Evolution of quasi-Bell states in a strongly coupled qubit-oscillator system and a study of complexity

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We study evolution of bipartite entangled quasi-Bell states in a strongly coupled qubit-oscillator system in the presence of a static bias. The system is well-described by the Jaynes-Cummings model [1] in the context of the rotating wave approximation that holds for a near resonance scenario, and a small qubit-oscillator coupling compared to the oscillator frequency. Recently, however, a variety of experimental situations pertaining to stronger coupling domain, where the rotating wave approximation is no longer valid, have been investigated. Various experimental realizations such as a nanomechanical resonator capacitively coupled to a Cooper-pair box [2], a quantum semiconductor microcavity undergoing excitonic transitions [3], a flux-biased superconducting quantum circuit that uses large nonlinear inductance of a Josephson junction to achieve ultrastrong coupling with a coplanar waveguide resonator [4] have been recently achieved. For a superconducting qubit an external static bias that removes the degeneracy of the effective potential of the oscillator and avoids crossings of the energy levels [5, 6] may be easily achieved.

On the other hand, for the coupled qubit-oscillator system the nonclassical quasi-Bell states are of much interest. They exhibit entanglement of microscopic atomic states and the photonic coherent states that can be regarded as mesoscopic for reasonably large values of the coherent state amplitudes. When the amplitude of the coherent states are large enough they are often called Schrödinger cat states as they introduce entanglement between a microscopic and a classical object. In the instance of cavity quantum electrodynamics they have been used [7] for generating so-called even or odd coherent states as well as more generalized configurations of mesoscopic field superposition states. Bell inequality tests involving these qubit-field entangled states have been recently proposed [8]. These states also play a crucial role in the non-destructive measurement [9] of the photon number in a field stored in a cavity. Moreover, it has been observed [6] that in the large coupling regime a state of the generic quasi-Bell type becomes the approximate ground state of the combined system.

To study the strongly coupled qubit-oscillator system we employ the adiabatic approximation [10, 6]. In the regime of large detuning and strong coupling, the adiabatic approximation that relies on the separation of the time scales characterized the high oscillator frequency and the (renormalized) low qubit frequency could be used. The fast-moving oscillator then adiabatically adjusts to the slow changes of the state of the qubit.

Though in the case of cavity electrodynamics the usual experimental set up is described by an unbiased qubit operated through its degeneracy point, the static bias of a superconducting qubit, as stated before may be easily varied, say, by operating a magnetic flux on a Josephson junction [11].

Using the adiabatic approximation [10], for a qubit coupled strongly with oscillator of high frequency is studied in the presence of a static bias. In particular, starting with Schrödinger cat state which is an entangled state of a qubit and a coherent-state of the harmonic oscillator, we find the time-evolution of the reduced density matrices of both the qubit and the oscillator.

The reduced density matrix for the qubit is expanded till the fourth power of coupling constant and is written in closed forms comprising of linear combinations of Jacobi theta functions [12]. The analytical results based on the theta function evaluations are found to be in good agreement with their series counterparts. The entropy of the system quantifying the entanglement is computed via the qubit reduced density matrix. In the bias-free condition and under the adiabatic approximation scheme, the entropy turns out to be time-independent. This is however the artifact of the adiabatic approximation which eliminates the off-diagonal blocks of the Hamiltonian assuming the coupling constant is not too large. Under these approximation, the reduced density matrix of the slow varying qubit becomes stationary retaining high value of the entanglement.

On the other hand, the reduced density matrix of the oscillator is employed for obtaining the phase space distributions such as the Husimi Q-function, which, in turn, is utilized for obtaining closed-form expressions of the expectation values of antinormally ordered operators [13], as linear combinations of Jacobi theta functions. Our closed-form evaluations of various physical quantities are compared with, and found to be good approximations of, their series values in the regime characterized by strong coupling as well as weak bias. Our present scheme of approximating the density and other physical quantities with closed form evalutions via Jacobi theta functions has been checked for the renormalized values of the coupling constant and the bias equaling 0.18 and 0.02, respectively. However retaining more terms in the evaluation of the perturbative series these results can be improved.

Our evaluation of the Husimi Q function allows as to study the complexity of the strongly coupled system. Complexity is a measure of delocalization of the Husimi distribution in phase space and it is computed, for instance by the inverse of second moment of the Husimi distribution [14]. We evaluate complexity of the strongly coupled qubit-oscillator system analytically, and find close agreement with the numerical evaluation. The contour plots have features pointing towards long-living metastable states. The correspondence of the complexity with the Wehrl's classical entropy[15] is also verified and found to be qualitatively similar.

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