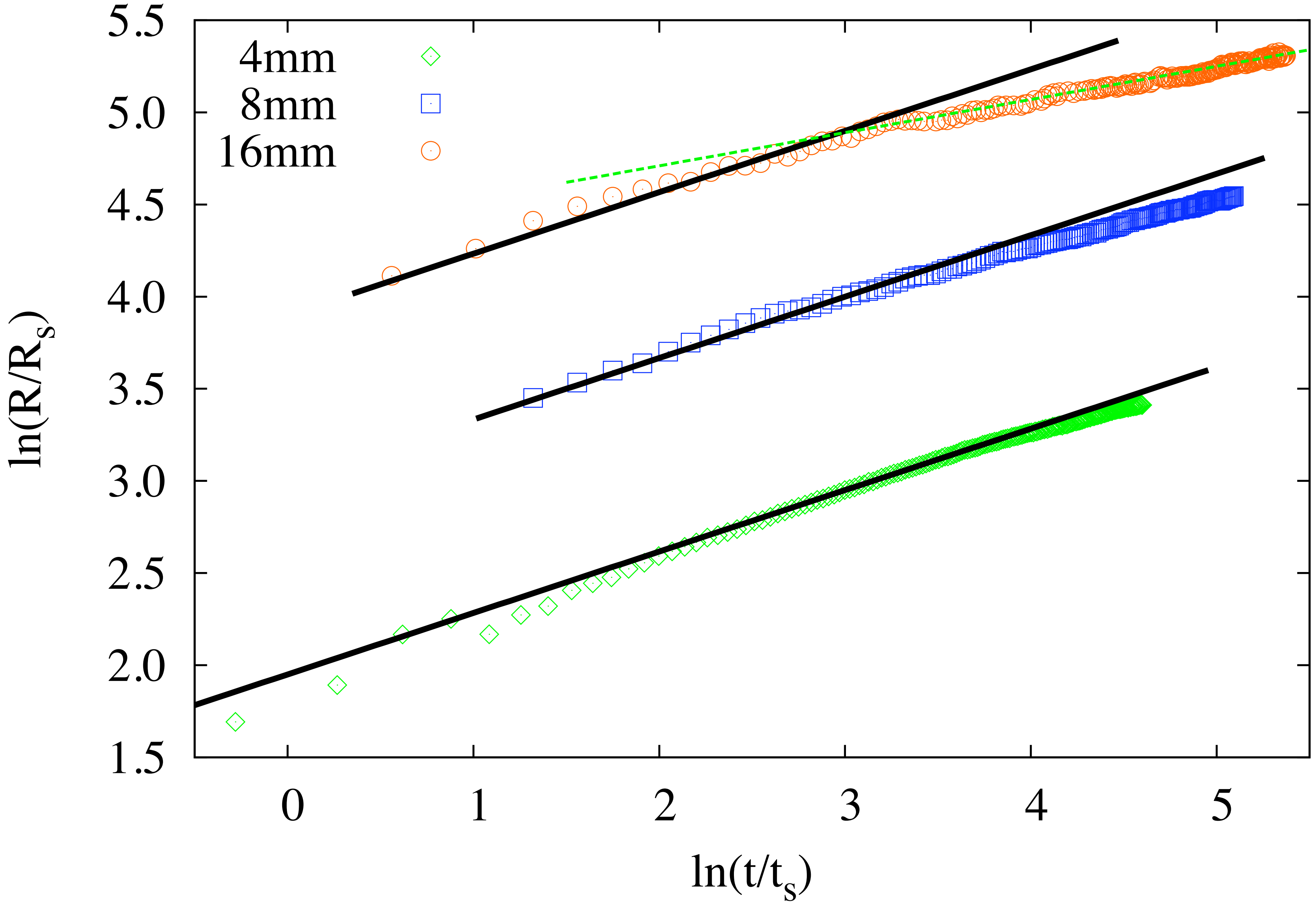


Experiments

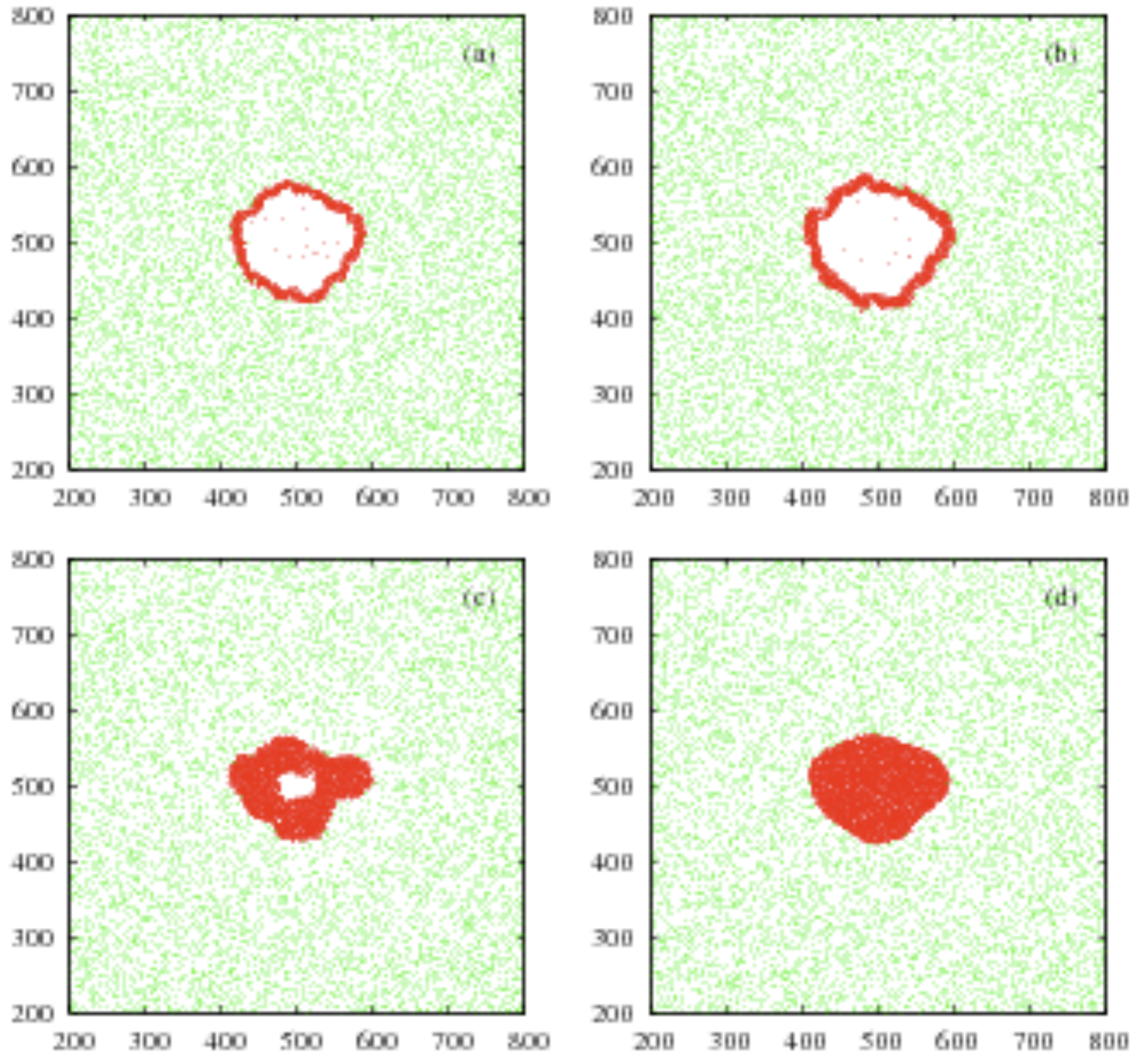


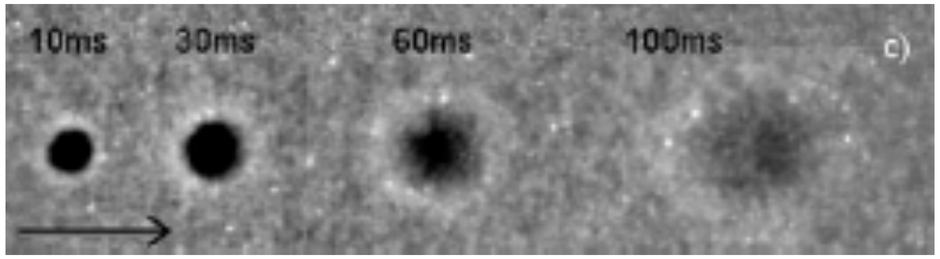
Boudet et al, PRL 2009

Data (Shock)

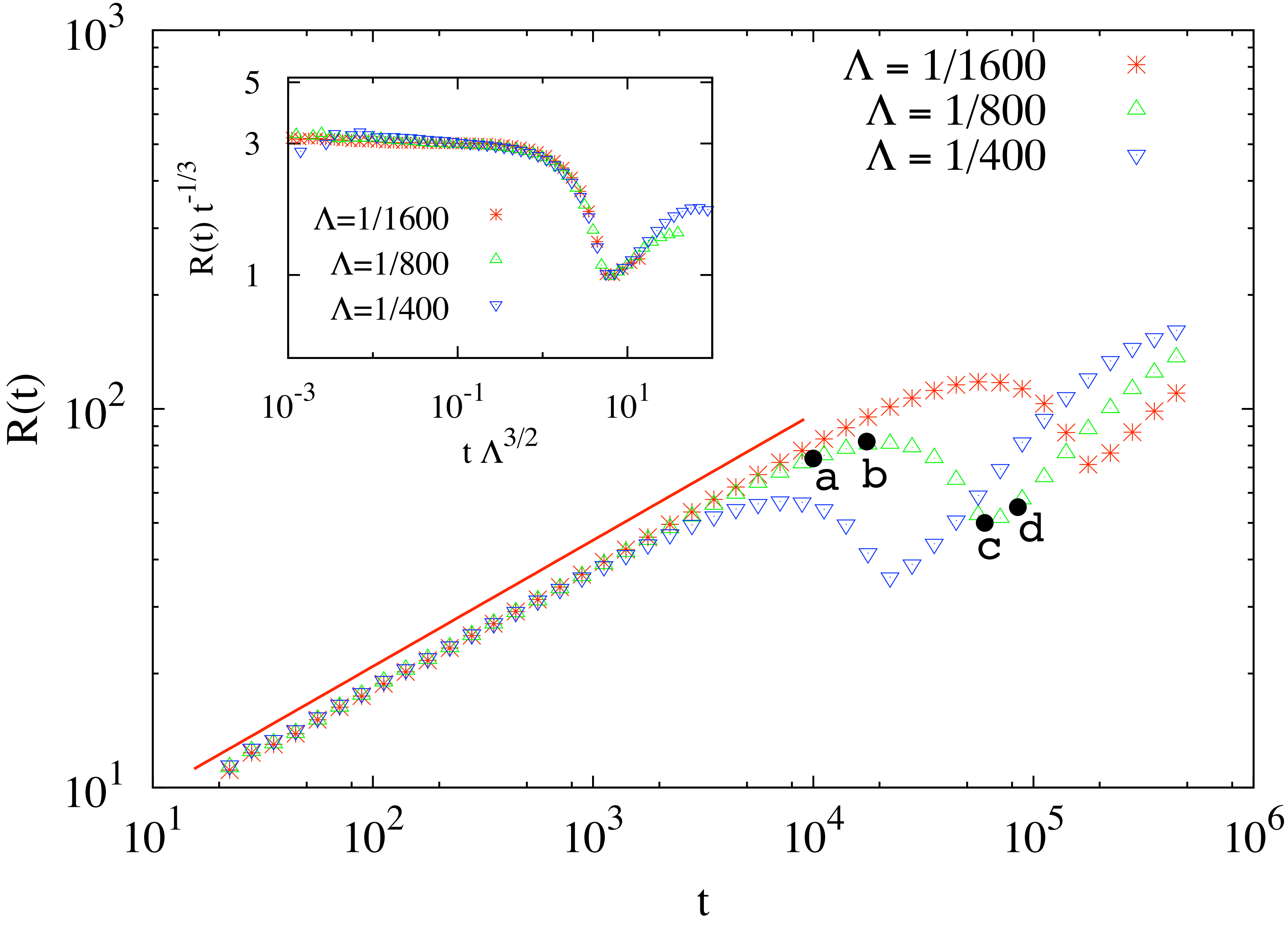


Non-zero ambient temperature

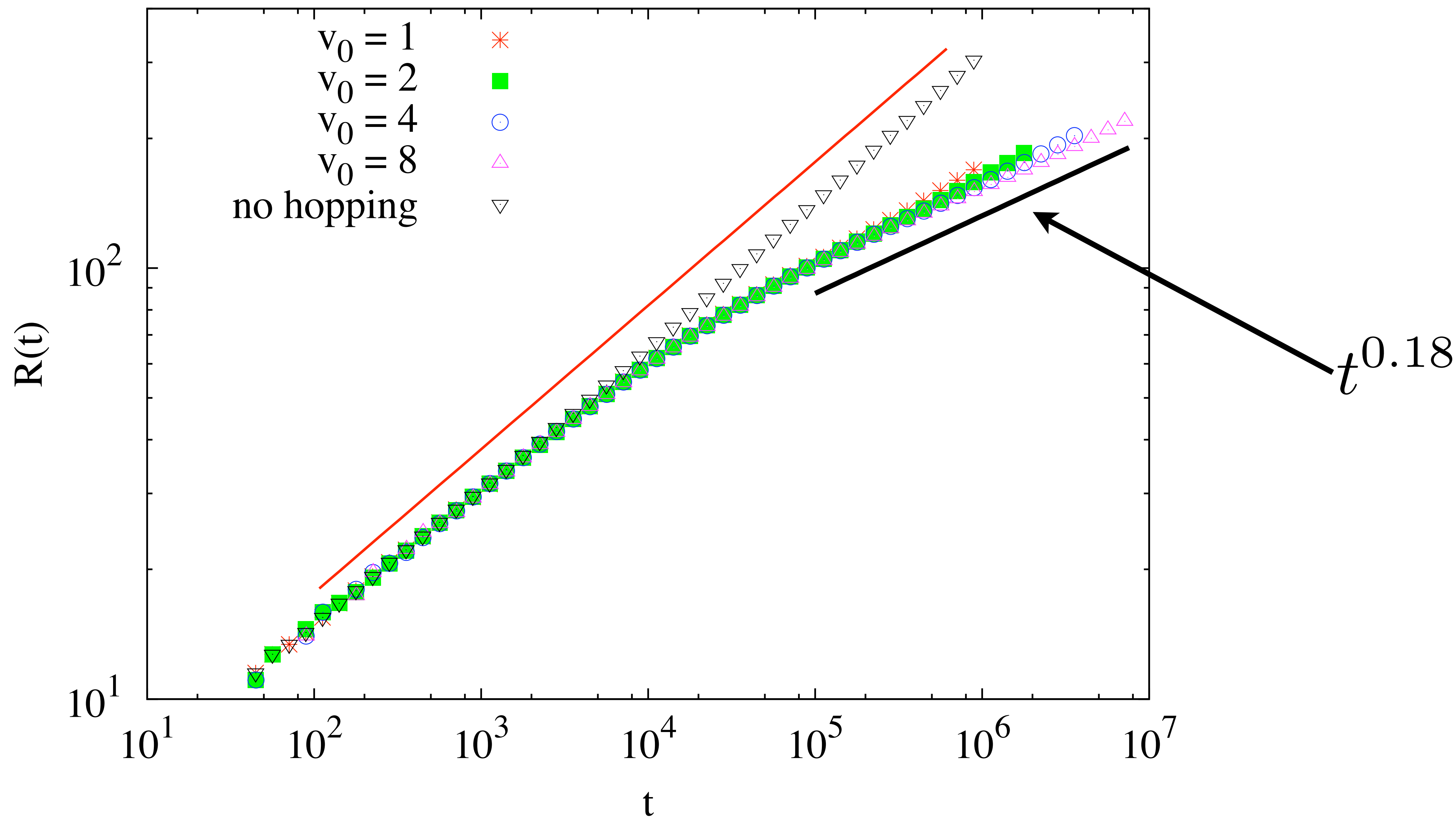




Non-zero ambient temperature

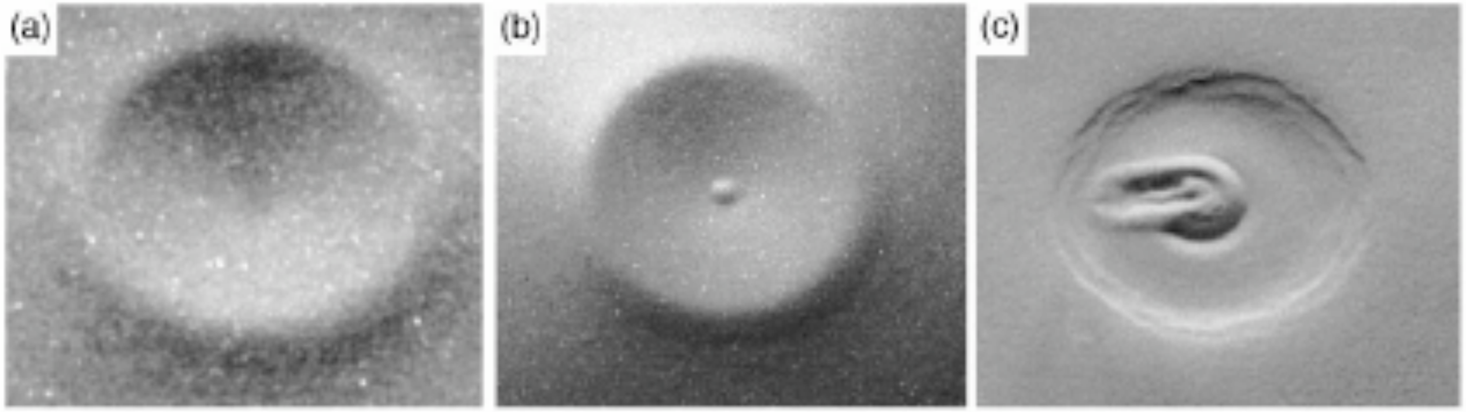


Model with escape rate



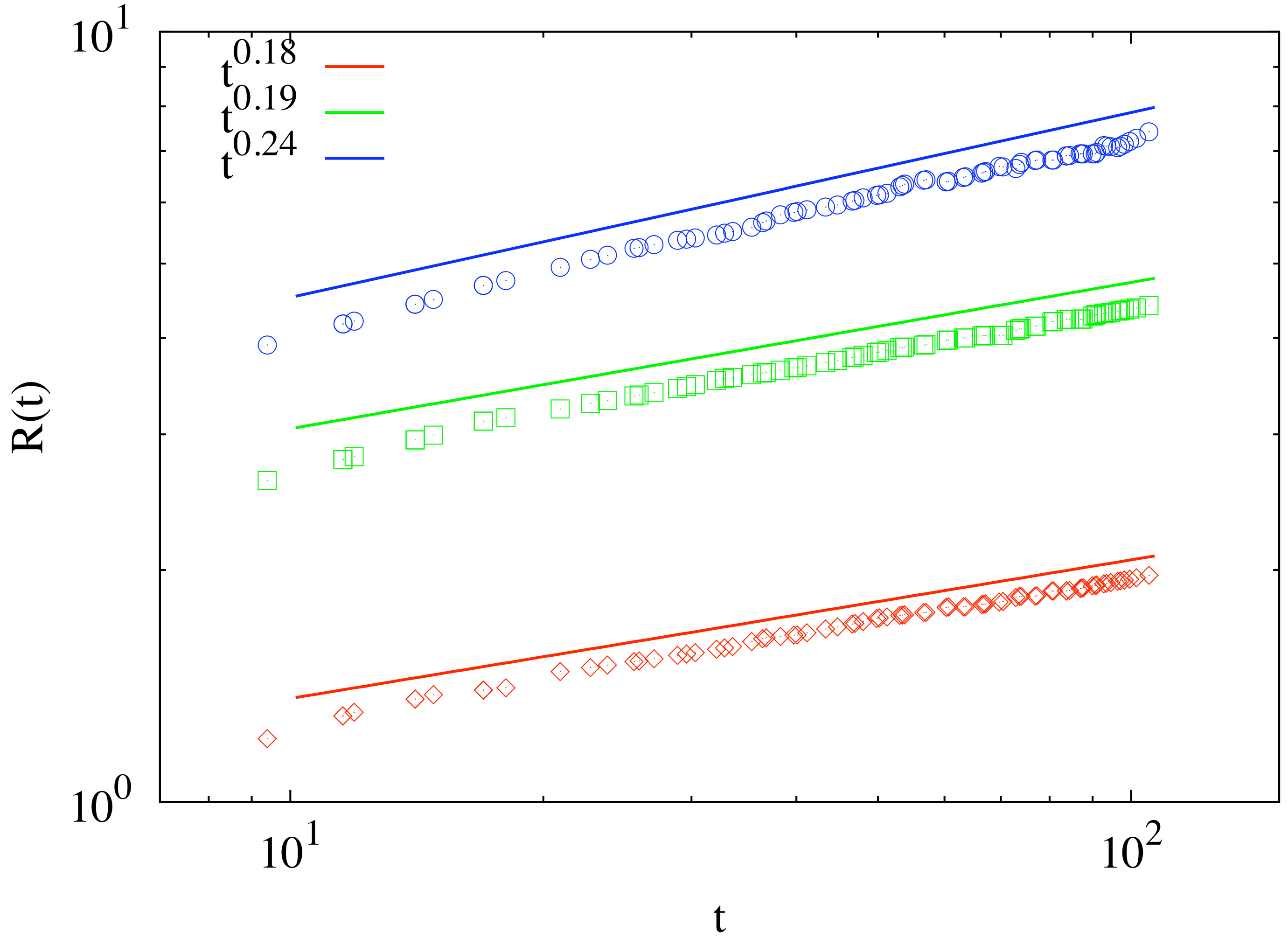
Experiment

Crater formation



Walsh et al, PRL 2003

Data (Crater)

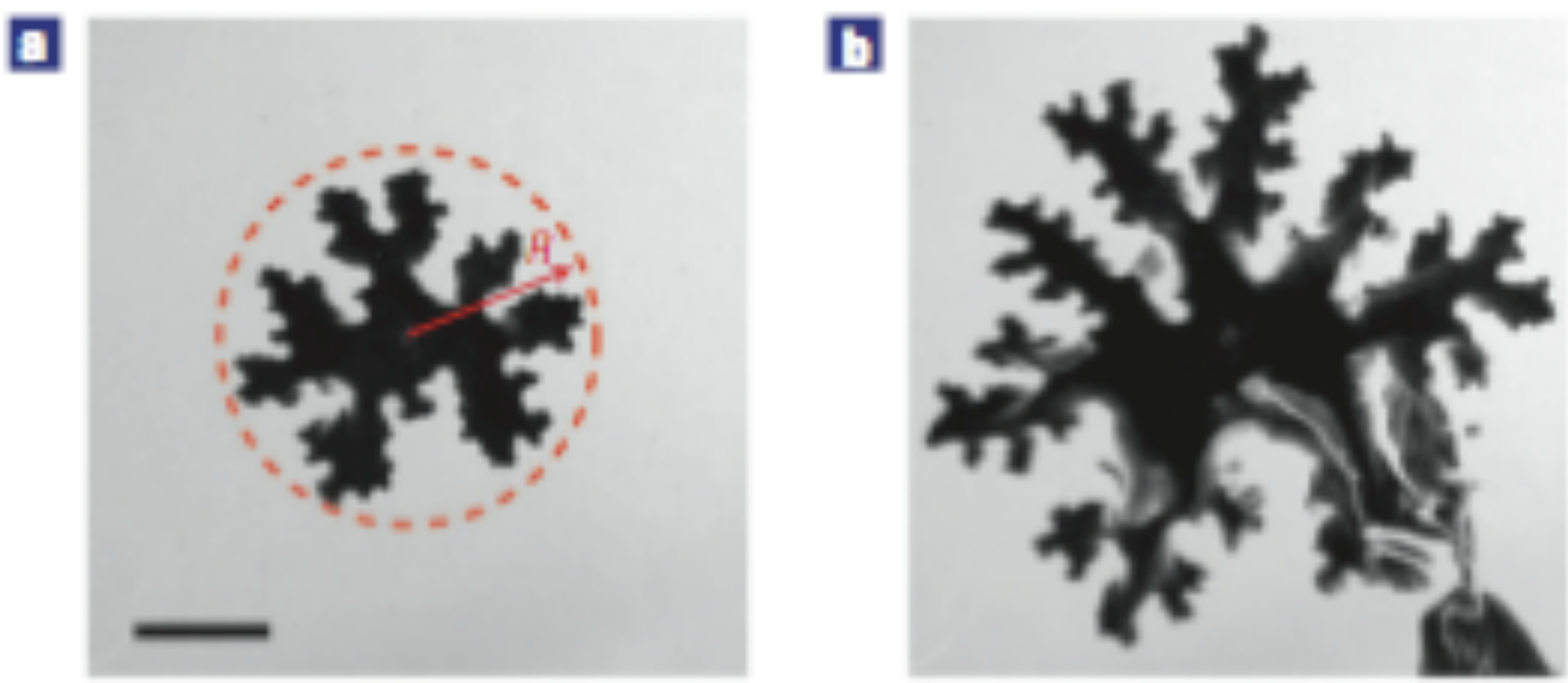


Summary & Outlook

- A generalization of the Taylor-Sedov problem
- Inelastic ⇒ clustering and band formation
- Conservation of radial momentum
- New exponents independent of r
- Describes experimental data well

Summary & Outlook

- Understanding crossovers
- Can the freely cooling gas be understood?
- Related experiments



Cheng et al, Nature Phys, 2008