Non-locality breaking qubit channels

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I. INTRODUCTION

Non-classical features of quantum theory are revealed in several ways. For continuous variable quantum systems, one may, in particular, look into the non-positivity of the Sudarshan-Glauber P distribution [1], [2] to infer that the associated state has 'non-classicality' in the sense of literatures in quantum optics. For finite dimensional quantum systems, there is no unique quantitative way of describing non-classicality in quantum states. To put things in a quantitative manner, we raise the following question in the setting of the dual picture of states: given a concept of non-classicality , how strong a quantum channel should be in order to bring a state of a single mode quantum system to a classical state? In order to quantify this strength, we may compare the capacity of the channel to break entanglement of two-mode states and that of breaking the non-classicality of any two-mode state— the channel being acting, in each case, on one of the two modes. In other words, how does one relate an entanglement-breaking channel with a 'non-classicality' breaking channel.

One of the important manifestations of non-classicality in composite quantum systems is non-locality. A (universal) 'non-locality breaking' channel is the one which, when acts on one subsystem of a composite system's arbitrary state, brings it to a state whose measurement statistics (on the individual subsystems separately) can be reproduced by a local hidden variable model (*i.e.,* the state is local). Once again, we can ask: how is an entanglement breaking channel related to a non-locality breaking channel? It is known that a state of a composite system is local iff it satisfies all possible local realistic inequalities—an impossibility to verify, in general [3]. Thus the ideal situation to characterize a non-locality breaking channel for a d dim. quantum system would be to figure out the necessary-sufficient condition for satisfiability of all the independent local-realistic inequalities for a $d \otimes d'$ system for all integers $d' \geq 2$—a seemingly impossible task! In fact, except for the Bell-CHSH inequality (in $d = d' = 2$ case [4], [5]), there is no such necessary-sufficient

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condition available so far [6]. In this scenario, we concentrate in this paper only on the necessary-sufficient condition for satisfiability of the Bell-CHSH inequality.

Our goal, in this paper, is to see how an entanglement-breaking qubit channel differs from a non-locality breaking qubit channel, even though in the later case, we only consider here the action of the channel – unlike in the case of the former – on one of the qubits of a two-qubit state. We confine ourselves to the case of two-qubits as no necessary-sufficient condition for the satisfiability of any local realistic inequality for a $2 \otimes d$ system is known so far for $d \geq 3$ – unlike the case when $d = 2$ [6].

II. UNIVERSALITY OF NON-LOCALITY BREAKING PROPERTY OF UNITAL QUBIT CHANNELS

Assuming that a unital qubit channel [7] breaks non-locality of the two-qubit maximally entangled state $|\beta\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, it can be shown—using a lemma on eigenvalues of positive semi-definite matrices [8]—that the same channel also breaks non-locality of any other two-qubit state, the action of the channel being taken on one of the two qubits (see section IV of ref. [9] for the details).

This result is at par with the entanglement-breaking condition of any qubit channel [10] to the extent that the dimension of the other subsystem is restricted to two.

It is interesting to note that ratio of the volume of all entanglement-breaking unital qubit channels with that of all unital qubit channels is 0.5, while for unital qubit non-locality breaking channels , we found the corresponding ratio to be about 0.95. Thus, almost any unital qubit channel is non-locality breaking while there is a 50% chance that it is entanglement-breaking.

III. NON-UNIVERSALITY OF NON-LOCALITY BREAKING PROPERTY OF NON-UNITAL QUBIT CHANNELS

Considering a particular non-unital qubit channel (which is not an extremal channel), which breaks the non-locality of the maximally entangled state $|\beta\rangle$, we show numerically that there is at least one two-qubit non-maximally entangled state whose non-locality is not broken by the channel (see section VI.A of [9] for the details).

On the other hand , if we consider the amplitude damping qubit channel (an extremal non-unital channel), given by $\$(|0\rangle\langle 0|) = |0\rangle\langle 0|$, $\$(|1\rangle\langle 1|) = p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$, $\$(|0\rangle\langle 1|) = \sqrt{1-p}|0\rangle\langle 1|$,

$\$(|1\rangle\langle 0|) = \sqrt{1-p}|1\rangle\langle 0|$ (for $0 \leq p \leq 1$), we have then shown numerically that such a channel is universally non-locality breaking for all $p \geq \frac{1}{2}$ (see section VI.B of ref. [9] for the details).

IV. UNIVERSALITY FOR NON-LOCALITY BREAKING OF CHANNELS UNDER CONSTRAINT

It is shown that if for a qubit channel \$, the Choi state $(I \otimes \$)(|\beta\rangle\langle\beta|)$ [11] is a local state (*i.e*,, it satisfies the Bell-CHSH inequality) then $(I \otimes \$)(\rho_{AB})$ is also a local state for every two-qubit state ρ_{AB} provided $Tr_B(\rho_{AB})$ is maximally mixed (see section VII of [9] for the proof). Our proof here also shows that the composition of a qubit channel, that breaks non-locality of two-qubit maximally entangled state, with any other qubit channel also does the same job.

V. DISCUSSION

It is clear that every entanglement-breaking qubit channel is also universally non-locality breaking but the converse is not true, in general. But we do hope that if one can figure out the class ($\mathcal{C}_d$, say) of all qubit channels each of which can be universally non-locality breaking with respect to some suitable tight local realistic inequality (or, a set of inequalities) for $2 \otimes d$ systems, then $\mathcal{C}_2 \supseteq \mathcal{C}_3 \supseteq \ldots$, and we conjecture here that the class $\mathcal{C}_\infty \equiv \lim_{d\to\infty} \mathcal{C}_\infty$ must be equivalent to the class of entanglement-breaking qubit channels — so that, in this asymptotic sense, there won't be any difference between the notion of entanglement-breaking and non-locality breaking .

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