Non-classicality and entanglement of symmetric multiqubit states

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Non-classical quantum states draw attention both from conceptual point of view and as a resource for quantum information technology. Tests to explore whether a quantum system exhibits *non-classiclaity* has gained renewed attention. These investigations provide insights into how quantum description of nature turns out be inevitable. Widely accepted notion of non-classicality emerged from quasi-probability distributions of quantum systems [1]. Failure of the Sudarshan-Glauber P-function [2] from being a true classical probability density offers a decisive signature of *non-classicality* in single mode quantum radiation. On the other hand, information theoretic concepts define *non-classical correlations* in a composite quantum system [4–6]. A comparison of these two celebrated approaches in bipartite bosonic states revealed that *the notion of nonclassicality stemming from P representation is inequivalent to that emerging from information-theoretic arguments* [7]. In this paper, we investigate the same question in multiqubit symmetric systems and we find that the notion of non-classicality emerging from the P representation of spins [8] is a necessary and sufficient condition for entanglement [3].

Sudarshan-Glauber P representation of an arbitrary quantum state *formally* resembles a statistical mixture of coherent states of radiation. For a single coherent state, the weight function of the P representation reduces to a δ function in the phase space. However, for a large class of quantum states the P function (weight function) cannot be interpreted as classical probability density as it can assume negative values or is more singular than delta function. Consequently, states with ill behaved P-function are referred to as non-classical. P representation has been extended for discrete spin states too [8]. It allows for a decomposition of the spin density matrix as a weighted sum of spin coherent states [8]. Classicality of spin states, based on well behaved P representation has been explored by Giraud et. al. [9]. Luis and Rivas [10] showed that spin squeezing [11] manifestly reflects the failure of P function to be a classical probability density. They also investigated simple operational procedures revealing the violation of classical statistical bounds by non-classical spin states. In addition to studying the non-classicality of single spin systems, Giraud et. al. [9] analyzed the implications of P representation on entanglement in bipartite spin systems. They showed that a bipartite system consisting of two qubits is separable if and only if their P function is positive. However, in the case of a coupled system consisting of spin-1/2 and spin-1, it was recognized that separable states too can exhibit non-classicality manifested in terms of their non-positive or singular P function. We have confined our attention on symmetric N qubit states [12] for our investigation. Our main result is the following: the set of all separable symmetric multiqubit states possess a well-behaved positive P representation [3]. Conversely, the existence of a positive P function for a symmetric multiqubit state implies its separability [3]. In other words, we recognize that non-classicality (characterized by P-function negativity) implies entanglement and vice versa in symmetric multiqubit states [3]. We also explore how P function negativity gets reflected in terms of operational tests of non-classicality (entanglement) in symmetric multiqubit states.

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