STANDARD LEPTOGENESIS

PROBIR ROY

SINP

GENERAL REF:

Büchmüller & Plumächer
– Phys. Rep. 320 (1999) 329

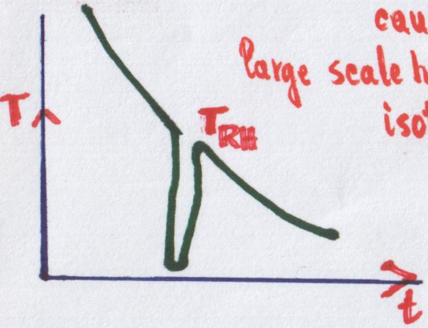
- Baryon asymmetry in concordance cosmology
- Baryogenesis in SM & BSM
- Heavy right-handed neutrinos & seesaw
- Standard leptogenesis with Majorana neutrinos
- Washout issues
- Boltzmann equations
- Sphaleronic conversion : $\Delta L \longrightarrow \Delta B$
- Conclusion

$c = \hbar = 1$; $k_B = 8.617 \times 10^{-5}\, eV \leftrightarrow 1$

• Baryon asymmetry in concordance cosmology (ΛCDM)

CC = BB $\longrightarrow$ temp. $\sim 10^{19}$ GeV

Books: Weinberg, Dodelson



$\downarrow$

Hubble expansion $\longrightarrow$ as per FRW + Λ-term

$\downarrow$

causality vs. large scale homogeneity, isotropy

Inflation $\longrightarrow$ rolling down of inflaton field, temp. 10^{14}–10^{16} GeV. $R(t) \propto e^{Ht}$. Primordial asymmetries diluted?

Reheating $\longrightarrow$ inflaton decay?

$\downarrow$

Radiation dominated Universe $\longrightarrow$ all elementary particles & quanta in chemical equilibrium (balance between creation & annihilation): thermal bath

$R(t) \propto t^{1/2}$

BSM

$n_\gamma = \frac{\zeta(3)}{\pi^2} \cdot 2 \cdot T^3$ Bose gas

Freeze-out of different particle species begins $\longrightarrow$ depending on strength and range of interactions

$\downarrow$

BBN $\longrightarrow$ $T \sim O(\text{MeV})$. ^{2}H, ^{3}He, ^{4}He, ^{7}Li ... form
$t \sim 3$ min.

$\downarrow$

Matter domination starts $\longrightarrow$ $T \sim O(10\,\text{eV})$

$R(t) \propto t^{2/3}$

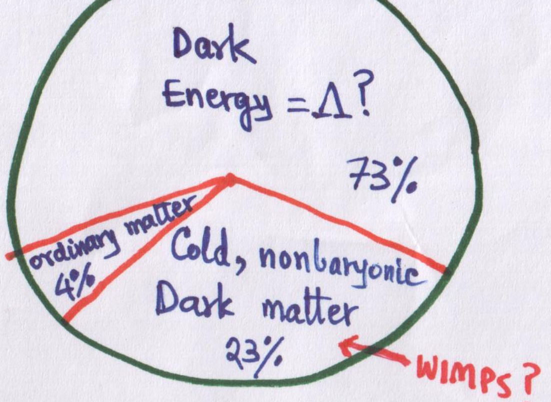
$\downarrow$

CMB decouples $\longrightarrow$ $T \sim 0.3$ eV, $t \sim 380000$ yrs., e^-'s, p's form H-atom

Λ-term takes over $\longrightarrow$ $T \sim O(1)$ meV, dark energy domination

TODAY $\longrightarrow$ $T \sim 0.24$ meV.

Universe energy budget



$\sim 0.1 - 2\%$ of DM is "hot" (neutrinos) $\longleftrightarrow$ $\sum_i m_{\nu_i} < 0.28\,\mathrm{eV}$ large scale galactic red-shift survey.

Particle – antiparticle production symmetric. Antimatter = ?

few $\bar{p}$'s in cosmic rays explained in terms of pair creation from interstellar dust or atmospheric nuclei

No credible mechanism of matter-antimatter separation.

No antimatter galaxies!

Review: Steigman, arXiv:0808.1122

Dimensionless measure of matter excess over antimatter: $\eta_B = \frac{n_B - \bar{n}_B}{n_\gamma} \simeq \frac{n_B}{n_\gamma} \quad \because \frac{\bar{n}_B}{n_B} < 10^{-9}$?

Range estimated from BBN $\longrightarrow$ observed abundance of light elements:

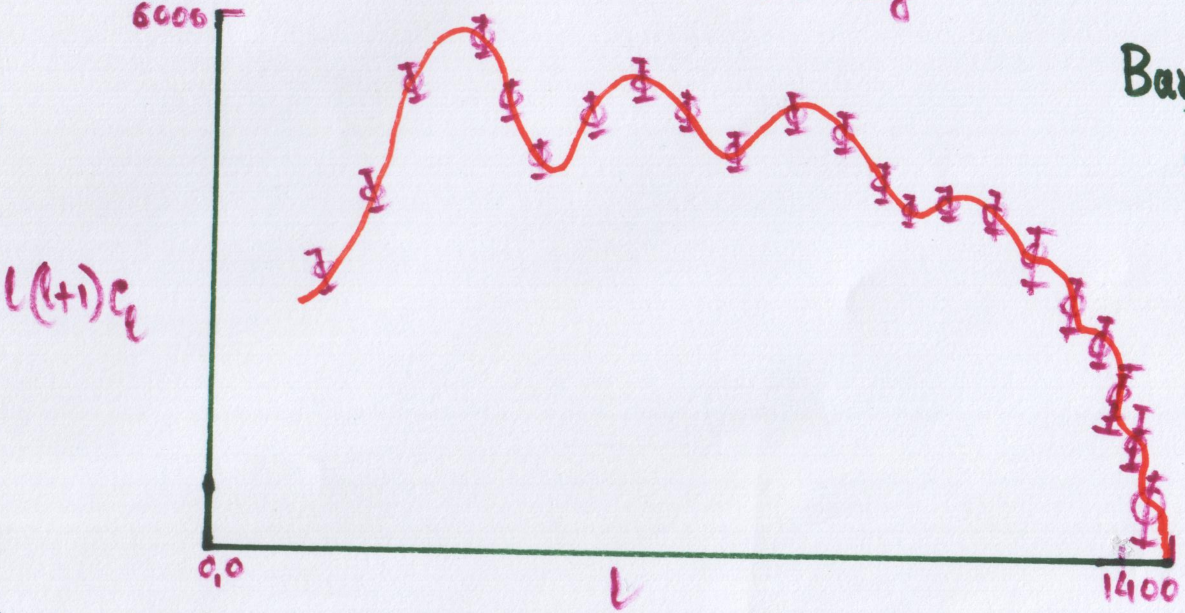
$$4.7 \times 10^{-10} \lesssim \eta_B^{BBN} \lesssim 6.5 \times 10^{-10} \text{ at } 95\% \text{ c.l.}$$

Better determination from CMB by WMAP & South Pole Telescope

On top of blackbody radiation angular anisotropies due to fluctuations in thermal bath.

$$\frac{\Delta T}{T} = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi), \quad C_l = \langle |a_{lm}|^2 \rangle$$: azimuthal avg.

Angular power spectrum of CMB



Baryon acoustic oscillations

Fit between theory & expt. depends on parameter η_B. Best fit $\Rightarrow$

$$\eta_B = (6.225 \pm 0.17) \times 10^{-10}$$

Sakharov conditions

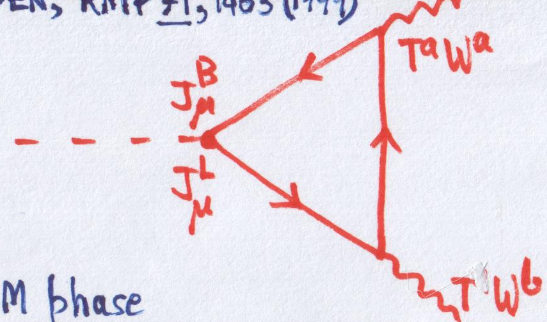
- $\not{B}$: need transition : $B=0 \longrightarrow B \neq 0$. $\quad B = \int d^3x\, J_0^B,\ J_\mu^B = \frac{1}{3}\sum_i (\bar{q}_{Li}\gamma_\mu q_{Li} - \bar{u}^c_{Li}\gamma_\mu u^c_{Li} - \bar{d}^c_{Li}\gamma_\mu d^c_{Li})$
- $\not{C}\not{P}$ & $\not{C}$ Need $|M(i\to f)|^2 \neq |M(\bar{i}\to\bar{f})|^2$ summed over d.f. $\quad L = \int d^3x\, J_0^L,\ J_\mu^L = \sum_i (\bar{l}_{Li}\gamma_\mu l_{Li} - \bar{e}^c_{Li}\gamma_\mu e^c_{Li})$
- Departure from thermal equilibrium or $\langle B(t)\rangle = \mathrm{Tr}\,\rho\, B(t) = \mathrm{Tr}\, e^{-\beta H} e^{iHt} B(0) e^{-iHt} = \langle B(0)\rangle$.

- **Baryogenesis in SM and BSM**

TRODDEN, RMP 71, 1463 (1999)

SM:

$\cancel{B}$ from 't Hooft anomaly in SM:



$$\partial^\mu J^B_\mu = \partial^\mu J^L_\mu = \frac{N_g}{32\pi^2}\left(g^2 W^a_{\mu\nu}\tilde{W}^{\mu\nu a} + g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon^{abc} W^b_\mu W^c_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad \tilde{B}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} B^{\rho\sigma}$$

$$\tilde{W}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} W^{\rho\sigma a}$$

$\cancel{C}$ from V–A weak interactions, $\cancel{CP}$ from CKM phase

$|\epsilon_{CKM}| \sim 2.3\times 10^{-3}$ from K^0–$\bar{K}^0$ transition

smaller from B^0–$\bar{B}^0$ transition

Departure from thermal equilibrium at EW phase transition.

$$\partial^\mu (J^B_\mu - J^L_\mu) = 0\ ;\ \partial^\mu (J^B_\mu + J^L_\mu) = 2N_g\,\partial_\mu K^\mu$$

↑ CS current

$$K^\mu = -\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta} W^a_\nu \left(\partial_\alpha W^a_\beta + \frac{g}{3}\epsilon^{abc} W^b_\alpha W^c_\beta\right) + \frac{g'^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta} B_\nu B_{\alpha\beta}$$

B+L nonconservation from nontrivial topological structure of nonabelian gauge theories.

$$[B+L](t_f) - [B+L](t_i) = \int_{t_i}^{t_f} dt \int d^3x\ \partial^\mu J^{B+L}_\mu = N_g\left[N_{cs}(t_f) - N_{cs}(t_i)\right] = N_g \Delta N_{cs}.$$

$$N_{cs}(t) = \frac{g^3}{96\pi^2}\int d^3x\ \epsilon_{ijk} W^{ai} W^{bj} W^{ck}$$

↑

Chern-Simon no. of nonabelian gauge field

N.B. $\int_{t_i}^{t_f} dt \int d^3x\ B_{\mu\nu}(x)\tilde{B}^{\mu\nu}(x) = 0.$

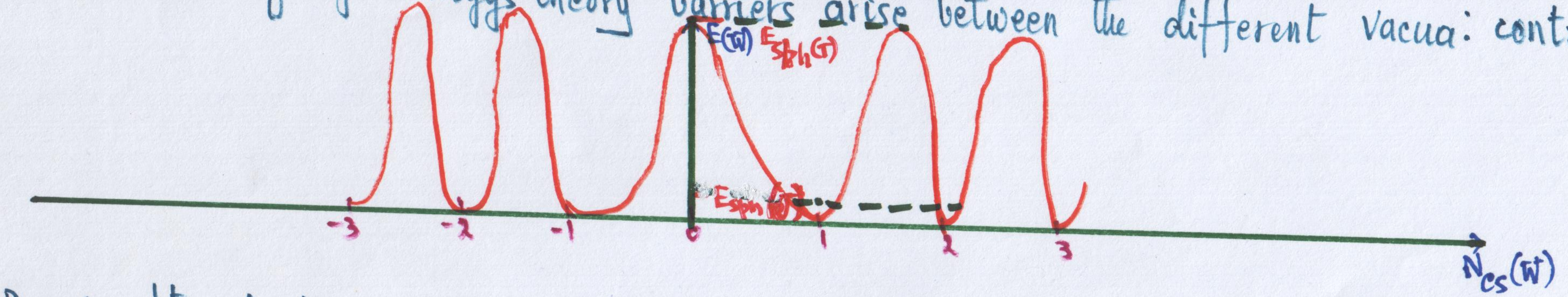
No CS no. for U(1) in (3+1) D.

→ Infinitely many degenerate ground states with $\Delta N_{cs} = \pm 1, \pm 2, \ldots$

Fermionless part of SM with time derivatives put to zero to study vacuum. Scale $W^a_{ij} \to g W^a_{ij}$.

$$H = \int d^3x \left[\frac{1}{4g^2} W^a_{ij} W^a_{ij} + (D_i \phi)^\dagger (D_i \phi) + \lambda \left(\phi^\dagger \phi - \tfrac{1}{2} \underset{\substack{\uparrow \\ 246\ \text{GeV}}}{v^2} \right)^2 \right] \qquad (1)$$

In such a gauge + Higgs theory barriers arise between the different vacua: controlled by $v(T)$.



Barrier ht slowly varying function of v. Set up fermionic Dirac eqns & study zero modes: $n_p - n_{\bar{p}}$ $\longrightarrow$ B, L. One finds $\frac{\Delta B}{N_g} = \Delta L_i = \Delta N_{cs} = \pm n$

$$\therefore \quad \Delta B = \Delta L = N_g \Delta N_{cs} = \pm 3n,$$

for 3 generations. Vac $\longrightarrow$ Vac transition: **change in B, L by multiples of 3.** $\frac{1}{3}B - L_e$, $\frac{1}{3}B - L_\mu$, $\frac{1}{3}B - L_\tau$ conserved

SU(2) instantons $\longrightarrow$ effective operator

B+L violation
B−L conservation

$$O_{B+L} = \prod_{i=1}^{3} (q_{Li}, q_{Li}, q_{Li}, l_i).$$

Effects B+L violation thru' 12 fermion operator induced transition, e.g.

$$\underset{B-L=-1}{u^c + d^c + e^c} \longrightarrow \underset{B-L=-1}{d + 2s + 2b + t + \nu_e + \nu_\mu + \nu_\tau}$$

T=0 tunneling probability $e^{-16\pi^2/g^2} \simeq 10^{-165}$ Negligible!

SPHALERONIC SOLUTIONS

Static Finite energy solitonic topological solutions to Hamiltonian (1). (Manton)

↳ charge 1/2.

↑ of Weinberg–Salam theory

Saddle point like solutions: can interpolate between different vacua $\longrightarrow \Delta B, \Delta L$ transitions.

Sphaleron energy

$$E_{sph.}(T) \simeq \frac{8\pi}{g} v(T) :$$

can exceed barrier heat at $T \gtrsim M_W \longrightarrow$ thermal transitions.

$$T < M_W : \quad \frac{\Gamma_{B+L}}{V} \propto \frac{M_W^7}{(\alpha_W T)^3} e^{-\beta E_{sph}(T)} \propto e^{-\frac{4 M_W}{\alpha_W T}} .$$

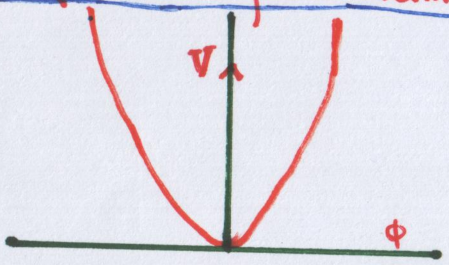
$$T \gtrsim M_W : \quad \frac{\Gamma_{B+L}}{V} \propto \frac{\alpha_W^5}{\ln \alpha_W} T^4 :$$

Unsuppressed & profuse.

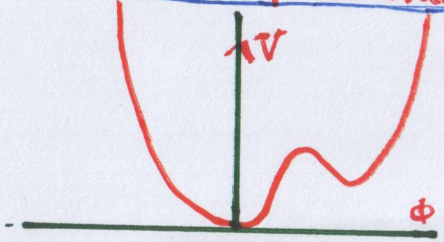
Thermal equilibrium for $100\,\text{GeV} < T < \alpha_W^4 M_P$.

Won't be washed out if $\Gamma_{B+L} < H$, happening at $T \sim M_W$.

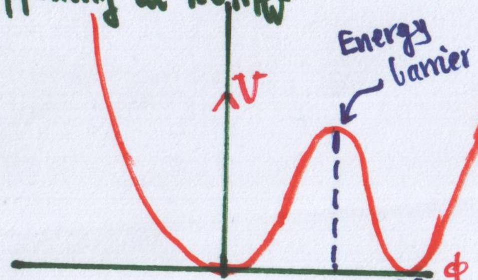
Departure from thermal equilibrium in EW phase transition



$T >> M_W$

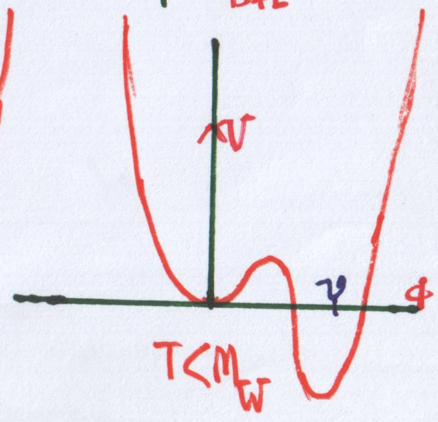


$T \gtrsim M_W$



$T \simeq M_W$ $v(T_c)$

↑ T_c



$T < M_W$

Strong first order phase transition needed.

$v(T)$: thermodynamic quantity

Strong : $\frac{\Delta v(T_c)}{T_c} > 1$.

First order : $\Delta v \neq 0$

Vacuum with $v(T)=0$ favoured energetically when $T>T_c$ and as $T \to T_c+$.
At $T<T_c$, one with $v(T)\neq 0$ more favorable. At $T=T_c$, both phases coexist

quantum tunneling from false ($v=0$) to true ($v\neq 0$) vacuum

Bubble formation ($v\neq 0$ state within $v=0$ vacuum)

growth till the whole spacetime filled up, completing phase transition.

As bubbles pass each pt. in space, order parameter keeps changing between $v=0$ & $v\neq 0$.
Departure from thermal equilibrium.

Problem with magnitude in SM

For first order phase transition (strong), need $m_H < 45$ GeV ✗ LEP

Also, CKM CP insufficient for $|\eta_b|$ during bubble nucleation.

$$\eta_B \simeq \frac{\alpha_W^4 T_c^3}{s} \delta_{CP}(T_c) \sim 10^{-8}\, \delta_{CP}(T_c)$$

$$\delta_{CP} = \frac{1}{T_c^{12}} (m_t^2-m_c^2)(m_c^2-m_u^2)(m_t^2-m_u^2)(m_b^2-m_s^2)(m_s^2-m_d^2)(m_b^2-m_d^2) \cdot J_{CP} \sim 10^{-20}$$

$T_c \to M_W$

$J_{CP} \simeq 10^{-3}$ ↑ $K_0-\bar{K}_0$

$$\eta_B \sim 10^{-28} \ll 10^{-10} \text{ (observed)}$$

SM electroweak baryogenesis does not work!

BSM

Good review: Riotto & Trodden, Ann. Rev. Nucl. Part. Sci. 49 (1999) 35.

① MSSM

Two Higgs doublets + slew of soft SUSY breaking terms.

- Trilinear in scalars
- Bilinear in Higgses
- Gaugino Majorana mass terms

all admit phases

$$W_{MSSM} = \mu \Phi_1 \cdot \Phi_2 + h^u \Phi_2 \cdot Q U^c + h^d \Phi_1 \cdot Q D^c + h^e \Phi_1 \cdot L E^c$$

strong restrictions from lack of observed d_e, d_μ.

Right η_β obtains from EW baryogenesis for $m_{\phi_{min}} \sim 120$-140 GeV & $\tan\beta \equiv v_u/v_d$ large (~ 50). V. restricted regions in parameter space.

Testable at LHC

② Affleck-Dine baryogenesis

Linked with inflation. Slepton & squark fields can roll down flat directions in inflationary potential and have coherent oscillations around minima with large $\langle B \rangle \neq 0 \neq \langle L \rangle$. At the end of inflation, with Hubble expansion, these oscillations can be stable and sustain large B & L asymmetries $\longrightarrow$ right kind of η_B, provided $m_{squark\,(stop)} \sim m_{slepton} \sim O(10^2\ \text{GeV})$.

③ GUT Baryogenesis

Review: Chen & Mahanthappa, IJMP A18 (2003) 5819

From $\cancel{B}$, $\cancel{C}$ & $\cancel{CP}$ of GUT interactions → characteristic scale 10^{15}–10^{16} GeV ← coupling unification ← lack of proton decay

Problem with dilution from inflation. Also, from sphalerons unless ∃ primordial B-L asymmetry.

SU(5) or SUSY SU(5) does not work well.

SO(10) does better!

→ spinorial $\underset{\sim}{16}$: $\psi_L(16) = (q_L, u^c_L, e^c_L, d^c_L, \ell_L, \nu^c_L)$

$\nu^c_L = \nu_R^c$ ↑ heavy SM singlet

Can have large Majorana mass

New idea ↓

④* Baryogenesis via leptogenesis

Define heavy Majorana fermion

$$|N\rangle = \frac{1}{\sqrt{2}} |\nu_R + \underbrace{\nu^c_R}_{\nu_L^c}\rangle$$

and give N a Majorana mass $M_N \sim m_{B-L}$ with $M_W << m_{B-L} << M_{GUT}$

$N \to \ell\phi, \ell^c\phi^\dagger$ can generate ΔL and $\cancel{CP}$ from different rates

$\Delta L \longrightarrow \Delta B$ by sphalerons at $T \sim M_W$.

Simplest to have N_i for generation i

- **Heavy right-handed neutrinos & seesaw mechanism**

Massive neutrinos mix like quarks: convention – basis with mass diagonal $l_\alpha^\pm$

Popular perception: light neutrinos Majorana particles $\longrightarrow \not\subset$ $0\nu\beta\beta$ decay

General $\mathcal{L}$ involving l_α^- & neutrinos: $\qquad l_{\alpha L} = \begin{pmatrix} \nu_\alpha \\ e_\alpha \end{pmatrix}_L$

$$\mathcal{L}^{\nu}_{\substack{\text{Yukawa} \\ +\text{mass}}} = -\underbrace{h^\nu_{\alpha i}}_{\substack{\text{Yukawa}\\ \text{coupling}\\ \text{matrix}}} \bar{l}_{L\alpha} \nu_{Ri} \phi - \frac{1}{2} \overline{\nu_{R\,i}^{c}} \underbrace{M_{ij}}_{\substack{\text{Majorana mass term,}\\ \text{allowed by } SU(2)_L \times U(1)_Y}} \nu_{Rj} + h.c.$$

EW symmetry breaking $\longrightarrow \langle \phi_0 \rangle = \frac{v}{\sqrt{2}} \longrightarrow$

$$m_l = h^l \frac{v}{\sqrt{2}}, \quad m_D = \frac{h^\nu v}{\sqrt{2}} \ll \underline{M}.$$

m_D & $\underline{M}$ 3×3 matrices in generation space

$\underline{M}$ $\downarrow$ has eigenvalues M_1, M_2, M_3.

Finally,

$$\mathcal{L}^\nu_m = -\frac{1}{2} \overline{\nu^c}_L M \nu_R - \bar{\nu}_L m_D \nu_R + h.c. = -\frac{1}{2} \left(\bar{\nu}_L \;\; \overline{\nu^c}_L \right) \begin{pmatrix} 0 & m_D \\ m_D^T & \underline{M} \end{pmatrix} \begin{pmatrix} \nu^c_R \\ \nu_R \end{pmatrix} + h.c.$$

Block diagonalization of complex symmetric matrix $\longrightarrow$ mass eigenstate fields ν^m from

$$\nu^m \simeq \left(U_\nu^T \nu_L^f + U_\nu \nu^{f\,c}{}_L \right) \frac{1}{\sqrt{2}}, \quad N = \left(\nu_R + \nu^c{}_R \right) \frac{1}{\sqrt{2}}$$

flavor eigenstate fields ν^f.

Mass matrices

$$m_\nu^{diag} \simeq -U_\nu^T\, m_D^T\, M^{-1}\, m_D\, U_\nu + O\left(m_D^3/M^2\right) \text{ terms} = \text{diag.}\,(m_1, m_2, m_3)$$

$$m_N \simeq M$$

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

↑
parametrization
PMNS

$$\Delta_{ij}^2 \equiv m_i^2 - m_j^2$$

$$\sqrt{\Delta_{21}^2} \simeq 0.009\ \text{eV}$$
$$\sqrt{|\Delta_{32}^2|} \simeq 0.05\ \text{eV}$$

$\sum_i m_i \leq 0.28\,\text{eV}$ ← cosmology
red shifts of large scale galactic survey.

$\theta_{12} \sim 34^\circ;\ 30^\circ < \theta_{12} < 36^\circ$
$\theta_{23} \sim 45^\circ;\ 37^\circ < \theta_{23} < 54^\circ$ $\Big|\ 3\sigma$
$\theta_{13} = \left(9^{+3}_{-5}\right)^\circ$

$M_i \sim 10^{12} - 10^{14}$ GeV → sub-eV ν-masses with $m_D \sim$ charged fermion masses

$\exists$ type II seesaw with Higgs triplet
$\exists$ type III seesaw with fermion triplet.

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\text{light } \nu_l \arrow[r, "\text{phases } \delta, \alpha_1, \alpha_2"] \arrow[d, "\text{seesaw}"'] & CP \arrow[d, "\text{sphaleronic transition}"] \\
\text{Heavy } M_R \arrow[r, "\text{leptogenesis}"'] & \eta_B
\end{tikzcd}$$

• STANDARD LEPTOGENESIS WITH MAJORANA NEUTRINOS

CP asymmetry from N_i decay

Reviews: Pilaftsis, hep-ph/9708235, 9812256

$$\mathcal{L}_I = -h^{\nu}_{\alpha i}\, \bar{\ell}_{L\alpha} N_{Ri}\, \phi + \text{h.c.}$$

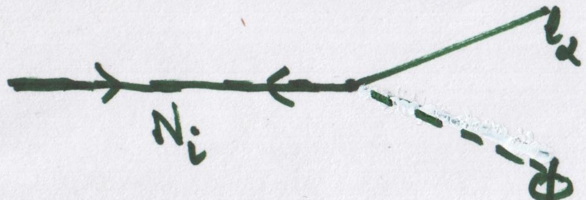
$$h^{\nu} = m_D v^{-1}$$

↑ crucial to seesaw

$N_i \to \phi\, \ell_{\alpha L}, \phi^{\dagger} \ell^{c}_{\alpha R}$: CP conjugate channels, but unequal rates because of $\cancel{CP}$

$$\Gamma(N_i) \equiv \sum_{\alpha} \left[\Gamma(N_i \to \phi\, \ell_{\alpha}) + \Gamma(N_i \to \phi\, \ell^{c}_{\alpha R}) \right]$$

Tree:

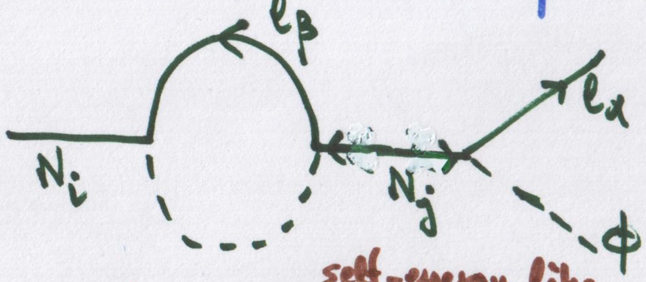


$$\Gamma^{\text{tree}}(N_i \to \phi^{\dagger} \ell^{c}_{\alpha R}) = \Gamma^{\text{tree}}(N_i \to \phi\, \ell_{\alpha}) = \frac{M_i}{8\pi} (h^{\nu\dagger} h^{\nu})_{ii}$$

$$\Gamma^{\text{tree}}(N_i) = \frac{M_i}{4\pi} (h^{\nu\dagger} h^{\nu})_{ii}$$

To lowest order, the difference arises from tree - 1 loop interference.

1-loop diagrams:



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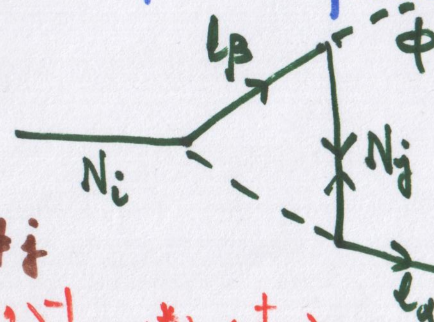


decay asymmetry (L, $\cancel{CP}$)

self-energy like

$i \neq j$

vertex correction

$$\epsilon_{i\alpha} = \Gamma^{-1}(N_i) \left[\Gamma(N_i \to \phi\, \ell_{\alpha}) - \bar{\Gamma}(N_i \to \phi^{\dagger} \ell^{c}_{\alpha}) \right] \simeq \frac{3}{16\pi} (h^{\nu\dagger} h^{\nu})^{-1}_{ii} \Big[\sum_{j \neq i} \frac{2}{3} (M_j^2 / M_i^2)^{-1} \, \mathrm{Im}\, h^{*\nu}_{\alpha i} (h^{\nu\dagger} h^{\nu})_{ji} h_{\alpha j}$$
$$+ \frac{M_i}{M_j}\, \xi\, (M_j^2 / M_i^2)\, \mathrm{Im}\, h^{*\nu}_{\alpha i} (h^{\nu\dagger} h^{\nu})_{ij} h_{\alpha j} \Big]$$

Each Im = 0 for $i = j$.

Loop function $\xi_i(x)$ has different forms in SM & MSSM ← additional diagrams

$$\xi_{SM}(x) = \frac{2}{3}x\left[(1+x)\ln\frac{1+x}{x} - \frac{2-x}{1-x}\right] ; \quad \xi_{MSSM}(x) = \sqrt{x}\left(\frac{2}{1-x} - \ln\frac{1+x}{x}\right).$$

With $\sum_\alpha$, the "self-energy" part vanishes. Also $\xi_i(x) \to 0$ as $x \to 0$, i.e. massless ($M_j = 0$) RH neutrinos cannot contribute

← sterile

Estimate

Davidson, Ibarra, hep-ph/0202239

Hamaguchi, Murayama, Yanagida, hep-ph/0109030

$$\epsilon_1 \lesssim \frac{3}{16\pi}\frac{M_1\sqrt{\Delta^2_{atm}}}{v^2} \simeq 10^{-6}\left(\frac{M_1}{10^{10}\,GeV}\right)\left(\frac{\sqrt{\Delta^2_{32}}}{.05\,eV}\right)$$

WASHOUT ISSUES

Interactions in / out of equilibrium if rates $>$ / $<$ expansion rate H of Universe

In radiation dominated Universe:

$$H(T=M_i) = \frac{\pi M_i^2}{M_P}\sqrt{\frac{g_*}{90}},$$

M_P ← 1.2×10^{19} GeV

rel d.f. →

$$g_* = \left(\sum_{\substack{\text{bosons}\\ i}} g_i + \frac{7}{8}\sum_{\substack{\text{fermions}\\ i}} g_i\right)\left(\frac{T_i}{T}\right)^3$$

$$g_* = \begin{cases} 106.75 & : SM \\ 228.75 & : MSSM \end{cases}$$

T_i = freeze-out temp. of ith species.

Survival or wash-out of CP asymmetry controlled by

$$K_i \equiv \frac{\Gamma_{N_i}}{H(T=M_i)} = \frac{\tilde{m}_i}{m_*}$$

being $<$ or $>$ 1.

$\tilde{m}_i$ → washout mass → or effective

m_* → equilibrium mass

When in / out of equilibrium, inverse decay $\phi\ell_\alpha \to N_i$ can / cannot wash out asymmetry.

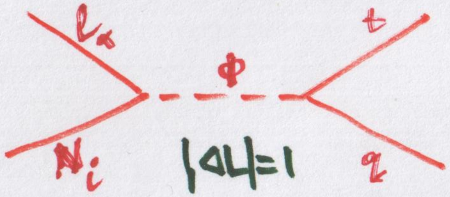
$\tilde{m}_i = M_i^{-1} (m_D^\dagger m_D)_{ii}$, $m_* = 4\pi H (T = M_i)$ Buchmüller & Plümacher

$\simeq 1.08 \times 10^{-3}$ eV for $M_i \sim 10^{12}$ GeV

Reasonable assumptions $\longrightarrow$

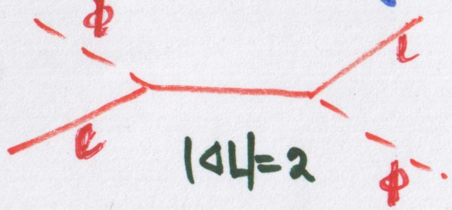
$$m_{lightest} < \tilde{m}_1 < m_{heaviest} .$$

Additional mechanisms for wash-out of ϵ_i : $2 \to 2$ scattering with $|\Delta L| = 1, 2$.



$|\Delta L| = 1$

$\ell_\alpha + N_i \to \begin{matrix} t\bar{q} \\ \bar{t} q \end{matrix}$



$|\Delta L| = 2$

$\ell_\alpha + \phi \to \ell_\alpha^c + \phi^\dagger$

$\ell_\alpha^c + \phi^\dagger \to \ell_\alpha + \phi$

Though CP inv. at tree level, these are $\cancel{t}$ and can dilute ϵ_i

Washout parameter κ

$$Y_L = \frac{n_L - \bar{n}_L}{s} = \kappa \frac{\epsilon_1}{g_*}$$

in the N_1-dominated scenario. For $1 < K < 10$, $\kappa \sim (2\sqrt{K^2+9})^{-1} \sim 10^{-1}$-$10^{-2}$: Partial washout

<u>Two extreme cases</u>

(1) $\kappa << 1$

Now $H^{-1} \Gamma_{ID} \sim (M_i/T)^{3/2} e^{-M_i/T} . K$

$H^{-1} \Gamma_{sc.} \sim (M_i/T)^5 . K$

For $T \lesssim M_i$, no washout

$n_\ell \neq n_{\bar{\ell}} \sim n_\gamma \leftarrow$ near-thermal equilibrium

(2) $\kappa >> 1$

equil. distrn. n_ℓ and $n_{\bar{\ell}}$ follow therm.

$\frac{d}{dt}(n_\ell - \bar{n}_{\bar{\ell}}) + 3H(n_\ell - \bar{n}_\ell) = 0 \to \Delta n_\ell \propto e^{-3Ht} \to 0$

Complete washout

• BOLTZMANN EQUATIONS

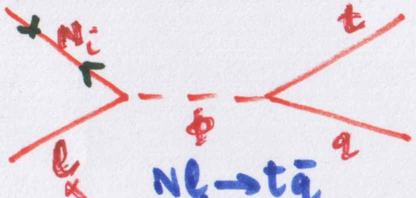
Buchmüller, di Bari, Plümacher, NP B643, 347 (2002).
Barbieri, Creminelli, Strumia, Tetradis, NP B575, 61 (2000).

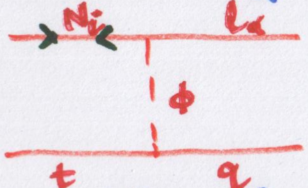
In expanding Universe, out-of-equilibrium decay of N_i treatable by Boltzmann eqns.

Relevant processes :

D: $N_i \longrightarrow \begin{matrix} \phi \ell_\alpha \\ \phi^\dagger \ell^c_\alpha \end{matrix}$; $\Delta B = 0,\ \Delta L = \pm 1$; ID: $\begin{matrix} \phi \ell_\alpha \\ \phi^\dagger \ell^c_\alpha \end{matrix} \longrightarrow N_i$

$|\Delta L| = 1$ scattering S:



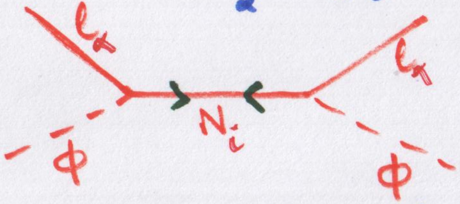


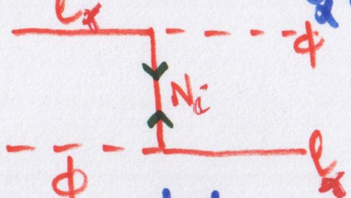
$N\ell_\alpha \rightarrow t\bar{q}$
$N\ell^c_\alpha \rightarrow \bar{t}q$

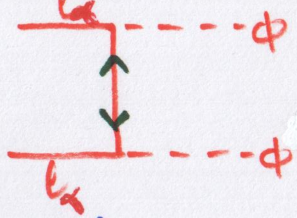
$Nt \rightarrow \ell^c_\alpha q$
$N\bar{t} \rightarrow \ell_\alpha \bar{q}$

$\Delta B = 0,\ \Delta L = \pm 1$
(no. density of N_i changes)

$|\Delta L| = 2$ scattering W:







Rates $\Gamma_D, \Gamma_S, \Gamma_W$

$\phi \ell_\alpha \rightarrow \phi^\dagger \ell^c_\alpha$, $\ell_\alpha \ell_\alpha \rightarrow \phi^\dagger \phi^\dagger$, $\ell^c_\alpha \ell^c_\alpha \rightarrow \phi\phi$

$\Delta B = 0,\ \Delta L = \pm 2$
(no. density of N_i does not change)

N_1-dominated scenario: $M_1 << M_{2,3}$

For $T \gtrsim M_1$, washout processes need to be strong enough to keep all N_1's in equilibrium in thermal bath.
Yet, as $T \longrightarrow < M_1$, they must become weak enough $\ni \epsilon^i_\alpha$ develops & sustains.

Let $z \equiv M_1/T$, $n_{N_1} =$ no. density of N_1-states, $N_{B-L} =$ no. density of B-L carrying states.

Define

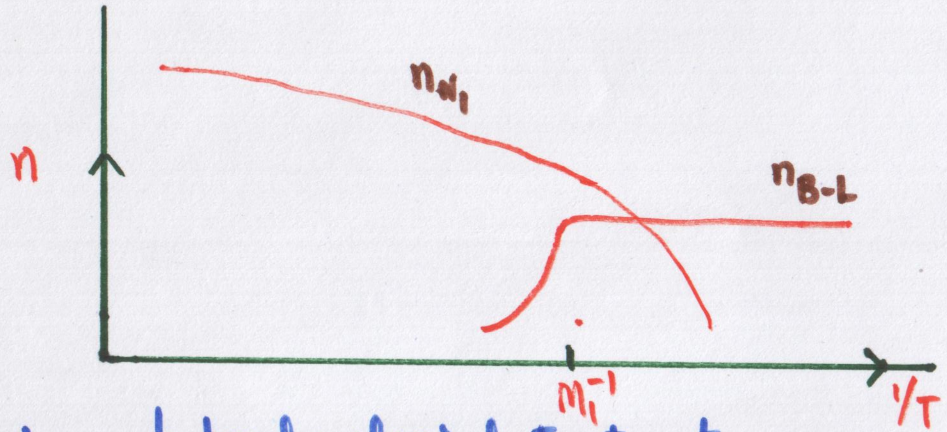
$$D \equiv \frac{\Gamma_D}{1+z}, \quad S \equiv \frac{\Gamma_S}{1+z}, \quad W \equiv \frac{\Gamma_W}{1+z}$$

Boltzmann eqns.

$$\frac{dn_{N_1}}{dz} = -(D+S)\left(n_{N_1} - n_{N_1}^{eq.}\right)$$

?

$$\frac{dn_{B-L}}{dz} = -\epsilon_1 (D+S)\left(n_{N_1} - n_{N_1}^{eq.}\right) - W n_{B-L}$$



2 shortcomings of purely classical treatment

- Collision terms (elastic scattering) in S-matrix & their quantum interference ignored.
- Quantum time evolution (i.e. quantum extension of B-eqns.) not considered.

• SPHALERONIC CONVERSION FROM ΔL TO ΔB

Buchmüller, Peccei, Yanagida, Ann. Rev. Nucl. Part. Sci. 55, 311 (2005)

Leptogenesis $\longrightarrow$ nonzero n_{B-L}.

Afterwards, $\exists$ weakly coupled thermodynamic plasma with T, V, μ.

consider in SM with one Higgs doublet.

Partition function:

$$Z(\mu, T, V) = e^{-\beta\left(H - \sum_i \mu_i Q_i\right)}.$$

Q_i = charge operator of field i

μ_i = chemical potential of field i

Extensive system: vol. can be factored out. Admits thermodynamic potential:

$$\Omega(\mu, T) = -\frac{T}{V} \ln Z(\mu, T, V).$$

$$B = \sum_i \left(2\mu_{q_i} + \mu_{u_i} + \mu_{d_i}\right)$$

$$L_i = 2\mu_{\ell_i} + \mu_{e_i}$$

$$L = \sum_i L_i$$

derivative $\Rightarrow$ asymmetry between particle and antiparticle no. densities.

$$n_i - \bar{n}_i = -\frac{\partial \Omega(\mu, T)}{\partial \mu_i} = \frac{1}{6} g T^3 \begin{cases} \beta\mu_i & \text{(fermions)} \\ 2\beta\mu_i & \text{(bosons)} \end{cases} + O\left[(\beta\mu_i)^3\right]$$

For a noninteracting gas of massless particles

$$\boxed{\begin{aligned} n_B - \bar{n}_B &= \tfrac{1}{6} B T^2 \\ n_{L_i} - \bar{n}_{L_i} &= \tfrac{1}{6} L_i T^2 \end{aligned}}$$

In the high temp. plasma elementary quarks, leptons, gauge & Higgs bosons do interact — via gauge and Yukawa couplings + $\exists$ sphaleronic interactions.

Conservation laws $\longrightarrow$ constraints on μ_i in thermal equilibrium.

Four sets of such constraints:

- Sphaleronic interactions due to $\mathcal{O}_{B+L} = \prod_i \overbrace{(q_{Li}, q_{Li}, q_{Li}}^{\text{color-avg}}, \ell_{Li}) \longrightarrow \boxed{\sum_i (3\mu_{q_i} + \mu_{\ell_i}) = 0}$

  $B=3,\ L=3;\ B-L=0$

- $SU(3)_c$ QCD instanton processes thru' $\mathcal{O}_{\text{instanton}} = \prod_i \overbrace{(q_{Li}, q_{Li}, u^c_{Li}, d^c_{Li})}^{\text{color avg}} \longrightarrow \boxed{\sum_i (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i})}$

  $B=0,\ L=0,\ B-L=0$

- Total Y of plasma must vanish, $\sum_i Y_i \mu_i = 0 \longrightarrow \boxed{\sum_i (\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i} - \mu_{e_i}) + 2\mu_\phi = 0}$

  $\overbrace{Y_{u^c_i} = -\frac{4}{3},\ Y_{d^c_i} = \frac{2}{3},\ Y_{q_i} = 1/3}^{\text{color-sum}}$

  $Y_{e^c_i} = 2,\ Y_{\phi_a} = 1 \rightarrow$ complex doublet

- Yukawa interactions

$$\left.\begin{array}{l} \mathcal{L}\, \bar d_{Ri}\, q_{Li}\, \phi^\dagger \\ \mathcal{L}\, \bar u_{Ri}\, q_{Li}\, \phi \end{array}\right| \text{color avg} \qquad \begin{array}{l} \longrightarrow \\ \longrightarrow \end{array} \qquad \begin{array}{l} \mu_{q_i} - \mu_{d_i} - \mu_\phi = 0 \\ \mu_{q_i} - \mu_{u_i} + \mu_\phi = 0 \end{array}$$

$$\mathcal{L}\, \bar e_{Ri}\, l_i\, \phi^\dagger \longrightarrow \mu_{\ell_i} - \mu_{e_i} - \mu_\phi = 0$$

If all Yukawa interactions in equilibrium, $L - \frac{B}{N_g}$ preserved.

Further, assume equilibrium among fermions of different generations. $\longrightarrow \boxed{\mu_{\ell_i} = \mu_\ell,\ \mu_{q_i} = \mu_q,\ \mu_{u_i} = \mu_u,\ \mu_{d_i} = \mu_d,\ \mu_{e_i} = \mu_e}$

From all above constraints, algebra $\longrightarrow$

$$\mu_e = \frac{2N_g+3}{6N_g+3}\mu_\ell,\quad \mu_d = -\frac{6N_g+1}{6N_g+3}\mu_\ell$$

$$\mu_u = \frac{2N_g+1}{6N_g+3}\mu_\ell,\quad \mu_q = -\frac{1}{3}\mu_\ell,\quad \mu_H = \frac{4N_g}{6N_g+3}\mu_\ell.$$

Return to expressions $B = \sum_i (2\mu_{q_i} + \mu_{u_i} + \mu_{d_i})$, $L = \sum_i (2\mu_{\ell_i} + \mu_{e_i})$ & substitute

$\longrightarrow$

$$B = -\frac{4}{3} N_g \mu_\ell , \quad L = \frac{14 N_g^2 + 9 N_g}{6 N_g + 3} \mu_\ell ,$$

or,

$$B = \frac{8 N_g + 4}{22 N_g + 13} (B-L) , \quad L = -\frac{14 N_g + 9}{22 N_g + 13} (B-L) .$$

Above with one Higgs doublet ϕ. With N_ϕ Higgs doublets $\longrightarrow$

$$B = \frac{8 N_g + 4 N_\phi}{22 N_g + 13 N_\phi} (B-L) , \quad L = -\frac{14 N_g + 9 N_\phi}{22 N_g + 13 N_\phi} (B-L) .$$

In baryogenesis via leptogenesis, $\langle B_{initial} \rangle = 0$, $\langle L_{initial} \rangle \neq 0$ and B-L conserved by sphalerons.

$\therefore$

$$\boxed{\langle B_{final} \rangle = -\frac{8 N_g + 4 N_\phi}{22 N_g + 13 N_\phi} \langle L_{initial} \rangle}$$

$$\langle L_{final} \rangle = \frac{14 N_g + 9 N_\phi}{22 N_g + 13 N_\phi} \langle L_{initial} \rangle .$$

Graphically :