

Lack of measurement independence can simulate quantum correlation even when signaling cannot

Manik Banik^{1,*}

¹*Physics and Applied Mathematics Unit,*

Indian Statistical Institute, 203 B.T. Road, Kolkata-700108, India

In a world described by quantum mechanics, one cannot have determinism, no-signaling and free will all together. Under these three assumptions the set of statistical correlations are restricted to the set of Bell local correlations. In standard Bell scenario any non-local correlation has a deterministic explanation if one either sacrifices free will or allows signaling. Recently in [Phys. Rev. Lett. **108**, 200401 (2012)] F. Buscemi generalizes the standard Bell scenario into ‘semi-quantum’ Bell scenario. Interestingly, in ‘semi-quantum’ Bell game scenario we show existence of correlations that cannot be obtained by sacrificing no-signaling by any amount, whereas such correlations can be obtained by giving up a small degree of free will.

Outcomes of different measurements, performed at spatially separated location, may be correlated. Correlation that satisfies three different assumptions, namely, *determinism*, *no-signaling* and *experimental free will* (or *measurement independence*) never violates the celebrated Bell inequality [1, 2], and these correlations are called *local realistic* correlation. All correlations observed in classical world satisfy Bell inequality. In quantum world performing different measurements at spatially separated location one can get correlations that violate Bell-CHSH inequality, which has been experimentally verified in [3]. Therefore if an experimenter enjoys total *free will* in choosing the measurements performed on his/her system then quantum mechanics is indeed in contradiction with *local realism*. Correlations that violate Bell inequality are known as *Bell nonlocal* correlations and the amount of nonlocality is quantified by the amount of Bell inequality violation [1, 2].

Interestingly if one is not strict with the above mentioned assumptions then also he/she can obtain *Bell nonlocal* correlations. This fact gives an operational way to quantify the

*Electronic address: manik11ju@gmail.com

amount of nonlocality in a given correlation. It has been shown that any *Bell nonlocal* correlation can be obtained by sacrificing *no-signaling* as well as sacrificing *measurement independence* in suitable amount. Therefore in Bell scenario a *Bell nonlocal* correlations that can be achieved by sacrificing *measurement independence* must be achieved by sacrificing *no-signaling*. In a recent work we will show that this is not true in general [4].

Bell nonlocal correlations can be well understood by considering the standard Bell game scenario. The standard Bell game scenario [1, 2] involves two spatially separated players, say Alice and Bob. A referee picks indices $s \in \mathcal{S}$ and $t \in \mathcal{T}$ at random and sends them separately to Alice and Bob, respectively. The two players must compute answers $x \in \mathcal{X}$ and $y \in \mathcal{Y}$, respectively, and send them to the referee, who will then pay them both. First, the players are told the rules of the game. Knowing the rules, the players are allowed to agree on any strategy and to share any variable $\lambda \in \Lambda$. The distribution $\rho(\lambda)$ of the shared variable λ must be independent of the referee's questions, say s and t ; otherwise the *measurement independence* assumption will not be satisfied. After starting the game the two players are forbidden to communicate with each other. Very recently, F. Buscemi [5] generalizes the standard Bell non-local game into 'semi-quantum' non-local game. In 'semi-quantum' non-local game referee picks indices $s \in \mathcal{S}$ and $t \in \mathcal{T}$ at random and sends quantum states τ^s and ω^t to Alice and Bob, respectively, without revealing the classical indices s and t . In this case, also, the free will assumption is applied i.e., the distribution $\rho(\lambda)$ of the shared variable λ is independent of referee's *quantum questions* and the constraint of no communication between the two players must satisfies.

In our work we show that in 'semi-quantum' non-local game scenario one can obtain correlations from entangled quantum states that can not be simulated by local operation assisted by classical communication, even if there is no limitation on classical communication, whereas such correlations can be simulated by sacrificing the *measurement independence* assumption. Thus in this 'semi-quantum' non-local game scenario in simulating the correlations achieved from entangled states, relaxation of *no-signaling* assumption has no use contrary to relaxation of *measurement independence* assumption. Therefore in 'semi-quantum' non-local game we find an operational distinction between two assumptions, namely *no-signaling* and *measurement independence*, which was not observed in the standard Bell game scenario.

In [6] Cavalcanti *et al* made a non-trivial extension of Buscemi's work. They have intro-

duced ‘quantum refereed steering games’ and have showed that all steerable states exhibit correlations in ‘quantum refereed steering games’ that can not be achieved by local operation with steering and shared randomness (LOSSR). In the context of our work it is an interesting question whether in ‘quantum refereed steering game’ one can find an operational distinction between *measurement independence* and *no-signaling*. We answer this question in negative by showing that any correlation achieved in ‘quantum refereed steering’ game can be simulated with the help of local operation assisted with finite amount of classical communications.

In an interesting discussion [7] of the measurement problem for a PR-box, J. Bub pointed out that, if Bob has access to a parameter or ontic state λ that completely specifies the output values for both of his possible inputs, or even specifies whether these output values are the same or different, then either there is a violation of no-signaling principle or Alice’s choice of input is not free but depends on the value of λ . Actually Bub’s conclusion holds for any standard Bell non-local correlation. Interestingly our result shows that in ‘semi-quantum’ non-local game there exists correlation whose deterministic explanation is possible only if free-will assumption is relaxed. Therefore in ‘semi-quantum’ non-local game scenario our result suggests further study of simulation protocol with reduced free-will.

**** the technical version of this work is available at [arXiv:1304.6425](https://arxiv.org/abs/1304.6425)**

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