Minimum Energy-Surface Required by Quantum Memory Devices

Wim van Dam
Department of Computer Science, Department of Physics
University of California, Santa Barbara, U.S.A.

Hieu D. Nguyen*
Department of Physics
University of California, Santa Barbara, U.S.A.

June 16, 2013

*Primary and corresponding author. Email: hdn@physics.ucsb.edu

Abstract

Introduction: We explore the physical requirements of storing information in a quantum mechanical device. Extrapolating Moore's law, we see that the technology used for information storage is quickly entering into the quantum regime where physical bits are small enough that their quantum behavior becomes significant. Therefore, understanding information storage according to the laws of quantum physics is valuable not only intellectually but also practically.

Here we analyze the cost of information storage and determine the optimal device storing a given amount of information while minimizing the cost in physical resources. Our approach is to define the cost as a joint product of energy and surface area and then minimize the cost among all interaction potentials within non-relativistic quantum mechanics. Our setting is for this reason different from Bekenstein's bound [J.D. Bekenstein, *Physical Review D,* Volume 30, pp. 1669 (1984)] or Lloyd's order-of-magnitude estimates of the ultimate limits of computation [S. Lloyd, *Nature,* Volume 406, pp. 1047 (2000) and *Physical Review Letters,* Volume 88, pp. 237901 (2002)]. Based on a number of plausible hypotheses, our result is an analytic expression of the lower bound to a device's surface area and energy when storing a given amount of information.

Our Result: Storing information is encoding different messages into a device's different stationary states according to a probability distribution whose Shannon entropy is the desired amount of information. Looking at a number of simple devices such as particle in a box, the hydrogen atom, and the simple harmonic oscillator, we notice that the average energy $\langle E\rangle$ and surface area $\langle r^2\rangle$ as functions of entropy can grow or decrease to arbitrarily small values, but their products $P := \langle E\rangle\langle r^2\rangle$ invariably increase as a function of the required entropy. Thus, we define the cost of information storage incurred by a device as the product of its expected surface area and energy and the cost of information storage as the minimum over all devices allowed within the non-relativistic quantum regime. Next, we find the form of the interaction potential based on assumptions of existence and uniqueness.

Having identified the optimal device, we derive a lower bound of the product of its average energy and surface area. Our main result is

$$P = \langle E\rangle\langle r^2\rangle \geq \frac{\hbar^2}{2m} d^2 \left(\mathrm{e}^{S/d} - 1\right)^2 ,$$

where P is the above mentioned cost, the product between the the average energy $\langle E \rangle$ and the surface area $\langle r^2 \rangle$, S the amount of encoded information, d the device's number of degrees of freedom, m the mass of each particle making up the device, and $\hbar \approx 1.056 \times 10^{34}$ Joule second is the reduced Planck constant.

Conclusion: Thinking classically, one would expect that information can be compressed in an arbitrarily small device without having to pay a price in energy. That is, we expect the greatest lower bound of P to be zero. However, our result shows that such an intuition is incorrect since it ignores the wave nature of particles that as they get compressed into a small space, their energy states shoot up due to the uncertainty principle. Our result also shows that P only seems that it should be zero because $\hbar$ is typically negligible in the classical regime. Moreover, as we compress information into fewer degrees of freedom, making S/d big, our result shows this endeavor to become unfeasible due to the cost rising exponentially. However, P scales only as a quadratic power in the number of degrees of freedom, indicating that it is more cost effective to keep the *information density* S/d fixed and bring in additional particles to store more information. In other words, as a function of the amount of information S, it is an exponentially bad idea to try to store this information in an analog manner using a single particle ($d = 1$) , when compared to the digital approach where the number of particles grows with S (i.e. when $S/d = 1$).

Reference and Acknowledgement: The technical article describing this work is: "Minimum Energy-Surface Required by Quantum Memory Devices", Wim van Dam, Hieu D. Nguyen, to appear in *Physical Review Letters,* arXiv:1212.3707. This material is based upon work supported by the National Science Foundation under Grant No. 0917244.