Entanglement and non-classicality of generalized quantum optical vortex states

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Entanglement is a fundamental characteristic of quantum mechanics. It is this feature of quantum mechanics which is being increasingly studied and analyzed to achieve a variety of quantum phenomena like quantum teleportation [1] and quantum information processing [2]. In the not so distant future these emerging technologies will define how we communicate and compute. Quantum optics [3, 4, 5, 6, 7]or more specifically photonic states of light are speculated to play an ever increasing role in this particular field. It is therefore important to study the entanglement features of various such states of light which can be generated and put to use for a practical model of quantum information processing.

In our present work we study the entanglement present in a class of generalized quantum optical vortex states [8, 9, 10, 11, 12, 13]. The states that we consider here are engineered by subtraction of photons from one of the modes of the output of a type II spontaneous parametric down conversion. These states have the form

$$|\xi\rangle_k^{(s)} = \mathcal{A}_k b^k \exp\left(\xi a^\dagger b^\dagger - \xi^* ab\right)|0,0\rangle \tag{1}$$

We call these two mode squeezed vortex (TMSV) states. Subtracting a photon from one of the modes is same as adding a photon in the other mode since the total number of photons in the two modes is a constant. If a photon is annihilated in one of the modes, it reappears at the other mode. The order of the vortex is determined by the difference in the number of photons between the two modes. If k photons are subtracted, the state, after some simplification, can be written as,

$$|\xi\rangle_k^{(s)} = \frac{e^{ik\theta}}{\cosh^2 r} a^{\dagger k}|\xi\rangle \tag{2}$$

An interesting feature of these vortex states is that there is always a fixed difference between the number of photons in the signal and the idler modes. The order of the vortex is determined by this difference in the number of photons between the two modes. In literature these are the so called pair coherent states [14, 15]. Pair coherent states provide an important example of non classical states of the two mode radiation field. It has been studied in detail for their non classical properties and as examples of EPR states [14].

Another interesting property of the TMSV states is the appearance of the vortex structure in

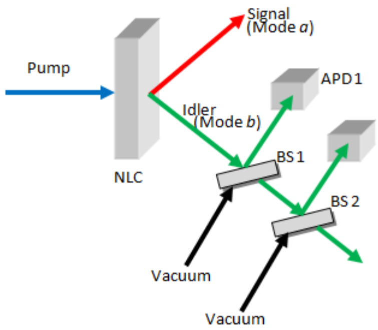

Figure 1: A schematic of the setup for producing TMSV state of order 2. Two 99% beam splitters(BS 1 and BS 2) are used for subtracting photons. NLC is a nonlinear crystal. SPDC generates the signal (mode a) and the idler (mode b). APD 1 and 2 are Avalanche Photo Diodes.

the quadrature space. Writing the bosonic creation operator $a^\dagger$ as $(x - \partial/\partial x)$, the quadrature distribution of the k photon subtracted state has the form,

$$\Psi_k^{(s)}(x,y) = \frac{e^{ik\theta}}{2^{k/2}\text{cosh}r}\left(x - \frac{\partial}{\partial x}\right)^k \Psi^{(s)}(x,y) \tag{3}$$

The presence of the vortex is pretty evident from the contour plot. The order can be determined from the number of singular points in the phase plot. It is interesting to note that the vortex structure in the quadrature distribution becomes less prominent with increasing order. It is thus important to study the variation of non classicality and entanglement with order and the squeezing parameter. We use the Wigner function [16] to study the non classicality of this state. For the current state under study it can be written as

$$W(\tilde{\alpha},\tilde{\beta}) = \frac{4}{\pi^2}(-1)^k \mathcal{L}_k\left[4|\tilde{\alpha}|^2\right]\exp\left[-2\left(|\tilde{\alpha}|^2 + |\tilde{\beta}|^2\right)\right], \tag{4}$$

where k is the number of photons subtracted. The presence of the negative regions is a signature of non classicality [17]. The volume of this negative region can be studied as a measure of non classicality. The cross correlaton between different quadratures of two different modes show interesting quantum inteference patterns. This can also be interpreted as a signature of entanglement. In support of this we study the entanglement present in this state with the help of logarithmic negativity [2, 18] and entanglement of formation [19, 20] or concurrence and compare the observations from the two approaches. The concurrence is a measure of the amount of entanglement needed to create the entangled state and defined as [21],

$$C(\rho) = \text{min}_{p_i,\psi_i}\sum_i p_i E\left(|\psi_i\rangle\langle\psi_i|\right) \tag{5}$$

The minimization is taken over all possible decompositions of pure states ψ_i with probabilities p_i, which taken together reproduce the density matrix ρ of the given mixed state. On the other hand, $E\left(|\psi_i\rangle\langle\psi_i|\right)$ is the entropy of entanglement [22], more commonly known as the von Neumann entropy [23], of the reduced density matrix. The von Neumann entropy is a well defined standard measure of pure state entanglement. For mixed states, it quantifies the amount of mixedness present between the constituent modes rather than the entanglement present between them. From our study of the von Neumann entropy we found out that the state changes from a

completely pure state to a completely mixed state with increasing value of the squeezing parameter. At $r = 3.2$, the state becomes completely mixed. This means that apart from squeezing the quadratures, the SPDC also results in mixing them.
We observe some surprising results when studying the logarithmic negativity and the concurrence. It was observed from our study of logarithmic negativity, that for two photon subtraction there was a very small rise in entanglement when compared to that of single photon subtraction. The entanglement falls sharply after reaching the peak. Further subtraction worsens the situation. For three photon subtraction the initial value falls to half. For four photon subtraction, the starting value is 0.042. However, our study of concurrence tells a different story. We observe that $C(\rho)$ increases with increase in r, the squeezing parameter. The rate of increase gradually slows down till it reaches the maximum value and then falls off rapidly to 0. It should be noticed that $C(\rho)$ attains its maximum value at $r \sim 2.1$. This means that the two modes become more and more entangled with increasing r until it reaches a maximum value beyond which the modes start becoming disentangled. We infer that there exists an optimum value of the squeezing parameter which when applied at the beam splitter produces maximum entanglement of the two modes. If we go on squeezing the modes more and more, they will no more be entangled. As we move to higher orders of the vortex, the range of values of r which produces maximal entanglement, starts broadening. At k=5, there exists a well defined range of r that produces maximal entanglement of the two modes. The entanglement falls off rapidly on both the ends of this range. So we state that the TMSV states become maximally entangled mixed states (MEMS) for a particular value of the squeezing parameter. It would thus be interesting to compare the properties of these states with other MEMS states in order to check the suitability for applications in the field of quantum information and computation.
Here we also present an interpretation of the quantum inteference patterns observed in our study of the cross correlations between different quadratures of the two different modes. It is observed that the interference is most prominent for $r = 2.1$. The interference effects decrease as we move away from $r = 2.1$ on either side which is exactly similar to what we obtained from the entanglement of formation. Broadening of the range of r which gives rise to a well defined interference fringe system with increase in the order of the vortex was also observed. Hence, we suggest that the quantum interefence effects arising from the Wigner function of the cross correlation between different quadratures of the two modes can be interpreted as a signature of the entanglement present between the two modes. It is seen that the number of fringes as well as the volume of the fringes change with changing r. However, the maximum number of negative fringes is restricted by the order of the vortex. Since the negativity of the Wigner function is interpreted as a measure of non classicality, we suggest, the volume of the negative part of the quantum interference fringes might be a possible candidate for quantifying entanglement and needs to be further studied.
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For figures, a complete list of reference and a more technical description please refer to our work: `http://arxiv.org/abs/1304.2887`

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