Effect of Bloch Vectors on Quantum Discord of Non-X State

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Abstract: When the Bloch vectors are perpendicularly oriented, the level surfaces of quantum discord of non-X state are formed by three "tubes" along orthogonal directions. The level surfaces shrink when the Bloch vectors are increased while they expand when the quantum discord are increased. For different Bloch vectors, the tube along one direction may disappear while tubes along other directions may still exist.

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1. Introduction

The entanglement plays an important role in quantum information processing [1]. Various quantum information processing tasks can be achieved based on entanglement. However, entanglement is not the only kind of correlation useful in quantum information theory. It was recently demonstrated that the absence of entanglement is not the sign of being quantum locality in quantum correlation[2]. For some cases, the quantum correlation can show some quantum properties of the system when the entanglement is zero[3]. So it is necessary to characterize and quantify quantum correlations.

To measure the quantum correlation, quantum discord (QD) was introduced by Ollivier and Zurek [4] and by Henderson and Vedral [5]. The QD is the difference between two expressions of mutual information for a quantum system and attracts much attention in recent years. It shows that QD may be a more general nonclassical correlation compared with entanglement, and only the state with zero QD represents purely classical correlation.

The dynamics of QD shows that QD is more robust than entanglement under the Markovian and non-Markovian dissipative processes. It is known that Bell-diagonal state[6] is a special X-state with zero Bloch vectors. Recently, QD for a class of two-qubit states with parallel nonzero Bloch vectors is investigated [7].

In this article, the quantum discord of non-X state with Bloch vectors in the *x* and *z* directions is investigated. The Bell-diagonal states [8] and X-states [9] are included as the special cases. The analytical expressions of the eigenvalues of a two-qubit state are derived. The deformation of the geometric objects, such as tetrahedron *T* and octahedron *O* are obtained. The level surfaces are presented when the Bloch vectors and the quantum discord are varied.

2. Quantum Discord for Non-X State

For a non-X state, if the Bloch vectors $\vec{r}$ and $\vec{s}$ are orthogonal vectors with $\vec{r}$ =(r,0,0) and $\vec{s}$ =(0,0,s), the state can be written in the matrix form

$$\rho=\frac{1}{4}\begin{bmatrix}1+s+c_3 & 0 & r & c_1-c_2\\ 0 & 1-s-c_3 & c_1+c_2 & r\\ r & c_1+c_2 & 1+s-c_3 & 0\\ c_1-c_2 & r & 0 & 1-s+c_3\end{bmatrix} \tag{1}$$

where c_1, c_2, c_3 are parameters orthogonal with each other. The quantum discord can be written as

$$Q(\rho) = S(\rho^B) + \sum_{i=1}^{4} \lambda_i \log_2 \lambda_i + \min[S(\rho|\{B_k\})] \quad (2)$$

where $S(\rho^B) = 1 - \frac{1-s}{2}\log_2(1-s) - \frac{1+s}{2}\log_2(1+s)$ is the von Neumann entropy, λ_i are the four eigenvalues of Eq. (1) and are functions of r, s, c_i.

The level surface of quantum discord is plotted in Fig. 1when the Bloch vectors are varied. In Figs. 1(a) and 1(b), the level surfaces of the discord are formed by three intersecting "tubes" along the orthogonal directions of c_1, c_2 and c_3. The diameters of the tubes decrease as c_1, c_2 and c_3 increases. When the Bloch vectors $|\vec{s}| = 0.3$ and $|\vec{r}|$ is increased from r=0.1 to r=0.5, the level surfaces (tubes) are squeezed towards the center of the coordinate. It is noted that the tube along c_2 axis disappears. That is, the tube along c_2 axis is squeezed much faster than the tubes along other axes when r is increased. At the same time, the tetrahedron T and the octahedron O are also shrunk to the center of the figure. In Figs. 1(a) and 1(c), the quantum discord is increased from $Q(\rho) = 0.03$ to $Q(\rho) = 0.15$. The tubes are expanded and cut off by the state tetrahedron T. Similar phenomenon is also found in Figs. 1(c) and 1(d). Meanwhile, both classical correlation C and the quantum mutual information I increase with the absolute value of c_i when the quantum discord Q, i. e., the difference between them keeps a constant.

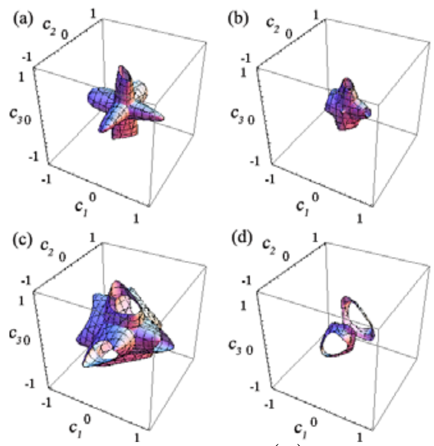

Fig. 1. Surfaces of quantum discord with $Q(\rho) = 0.03$, s=0.3, (a) r=0.1, (b) r=0.5; and $Q(\rho) = 0.15$, s=0.3, (c) r=0.1, (d) r=0.5.

3. Conclusion

The level surfaces of quantum discord for a class of two-qubit non-X states are investigated when the Bloch vectors $\vec{r}$ and $\vec{s}$ are perpendicularly oriented. The geometric objects of tetrahedron T and octahedron O are deformed. For some parameters and certain directions of Bloch vectors, the states are reduced to either Bell-diagonal state or X-state. The level surfaces of the quantum discord are formed by three "tubes" along three orthogonal directions of c_1, c_2 and c_3. The level surfaces shrink to the center when the Bloch vectors are increased. The tube along c_2 axis disappears since it is squeezed much faster than those along other axes when r is increased. The level surfaces are expanded and cut off by the state tetrahedron T when the quantum discord is increased.

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