Generation of free-traveling optical trio coherent states

M. D. Truong[1], Q. D. Tran[1], A. B. Nguyen[2,3], J. Kim[3]

[1] Center for Theoretical and Computational Physics, 32 Le Loi, Hue, Vietnam
[2]Institute of Physics, 10 Dao Tan, Hanoi, Vietnam
[3]Korea Institute for Advanced Study, 207-43 Cheongryangni 2-dong, Dongdaemun-gu, Seoul 130-722, Korea

Trio coherent states can be useful for continuous variables quantum tasks, so their generation is of primary importance. There exist schemes to generate stable TCSs in terms of vibrational motion of a trapped ion inside a crystal. However, to perform quantum communication and distributed quantum computation the states should be shared beforehand among distant parties. That is, their modes should be able to be directed to different desired locations in space. In this work, we propose an experimental setup to generate such free-traveling TCSs in terms of optical fields. Our scheme is feasible because it uses standard physical resources such as coherent states, balanced beam-splitters, phase-shifters, nonideal on-off photodetectors and realistic weak cross-Kerr nonlinearities, without the need of single photons or homodyne/heterodyne measurements. We study dependences of the fidelity of the state generated by our scheme with respect to the target TCS and the corresponding generation probability on the parameters involved. In theory, the fidelity can be perfect for whatever (scaled) weak nonlinearities τ and low photodetector efficiency η, provided that the amplitude $|\alpha|$ of an input coherent state is large enough, namely, $|\alpha| \geq 5/(\sqrt{\eta}\tau)$.

Trio coherent states (TCSs) [1] are non-Gaussian three-mode entangled states which can be served as a useful resource for continuous variables quantum tasks. Their nonclassical properties were investigated [2] and stable TCSs can be produced for the vibrational motion of an ion trapped in a 3D isotropic harmonic potential [3]. However, such vibrational TCSs are confined inside a crystal. Since for performing quantum communication and distributed quantum computation the states should be shared beforehand among distant parties, we are here to propose an experimental setup to generate optical TCSs that can freely travel in space.

Our scheme uses coherent states as inputs and employs standard optical elements such as balanced beam-splitters, phase-shifters and nonideal on-off photodetectors. As coherent states are classical and classical states cannot be transformed into nonclassical ones by means of linear elements, we need a kind of nonlinearity to generate the desired TCS. Namey, cross-Kerr nonlinearities are exploited in our generation scheme.

Four coherent states with amplitudes ξ, ξ, ξ and $\alpha\sqrt{2}$ are used as input modes 1, 2, 3 and 4, respectively, where ξ is the amplitude of the TCS to be generated and α may be varied to manage the fidelity and success probability. First, the coherent state 4 is mixed with a vacuum state (mode 5) on a 50:50 beamsplitter B1. After B1, mode 5 is further mixed with another vacuum state (mode 6) on a second 50:50 beamsplitter B2. Next, mode 1 (2, 3) passes together with mode 4 (5, 6) through a cross-Kerr medium with nonlinearity χ. Then, mode 5 is successively mixed with mode 6 and mode 4 on third and fourth 50:50 beamsplitters B3 and B4. After that modes 1 and 4 pass through another cross-Kerr medium with nonlinearity $-\chi$. Finally, two photodetectors D1 and D2 are arranged to detect photon numbers of modes 4 and 6, respectively.

We first analyze theoretically the case when the two photodetectors are ideal, i.e., they can resolve the incoming photon number. The calculation shows that the fidelity vanishes whenever either or both photodetectors fires. That is, we would succeed only when both the photodetectors register no photons. This implies that photon number resolving detectors are unnecessary.

We thus proceed to considering on-off photodetectors with efficiency η ($\eta < 1$ implies nonideal on-off photodetectors, while the ideal ones have $\eta = 1$). As a result, the silence of both the on-off photodetectors heralds generation of the target TCS in the output modes 1, 2 and 3, that can be directed to anywhere in space.

We have derived explicit analytical expressions for the success probability and fidelity of the generated state with respect to the desired TCS, which turn out to depend effectively on $\eta|\alpha|^2\tau^2$ ($\tau = \chi t$ with t the time the two modes propagate through the cross-Kerr medium), rather than η, α and χ separately. In theory, the fidelity can be of 100% for whatever weak nonlinearities χ and low photodetector efficiency η, provided that the amplitude $|\alpha|$ of an input coherent state is large enough, namely, $|\alpha| \geq 5/(\sqrt{\eta}\tau)$. However, the price to pay for a high fidelity is a reduce success probability, a fact that cannot be avoided in many probabilistic schemes for quantum state engineering. Yet, the fact that our scheme is nondeterministic causes no problems, because the TCSs we generate will be supplied off-line for a subsequent given quantum task. So we can repeat the whole process until success.

Our scheme is feasible since it does not require strong nonlinearities and high detector efficiency, provided the amplitude of the input mode 5 is large enough. For example, even with an inefficient on-off photodetectors with

$\eta \sim 0.8$, a laser pulse with about 10^6 mean photon numbers (i.e., $|\alpha| \sim \mathcal{O}(10^3)$) would compensate the smallness of the cross-Kerr nonlinearity $\tau \sim \mathcal{O}(10^{-3})$. Such weak (but not tiny) cross-Kerr nonlinearities can potentially be engineered in practice within the present technologies using various means such as doped optical fibers, cavity quantum electrodynamics, electromagnetically induced transparency, etc.. (see, e.g., [4]). This means that strong cross-Kerr nonlinearities are not at all a compulsory precondition and weak cross-Kerr nonlinearities could be very useful for quantum information processing and quantum computing [5]. Furthermore, not as in many other schemes/protocols, we require neither single-photon sources nor homodyne/heterodyne measurements that would raise the delicate issues associated with single-photon Kerr nonlinearities [6], adding a 'plus' to the feasibility of our scheme.

Full details can be seen in the manuscript [7].

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