Hardy's argument for temporal nonlocality

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For testing the existence of the superposition of macroscopically distinct quantum states, Leggett and Garg [1] put forward the notion of *macrorealism*. This notion rests on the classical paradigm [2, 3] that (i) physical properties of a macroscopic object exist independent of the act of observation and (ii) any observable can be measured non-invasively, i.e., the ideal measurement of an observable at any instant of time does not influence its subsequent evolution.

These original assumptions of [1], namely the assumptions of 'macroscopic realism' and 'noninvasive measurability', have been generalized to derive a temporal version of the Bell-CHSH inequality irrespective of whether the system under consideration is macroscopic or not [4, 5]. Unlike the original Bell-CHSH scenario [6] where correlations between measurement results on two distantly located physical systems are considered, temporal Bell-CHSH inequalities are derived by focusing on one and the same physical system and analyzing the correlations between measurement outcomes at two different times under the following two assumptions: (i) *Realism*: The measurement results are determined by hidden properties of the particles carried prior to and independent of observation and (ii) *Locality in time*: The result of measurement performed at time t_2 is independent of any measurement performed at some earlier or later time t_1.

These inequalities get violated in Quantum Mechanics and thereby give rise to the notion of *entanglement in time* which has been a topic of current research interest [3, 4, 7–11]. Interestingly, the original argument of Hardy, which establishes the incompatibility of Quantum Theory with the notion of local-realism [12, 13], can also be used to reveal this time-nonlocal feature of quantum states [7–9]. Recently, Hardy's argument has been studied in the case of two observables setting at each time of measurement [7, 8]. It has been shown there that the maximum probability of success of this argument can assume up to 25% for a spin-$\frac{1}{2}$ particle [7–9], the experimental verification of the above fact followed soon after in [9].

Hardy's argument was generalized by Clifton and Niemann [14] to show the spatial nonlocal feature of two spin-s systems. This argument was later reduced to its minimal form by Kunkri and Choudhary [15]. Inspired by [15], we write below the temporal version of Hardy's nonlocality conditions for d-level systems.

Consider a single d-level physical system on which an observer (Alice) chooses to measure one of two observables $\hat{A}_1$ or $\hat{A}_2$ at time t_1, whereas at a later time t_2, another observer (Bob) [16] measures either of the two observables $\hat{B}_1$ and $\hat{B}_2$. Consider now the following set of conditions on the probabilities for Alice and Bob to obtain outcomes a_i and b_j when measuring observables $\hat{A}_i$ and $\hat{B}_j$ respectively; $i,\, j \in \{1, 2\}$:

$$\text{prob}(\hat{A}_1, a_1\ ;\ \hat{B}_1, b_1) = 0, \tag{1}$$

$$\text{prob}(\hat{A}_1, \neg a_1\ ;\ \hat{B}_2, b_2) = 0, \tag{2}$$

$$\text{prob}(\hat{A}_2, a_2\ ;\ \hat{B}_1, \neg b_1) = 0, \tag{3}$$

$$\text{prob}(\hat{A}_2, a_2\ ;\ \hat{B}_2, b_2)\ > 0. \tag{4}$$

The first condition says that if Alice chooses to measure the observable $\hat{A}_1$ and Bob chooses observable $\hat{B}_1$, he will not obtain b_1 as measurement result whenever Alice has detected the measurement value a_1. The remaining equations can be analyzed in a similar manner ($\neg a_i$ denotes a measurement with any result other than a_i and similarly $\neg b_j$ denotes a measurement with any result other than b_j). These four conditions together form the basis of the temporal version of Hardy's argument for d-level physical systems. This version of Hardy's argument makes use of the fact that not all of the conditions (1)-(4) can be simultaneously satisfied in a time-local realistic theory, but they can be in quantum mechanics (see ref. [17] for the details).

In this work, we study the above mentioned version of Hardy's argument for arbitrary observables of the system and find that the maximum success probability of this argument remains 25% irrespective of the dimension of the system (for details, we refer to [17]). This is in sharp contrast with the findings of reference [7] where for spin observables it has been stated that the maximum probability of success of Hardy's argument decreases with increase in spin value of the system involved. We also discuss the reason of this discrepancy.

Thus this temporal nonlocality persists as opposed to the idea that quantum systems with higher dimensional state space behave more classical which was put forward in [14]. Our result is at par with the findings for spatially separated systems where the success probability for Hardy's argument is also independent of the dimension of systems' Hilbert space [18, 19]. Moreover, contrary to the implications from [7], our result yields the possibility to probe the existence of quantum superpositions for macroscopic systems by means of Hardy's argument and thus independent of the Leggett-Garg inequality.

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[5] There is one more difference: while in [1], the observer measures a single observable at different times under the supposition that its value is independent whether or not a measurement (of the same observable) was performed at some earlier or later time, in [4] at any given time the observer has a choice between two (or more) different measurement settings. The value assigned to an observable $\hat{A}$ at time t_1 is independent of whether or not a measurement was performed at some different time t_0 or which observable at that time was measured.

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