Joint weak value for all order coupling

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In recent times the issue of `weak measurement'(WM) in quantum mechanics (QM) has gained a significant and wide interest in realizing apparently counterintuitive quantum effects. This path-breaking idea was originally proposed by Aharonov, Albert and Vaidman(AAV) [1] that introduced a measurement scenario in QM so that the empirically measured value (coined as `weak value') of an observable can be seemingly weird in that it yields results going beyond the eigenvalue spectrum of the measured observable. Traditionally, WM comprises three steps; preparation of the particles in a quantum state(termed as pre-selection), a small perturbation for producing the weak correlation between the system and pointer variables, and the selection of the particles in a particular quantum state known as post-selection. Of late, this apparently counterintuitive quantum effect has gained wide interest in studying different physical phenomena, both theoretically (see, for example [2]) and experimentally [3]. Apart from providing new insight into conceptual quantum paradoxes, WM has several practical applications.

Interestingly, the weak value can also be complex, yet not unphysical; the imaginary part of the complex weak value produces the pointer displacement in the space, in which the perturbation is introduced, and the real part produces the pointer displacement in the conjugate space. In other words, measured displacement of final meter state is proportional to the weak value induced pointer shift, while the coupling is taken to be very weak. This simple correspondence breaks down, if the higher order expansion of the perturbation is considered, instead of first order used by AAV. This is due to the presence of many peaks [2] in the probability distribution of the post-selected meter state in such a semi-weak case. Thus, for higher order expansion, the well-known definition of weak value of an observable is merely a conditional average value, which has no bearing on the weak value in the usual sense of AAV. The standard formalism fails to predict any result when pre-selected and post-selected states are orthogonal because the usual weak value in such a case is undefined. An exact treatment does not have this problem which shows the exact post-selected meter distribution for orthogonal pre and post-selected states.

In recent times, the joint weak value of two observables has been demonstrated both theoretically [4-6] and experimentally [7]. Among other intriguing implications, measurement of joint weak value is particularly interesting because a strong von Neumann measurement of joint mean values requires a nonlinear Hamiltonian, which is a difficult task to achieve. Resch and Steinberg [4] first introduced a scheme, where the joint weak value of two commuting observables, corresponding to two pointer position variables can be measured from the statistics of the correlations between pointer variables of the post-selected pointer state. Specifically, they found that the joint meter displacement is proportional to the joint weak value of two observables, if second order expansion of the coupling is taken into account. Very recently, it is argued [6] that the higher order moments of the observables can be extracted using an orbital angular momentum state described by the Laguerre-Gauss mode but inaccessible with Gaussian pointer.

We may note here that in all the existing literature demonstrating joint weak value of two observables, the coupling strength is restricted to the second order. In this paper, we extend such formulation by providing a complete treatment of joint weak measurement scenario for all order of the coupling, which allows us to reveal several hitherto unexplored interesting features. In order to do this, the pointer state is taken to be a two-dimensional Gaussian. Our treatment reveals two main results. First, one can infer joint weak value for any given strength of the coupling, if a suitable selection of particular sub-ensemble of the system is made. The standard joint weak value then appears at the limit of second order expansion of the coupling. Second, a particularly surprising result is that, the single pointer displacement can provide the information about the joint weak value, if at least fourth order of the coupling is taken into account. These important features were unnoticed due to the lack of all order treatment of the joint weak measurement which we provided in this paper. Interestingly, in our treatment, the imaginary and real parts of the weak value of both the observables can be simultaneously extracted. We also showed that the higher order moments of single-particle operators is an inaccessible quantity with Gaussian pointer states only as opposed to a recent claim [6].

As regards the implication of our scheme, we may note that joint measurements are of great importance in the field of quantum information because it contains the information about quantum correlations between different degrees of freedom. A strong measurement of joint mean values requires a nonlinear Hamiltonian, which in difficult to engineer. Interestingly, a single pointer variable contains the information of joint weak value in our scheme. Another implication of our study is that the amplification of the signal (key motivation in studying WM) can be enhanced using Joint weak value which in turn improves the precession of estimating a small parameter. Our study can reveal how the negative joint weak probabilities emerge in the quantum paradoxes involving weak measurement. We may remark that the results demonstrated here can be empirically tested by the existing experimental techniques.

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