Pathways to Quark Lepton Unification

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Introduction

- Quest for the unified description of quarks and leptons is an old subject dating back to 1973 (J. C. Pati and A. Salam, Phys. Rev. **D8** 1240 (1973))

- This search has become more focused in recent years. We now have much more precise information on the quark and lepton mixing and their masses. This has made it possible to test previously accepted pictures and reject some of them!

- This talk is devoted to a review of the present status of various approaches to a unified description of quarks and lepton masses and mixing angles. We will discuss

- Present information on fermion masses and mixing angles

- Introduction to GUTs

- Flavour symmetries;

  - Origin and type of flavour symmetries
  - Possibility of integration into a unified framework

- Fermion masses in $SO(10)$ theories

Experimental Information

Why unification?

• We know that gauge couplings unify at a high scale in Supersymmetric theories

• Hint in favour of yukawa unification:

At the weak scale:

$m_b = 4.2$ GeV $m_\tau = 1.77$ GeV

At M_{GUT}:

- $m_b = m_\tau \approx 1.74$ GeV
- $\frac{m_t}{m_b} = \tan\beta$
- $\frac{3m_s}{m_\mu} = \frac{m_d}{3m_e} \approx 1$

Obstacles in unification :

Neutrino masses need not be hierarchical. They may follow the pattern

$$m_{\nu_1} \approx m_{\nu_2} \approx 0.05\text{eV} \gg m_{\nu_3}$$

OR

$$m_{\nu_1} \approx m_{\nu_2} \approx m_{\nu_3} \approx 0.3\text{eV}$$

OR

$$m_{\nu_3} \approx 0.05\text{eV} \qquad m_{\nu_2} \approx 0.01 \text{ eV} \gg m_{\nu_1}$$

$$-\mathcal{L}_{weak} = \frac{g}{\sqrt{2}} \left(\overline{u} V_q d_L + \overline{\nu}_L V_l^{\dagger} l_L \right) W + \text{H.C.}$$

$$V_f = \begin{pmatrix} c_{12}^f c_{13}^f & s_{12}^f c_{13}^f & s_{13}^f e^{-i\delta^f} \\ -(s_{12}^f c_{23}^f + c_{12}^f s_{23}^f s_{13}^f e^{i\delta^f}) & c_{12}^f c_{23}^f - s_{12}^f s_{23}^f s_{13}^f e^{i\delta^f} & s_{23}^f c_{13}^f \\ s_{12}^f s_{23}^f - c_{12}^f c_{23}^f s_{13}^f e^{i\delta^f} & -(c_{12}^f s_{23}^f + s_{12}^f c_{23}^f s_{13}^f e^{i\delta^f}) & c_{23}^f c_{13}^f \end{pmatrix}$$

$$\begin{aligned} \sin\theta_{12}^q &= 0.2252 \pm 0.0009 & \sin^2\theta_{12}^l &= 0.316 \pm 0.016 \\ \sin\theta_{23}^q &= (40.6 \pm 1.3).10^{-3} & \sin^2\theta_{23}^l &= 0.51 \pm 0.06 \\ \sin\theta_{13}^q &= (3.89 \pm 0.44).10^{-3} & \sin^2\theta_{13}^l &< 0.04 \\ \delta^q &= (56.31 \pm 10.24)^0 & \delta^l &= ? \end{aligned}$$

(A) Difference in the mixing pattern may be taken to mean that quarks and leptons are fundamentally different. Even in this case one may need to find special leptonic symmetries which lead to “magic values” for the neutrino mixing angles

$$\sin^2\theta_{12} = \frac{1}{3} \quad ; \quad \sin^2\theta_{23} = \frac{1}{2} \quad ; \quad \sin^2\theta_{13} = 0$$

(B) Alternatively, quarks and leptons may be unified in some framework which allows for completely different mechanisms for the quark and lepton mass generation. This may lead to different mixing and mass patterns for neutrinos.

A Quick Introduction to GUT

Grand unified theories provide group theoretical relations between various "charges" of the quarks and leptons. The first example of this was the idea of Pati and Salam who integrated $B - L$ and $SU(3)_c$ into an $SU(4)$ group. Three coloured quarks and a lepton are put into the 4 dimensional representation of $SU(4)_{PS}$. $B - L$ is a (tracelss) generator of $SU(4)$ and thus for 4 dimensional object it looks like

$$(x, x, x, -3x)$$

This explains why Baryon number of quarks is -1/3 times the lepton number.

The electric charge is given in $SU(4) \times SU(2)_L \times SU(2)_R$ as

$$Q = T_{3L} + T_{3R} + 1/2(B - L)$$

Alternative possibility (Rajasekaran and Roy, Pati and Salam) is that color is broken and

$$Q = T_{3c} + T_{8c} + T_{3L} + Y$$

Quark now have integer charges *e.g.*

$$(u, u, u) \quad \text{carry charges} \quad (0, 1, 1)$$

and theory looks completely different. It was shown that

- In the deep inelastic regions, $Q^2 \gg m_g^2$, one sees the average charges of the quarks and theory looks the same as QC

- In the intermediate regions of q^2 and above the color threshold, these two alternatives are different and very extensive investigations (Rajasekaran,Rindani,.....) was required to rule out this alternative.

Quick look at $SU(5)$

- 15 fermions

$$(u, u, u, d, d, d, u^c, u^c, u^c, e^c) \quad + \quad (d^c, d^c, d^c, \nu, e)$$

fall in

$$10 + \overline{5}$$

representation of $SU(5)$ (Georgi and Glashow (1974)).

- Gauge couplings of $SU(3) \times SU(2) \times U(1)$ unify at $M_X \approx 2 \times 10^{16}$ GeV in the supersymmetric $SU(5)$.

- The fermion masses are obtained from $\overline{5}+45$ Higgs fields. If only 5_H is present then

$$M_d = M_l^T$$

This relation has two important implications

- It implies b-τ unification $m_b = m_\tau$
- If the second generation masses come from 45_H then the above relation gets modified and one finds

$$3m_s \approx m_\mu$$

- With additional assumption, it can also explain why leptonic mixing angle is large and the quark mixing angle is small. Example:

$$M_l = M_d^T = \begin{pmatrix} 0 & \epsilon \\ \rho & 1 \end{pmatrix}$$

  $\epsilon < \rho \approx 1$ simultaneously explains small quark mixing $\sim \epsilon$ and large $\sim \rho$ leptonic mixing.

$SU(5)$ has no room for ν_R and more appropriate framework is $SO(10)$.

ν_L obtain their masses in the presence of ν_R through the seesaw mechanism:

$$M_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix} \qquad \Rightarrow M_\nu \approx m_L - m_D M_R^{-1} m_D^T$$

In the left right symmetric theories

$$m_L M_R \sim M_W^2$$

and largeness of M_R explains smallness of M_ν.

Neutrino mixing pattern is decided by

$$\mathcal{M}_{\nu f} \equiv V_l^T M_\nu V_l$$

$\mathcal{M}_{\nu f}$ can be partially reconstructed from experiments since

$$\mathcal{M}_{\nu f} \equiv V^{l*}\text{Diag.}(m_1, m_2, m_3)V^{l\dagger}$$

Even a partial knowledge of $\mathcal{M}_{\nu f}$ is helpful in uncovering basic symmetries of the leptonic world. For example, we know that $(0, 1/\sqrt{2}, 1/\sqrt{2})^T$ is an eigenvector of $\mathcal{M}_{\nu f}$. This implies

$$\mathcal{M}_{\nu f} = \begin{pmatrix} X & A & A \\ A & B & C \\ A & C & B \end{pmatrix}$$

This leads to the solar mixing angle

$$\tan 2\theta_s = \frac{2\sqrt{2}A}{B+C-X}$$

and more specific forms for $\mathcal{M}_{\nu f}$ also follow from this.

$$\theta_s = 45^0 \Rightarrow \mathcal{M}_{\nu f} = \begin{pmatrix} B+C & A & A \\ A & B & C \\ A & C & B \end{pmatrix} \quad \sin^2\theta_s = \tfrac{1}{3} \Rightarrow \mathcal{M}_{\nu f} = \begin{pmatrix} B+C-A & A & A \\ A & B & C \\ A & C & B \end{pmatrix}$$

These mass matrices are invariant under a $Z_2 \times Z_2$ symmetry. One of these is the μ-τ interchange symmetry. The others are

$$S_{BM} = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix} \quad S_{TBM} = 1/3 \begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}$$

•The $Z_2 \times Z_2$ symmetry is an effective symmetry of $\mathcal{M}_{\nu f}$

•One would like to obtain this symmetry starting from a more general symmetry G imposed on the lagrangian.

• Fairly realistic and attractive models based on A_4, S_3, S_4 and more complicated ones are proposed in the leptonic sector

From leptonic symmetries to quark lepton unification

- Find some underlying symmetries which lead to above forms for $\mathcal{M}_{\nu f}$ and at the same time lead to small quark mixing angles.

- There are two phenomenological approaches for the unified description
  - Quark lepton Complementarity
  - Universal mass matrices

Quark-Lepton Complementarity

This is motivated by the empirical relation

$$\frac{\pi}{4} = \theta_s + \theta_c$$
$$(34 \pm 3.0)^0 + 13^0$$

One assumes that

- The neutrino mass matrix has the bi-maximal form. leading to

$$U_\nu = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & 1/-\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}$$

- $SU(5)$ relation $M_d = M_l^T$ holds

- Cabibbo angle arises essentially from the diagonalization of M_d giving

$$U_l = \begin{pmatrix} \cos\theta_c & -\sin\theta_c, 0 & \\ \sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The leptonic mixing matrix $V_l = U_l^\dagger U_\nu$ then implies

$$\begin{aligned} \theta_{solar} &= \frac{\pi}{4} - \frac{1}{\sqrt{2}}\theta_c \ , \\ 34^0 &\approx 45^0 - 9^0 \ , \end{aligned}$$

In practice, it is difficult to meet all the assumptions. Most models are based on the left right symmetric theories. Recent realization in $SO(10)$ (K. M. Patel)

Universal Mass Matrices

One can obtain examples where all the fermion mass matrices have universal structures and the seesaw mechanism produces difference. One example is approximate μ-τ symmetry

$$M_f = \frac{m_{3f}}{2}\begin{pmatrix} 1 & 1+\lambda_F \\ 1+\lambda_F & 1 \end{pmatrix} \quad (f = u, d, l) \quad m_D = \frac{m_{3D}}{2}\begin{pmatrix} 1-\epsilon_D & 1+\lambda_D \\ 1+\lambda_D & 1+\epsilon_D \end{pmatrix}$$

$$M_R = \frac{M_3}{2}\begin{pmatrix} 1 & 1+\lambda_R \\ 1+\lambda_R & 1 \end{pmatrix}$$

$$\lambda_f \approx 2\frac{m_{2f}}{m_{3f}}; \quad f = u, d, l, D, R$$

and only source of the μ-τ breaking is

$$\epsilon_D \sim \frac{m_{2D}}{m_{3D}} \ll 1$$

. This implies after seesaw

$$M_\nu \approx \begin{pmatrix} B(1-\epsilon_\nu) & C \\ C & B(1+\epsilon_\nu) \end{pmatrix}$$

with

$$\epsilon_\nu \approx -\frac{2\epsilon_D\lambda_D}{\epsilon_D^2+\lambda_D^2} \approx 1$$

As a result, mixing is suppressed in M_ν; M_l leads to nearly maximal $\theta_{23} \approx \frac{\pi}{4}$.

Fermion unification and $SO(10)$

$SU(5)$ is not suitable for a unified description of fermion masses because

- Fermions are put in two separate representations $\overline{5}$ and 10
- No room for the RH neutrinos

Fermions are assigned to a single representation in $SO(10)$

$$16 = 10 + \overline{5} + 1$$

Fermion masses arise from

$$16 \times 16 = 10 + 126 + 120$$

Simplest choice is $10_H = 5_H + \overline{5}_H$

$$16_F Y_{10} 16_F 10_H$$

- No neutrino masses
- No mixing. BUT
- $Y_b = Y_\tau = Y_t$

Minimal $SO(10)$ model

$$16_F(Y_{10}10_H + Y_{126}\overline{126}_H)16_F$$

This implies

$$\begin{aligned} M_d &= F + H; & M_u = rH + sF; && M_l = H - 3F\ , \\ M_D &= rH - 3sF; & M_L = r_L F; && M_R = r_R F\ . \\ H &\equiv Y_{10}\langle 1,2,1\rangle_{10} & F \equiv Y_{126}\langle 1,2,1\rangle_{126} && \end{aligned}$$

- In the limit $F = 0$ one obtains $M_d = M_l \Rightarrow b$-τ unification
- F contributing to the second generation masses $\Rightarrow 3m_s = m_\mu$
- $M_L \sim M_d - M_l$ This implies

$$\mathcal{M}_\nu^{II} \sim m_b \begin{pmatrix} V_{cb} & V_{cb} \\ V_{cb} & 1 - m_\tau/m_b \end{pmatrix}$$

Thus b-τ unification enhances atmospheric mixing angles compared to the quark mixing!

- This mechanism also explains why θ_{13} and $\frac{\Delta_{sol}}{\Delta_A}$ are small.

$$\mathcal{M}_{\nu f} \approx \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}$$

- Predicts θ_{13} near the present limit

	A	**B**	**C**	**D**	**C1**
Observables	Pulls obtained for best fit solution				
(m_u/m_c)	0.0486938	−0.180782	0.0653101	0.0053847	0.0467579
(m_c/m_t)	1.22599	0.130589	0.246294	0.146932	0.297256
(m_d/m_s)	−0.229546	−0.730641	0.223201	−0.748148	−2.2904
(m_s/m_b)	−0.932536	−0.886438	−0.977249	−1.05766	0.735548
(m_e/m_μ)	0.0340323	0.442759	0.103692	−0.476364	0.0649144
(m_μ/m_τ)	0.310305	−0.526529	0.881934	0.938701	0.705648
(m_b/m_τ)	−0.486477	−0.194215	0.0172182	−0.34079	0.789868
$\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$	0.122267	−0.10063	−0.00563647	−0.120429	−0.180164
$\sin\theta^q_{12}$	0.0432634	0.227948	0.0186715	0.084149	0.130301
$\sin\theta^q_{23}$	−0.281221	−0.0401177	−0.167224	0.0649082	−0.273222
$\sin\theta^q_{13}$	1.37864	−0.275689	0.926186	0.559003	1.48675
$\sin^2\theta^l_{12}$	−0.0528379	−0.0598219	−0.38133	−0.172148	−0.746107
$\sin^2\theta^l_{23}$	−1.22555	−1.27077	−1.43475	0.0548963	−1.99485
$\delta_{CKM}[^\circ]$	−0.291137	0.397159	−0.350422	−0.755859	−0.956628
χ^2_{min}	**6.3479**	**3.7962**	**5.0715**	**3.8665**	**14.789**

Problems with the minimal model

- Minimal model: $10_H + \overline{126}_H + 210_H + 16_F$
- The fermion masses require the presence of an intermediate scale$\sim 10^{11-12}$ GeV. This conflicts with the gauge coupling unification
- Large atmospheric mixing requires $b-\tau$ unification in the type-II dominated scenario. This can occur only for some range of parameters. Set of parameters which do not show b-τ unification do not give good fits to fermion masses.
- Minimization of the full superpotential shows that the type-II seesaw cannot be the main source of neutrino masses in a large parameter space

Various non-minimal models are considered which avoid some of the problems mentioned above.

Quasi-degenerate Neutrinos

Why Quasi-degenerate Neutrinos?

- If neutrinos are QDG at high scale then small mixing angles can get enhanced to large mixing angles at the electroweak scale (Balaji, Mohapatra, Dighe, Rajasekaran, Parida......)

- It is natural to expect large mixing angles with QDG. If neutrinos are exactly degenerate then
$\mathcal{M}_{\nu f} = \text{Unitary Symmetry Matrix} = m_0 U(\theta, \phi, \alpha)$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta & 0 \\ s_\theta & -c_\theta & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix}$$

This $\Rightarrow \theta_{13} = 0; \theta_{12} = \theta; \theta_{23} = \phi/2$
All perturbations which lift degeneracy without changing the mixing will naturally explain the observed mixing pattern!

Obtaining QDG Neutrinos

Any G which has 3-dimensional representation and admits $3 \times 3 = 1 + ...$ can lead to degenerate neutrinos.

But one would like to break degeneracy and introduce mixing. This is non-trivial. Known examples are based on A_4 (Ma and Rajasekaran; Babu Ma and Valle), $O(3)$....

In type-I seesaw $M_\nu = m_D M_R^{-1} m_D^T$

If a symmetry exists which leads to

$$M_R \approx m_D^T m_D + \text{corrections......}$$

then one would get quasi-degenerate neutrinos.

Quasi Degenerate neutrinos in $SO(10)$

Consider $SO(10)$ model with $10_H, 1\bar{2}6_H$

$$M_d = F + H; \quad M_u = rH + sF; \quad M_l = H - 3F$$
$$M_D = rH - 3sF; \quad M_L = r_L F; \quad M_R = r_R F$$
$$\mathcal{M}^I_\nu = M_D M_R^{-1} M_D^T \approx \frac{r^2}{r_R}(H - 3s/rF)F^{-1}(H - 3s/rF)$$

Let us supplement this with an ansatz

$$\boxed{F = aH^2} \quad :$$

This $\Rightarrow \mathcal{M}^I_\nu \approx \frac{r^2}{ar_R}(I - O(s/r))$

- If the contribution from H (10-plet Higgs) dominates (limit $s \to 0$), then,
  - Correct $b - \tau$ unification is obtained which is favoured by the data extrapolated at GUT scale.
  - CKM matrix is unity.
  - Lepton mixing angles are determined from the diagonalization of symmetric unitary matrix U, and one gets $\theta_{23} = \phi$, $\theta_{12} = \frac{\theta}{2}$ and $\theta_{13} = 0$.

Switching on 126 contribution leads to

- Departure from degeneracy
- Masses of the first two generations

A very good fit to all fermion masses and mixing can be obtained in this framework.

A model for $F = aH^2$

Flavour group $G = SO(10) \times O(3) \times U(1)$
Original Fields:

$$\psi(16, 3, x), \phi_{10}(10, 1, -(x+y)), \phi_{\overline{126}}(\overline{126}, 1, -2y)$$

Additional Fields:

$$\Psi_V(16, 3, y), \Psi_{\overline{V}}(\overline{16}, 3, -y), \eta(1, 5, -\frac{1}{2}(x+y))$$

The general superpotential invariant under G is

$$W = M\Psi_{\overline{V}}\Psi_V + \beta\Psi_V\Psi_V\phi_{\overline{126}} + \gamma\Psi_V\psi\phi_{10} + \frac{\delta}{M_P}\Psi_{\overline{V}}\eta^2\psi + \frac{\delta'}{M_P}Tr\eta^2\Psi_{\overline{V}}\psi + ...$$

The effective theory after integration of heavy vector-like field is

$$W_{eff} \approx \beta\psi\xi^2\psi\phi_{\overline{126}} + \gamma\psi\xi\psi\phi_{10}$$

where,

$$\xi_{ab} \equiv \frac{\delta}{MM_P}(\eta^2_{ab} + \frac{\delta'}{\delta}Tr\eta^2\delta_{ab})$$

SOME REMARKS:

• The presently available information on neutrino mixing angles point to "magic values" of mixing angles

• They may arise from special leptonic symmetries

• Mixing angles may arise due to underlying dynamics: Grand Unified theories and seesaw mechanism can lead to this dynamics. In particular, there exist attractive $SO(10)$ models which naturally explain differences in quark and lepton masses as well as mixing angles.

• In the dynamical approach the exact "magic values" for mixing angles are not expected to hold.

• Even in the symmetry approach, symmetry breaking would leave its traces:
Small $\theta_{13} \leq 0.02$ for normal or inverted hierarchy
Relatively large θ_{13} for quasi degenerate spectrum

• It may be difficult to decide which approach is correct purely from experiments.

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