Solution for the discrimination of three qubit-mixed quantum states

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The goal of quantum state discrimination is to discriminate the quantum states of a given set as well as possible. In fact, in classical physics, every state can be orthogonal to each other and therefore perfectly distinguished [1]. However, in quantum physics, a state cannot be perfectly discriminated because of the existence of non-orthogonal states [2–4]. Quantum state discrimination [5] is classified into minimum error discrimination, originally introduced by Helstrom [2],unambiguous discrimination [6–8], and maximum confidence discrimination [9]. The purpose of minimum error strategy is to find the optimal measurement and the minimum error probability (or guessing probability) for arbitrary N qudit-mixed quantum states with arbitrary priori pobabilities. In the case of $N = 2$, regardless of the dimension, the Helstrom bound [2] gives an analytic solution to the problem. In the $N = 3$ case, the analytic solution for the pure qubit states is provided by [10, 11]. In [12] the analytic solution for the mixed qubit states is considered without the necessary and sufficient conditions for a solution. In other words, full understanding for the discrimination of 3 qubit-mixed quantum states has not yet been provided.

The von Neumann measurement [13] is used for optimal measurement for linearly independent quantum states . However, if the given quantum states are linearly dependent, the von Neumann measurement may not be optimal. Therefore, the Positive-Operator-Valued-Measure (POVM) should be used for arbitrary quantum states. From the point where POVM can be used as the measurement and the probability to guess the quantum states correctly becomes convex, the minimum error discrimination problem may be solved by convex optimization[14]. Other efforts to solve it have been made using the dual problem [15] or complementarity problem [16].

By applying qubit state geometry to the optimality conditions for the measurement operators and complementary states, Bae [17] obtained a geometric method to find the guessing probability and the optimal measurement for some special cases. However, they did not include the case where the optimal measurement cannot be POVM when every element is nonzero. In this paper, we show that the case where the optimal measurement cannot be POVM and where every element is nonzero can be understood through the existence of parameters satisfying the geometric optimality conditions [16]. We also clarify the meaning of these geometric conditions. Through the conditions and an inductive approach, we propose a method to discriminate arbitrary N qubit-mixed quantum states with arbitrary a priori probabilities. In this method, we define the intrinsic polytope for discrimination problems. When the polytope becomes a point, line segment, or triangle, we find the guessing probability, the necessary and sufficient condition for the exact solution, and the optimal measurement analytically. By the number of the extreme points for the intrinsic polytope and the geometric optimality conditions, we can provide a complete analysis for discrimination of the 3 qubit-mixed state. We also obtain its guessing probability and optimal measurement.

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