Asymptotic minimization of decoherence for ion trapping

Samyadeb Bhattacharya[a] and Sisir Roy[b]

[a,b]Physics and Applied Mathematics Unit, Indian Statistical Institute,
203 B.T.Road, Kolkata 700108, India

Abstract: Decoherence time has been calculated for an optical ion trap in a bistable potential model. Comparison has been made between decoherence time and Zeno time for double well potential as a special case. Zeno time is considered as a lower limit of decoherence time for sustainable quantum coherence. Equality of the respective timescales provides a certain transitional temperature, below which decoherence can be asymptotically minimized.

Keywords: Dissipative quantum system, Decoherence, Zeno time .

The complete version of this work can be found in arXiv:1301.1127 . This work has been accepted for publication in Modern Physics Letters B.

In recent times, optical ion traps has been prepared to construct quantum logic gates [1, 2]. In those systems, quantum decoherence plays a very significant negative role [3]. The system loses information due to environmental interaction. Since it is impossible to disentangle the system from the environment, decohering effect cannot be eradicated completely. Our main effort in this work is to find the conditions, under which the decohering effect due to environmental interaction can be minimized. Zeno dynamics plays a significant role in this attempt of decoherence minimization [4, 5]. Quantum Zeno effect is the phenomenon of complete freezing of decay dynamics, due to frequent (almost continuous) measurements [6]. In our understanding, these two phenomena (Decoherence and Zeno effect) have got a very intrinsic reciprocal relation. Whenever frequent non-selective measurement dominates the time evolution of the state, the system is forced to evolve in a subspace of the total Hilbert space, which is called "Zeno subspace". Probability leakage is not possible between these invariant Zeno subspaces. So each of these subspaces can be considered as some reduced isolated system. Due to their isolated nature, the process of decoherence can be halted within these Zeno subspaces. Now, if the environmental interaction is very strong, extreme decoherence may not allow the Zeno subspaces to sustain. So Zeno effect, characterized by it's corresponding time scale (Zeno time), gives a certain lower limit to decoherence, below which it is uncontrollable. If the decoherence time is smaller than the Zeno time, then the process of decoherence dominates the scenario. Exploiting this condition, in this work, we have formulated the procedure to compare the respective time scales of decoherence and Zeno effect and come up with a certain transitional temperature, below which it is possible to minimize decoherence asymptotically.

Here we have considered an asymmetric double well potential approximated as a two-state system with considering only the ground states of the wells separated by an asymmetry energy ϵ. We construct our model on the demonstration of a quantum logic gate prepared by a trapped ion laser cooled to the zero point energy [1]. In this particular case, the target qubit is spanned by two $^2S_{1/2}$ hyperfine ground states ($|\uparrow\rangle$ and $|\downarrow\rangle$ states) of a single $^9Be^+$ ion separated by $\nu_0 = 1.250$ GHz. We set these two energy levels as the ground states of the two wells of the double well potential separated by the asymmetry energy $\epsilon = h\nu_0$. The control qubit $|n\rangle$ is spanned by the first two states of trapped ion ($|0\rangle$ and $|1\rangle$), which can be identified by the first two states of each well approximated as harmonic oscillators. These two states are separated by $\nu_x \simeq 11$ MHz. So the basic four eigenstates are given by $|n\rangle|S\rangle = |0\rangle|\uparrow\rangle, |0\rangle|\downarrow\rangle, |1\rangle|\uparrow\rangle, |1\rangle|\downarrow\rangle$.

To calculate the Zeno time, we have used the formalism of retarded Schrödinger difference equation, originally developed by Caldirola and Montaldi [7]. Without going into the technicalities, now we directly come to the principal findings of our work. The ratio of decoherence time and Zeno time is found to be

$$\frac{\tau_{dec}}{\tau^Z} = \frac{2\hbar}{wK_BT}\sqrt{\frac{3\epsilon}{m}} \tag{1}$$

where w, T, ϵ and m are the distance between the wells, temperature, the asymmetry energy and the mass of the particle respectively. Preservation of quantum coherence leads us to conclude that the Zeno time represents a certain lower limit to decoherence time, under which the process of decoherence is uncontrollable and the system loses it's "quantumness". So from eqn.(1), we find that imposition of this lower limit to decoherence time leads us to a certain transitional temperature

$$T_{tran} = \frac{2\hbar}{wK_B}\sqrt{\frac{3\epsilon}{m}} \tag{2}$$

Above this temperature, the process of decoherence cannot be controlled.

Robust quantum memories are essential to realize the potential advances in quantum computation. Optical ion trap can be realized as a quantum storage device. But it is also essential to protect the information, which can be lost due to environmental interaction in the form of decoherence. So it is very important to control the decohering effect in order to build an effective ion trap quantum computer. In this work, we have dealt with the question that whether and under what condition environment induced decoherence can be minimized. As we have discussed that in our understanding, the intrinsic relation between decoherence and Quantum Zeno effect can be exploited in this aspect. Frequent nonselective measurement forces the system to evolve in the reduced zeno subspaces, which can be considered as some “quasi-isolated” system. If the Zeno effect is strong enough, so that the reduced subspaces remains quasi-isolated even under the influence of environmental interaction, effect of decoherence can be controlled. Based on this theoretical understanding, we have calculated a certain transitional temperature, by comparing the decoherence and Zeno timescales. It is clear from the above analysis that below this transitional temperature we can increase the decoherence time by controlling the parameters (w, ϵ). Hence we can minimize the decohering effect asymptotically, though it can never be eliminated completely.

---

[1] C.Monroe et.al.; Phys.Rev.Lett. **75**, 4714(1995).

[2] J.I.Cirac and P. Zoller; Phys.Rev.Lett. **74**, 4091(1995).

[3] W.H. Zurek; Progress in Mathematcal Physics; **48**; 1 (2007).

[4] L. Viola and S. Lloyd; Phys. Rev. A **58**, 2733 (1998).

[5] J.D. Franson, B.C. Jacobs and T.B. Pittman; Phys. Rev. A **70**, 062302 (2004).

[6] B.Mishra and E.C.G.Sudarshan; J.Math.Phys. **18**, 756(1977).

[7] P. Caldirola and M. Montaldi; Nuovo Cimento Soc. Ital. Fis., B **53**, 291 (1979).