Purification and retrieval of entanglement via single local filtering

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Quantum entanglement is a key resource for quantum information processing, quantum teleportation and communication, quantum cryptography and such other areas. But its fragility to environmental interaction is a cause of great concern for all its technological applications. It is well known that when a pure entangled state interacts with the environment, in addition to reduction in its entanglement, it loses its coherence also thus becoming a mixed state [1]. Purification [2] and concentration of entanglement [3, 4] has thus been an issue of great importance. Several protocols including collective measurements, Local Quantum Operation with Classical Communications (LQCC) and local filtering operations are proposed to perform this task [2–14]. In continuation of this stream of work, we have analyzed the effect of *single* local filtering operation with respect to purification and concentration of entanglement in quantum states.

In the first part of the work, we have examined the effect of filtering on a single qubit of the 3-qubit states belonging to two inequivalent SLOCC families. Filtering is a non-trace-preserving map that is seen to be capable of increasing entanglement with some probability and it represents a dichroic environment the extreme examples of which are polarizers [9]. We have shown here that redistribution of entanglement takes place between the subsystems of 3-qubit W state ($|W\rangle$) and superposition of W, obverse W states ($|W\bar{W}\rangle$). In both the states, we have illustrated the purification of that part of the state not acted upon by the filtering operator. Inspite of both 3-qubit GHZ and $|W\bar{W}\rangle$ belonging to the same SLOCC class [15], the subsystems of GHZ states remain disentangled even after single local filtering whereas the entanglement of that subsystem of $|W\bar{W}\rangle$ not directly affected filtering increase at the expense of the other part. Both purification and nature of entanglement transfer are shown to be dependent on the filtering parameter.

In the second part of the work, we have demonstrated the probabilistic retrieval of entanglement after Entanglement Sudden Death (ESD) [16, 17], caused due to single local filtering on a non-decohering qubit of both $|W\rangle$, $|W\bar{W}\rangle$ subjected to depolarizing noise on their remaining two-qubits. For all values of the filtering parameter, $0 \leq k \leq 1$, delay in the onset of ESD and increase in the subsystem entanglement is seen for $|W\bar{W}\rangle$ but in the case of $|W\rangle$, such a phenomenon happens only in the range $0 \leq k \leq 0.5$. For $0.5 < k < 1$, an early onset of ESD and decrease in entanglement of that part of the system not directly affected by filtering is observed in $|W\rangle$. This knowledge of the filtering-parameter dependence of the reversal of ESD (or otherwise) in the subsystems of a composite quantum state will help in choosing the right filter for the task under consideration. In this context, we wish to mention the work of M. Siomau and A. A. Kamli [14] where they have shown the effect of a local filter on the probabilistic retrieval of entanglement after ESD due to Generalized Amplitude Dampling [14, 18]. They have illustrated this result for 3-qubit W state and 4-qubit cluster states choosing a particular filtering parameter. Our choice of the depolarizing noise is in view of the observation that it is more effective in causing sudden death of entanglement in quantum states [19]. In [20], we have provided both analytical and graphical illustrations of all the results mentioned. The results are extendible to multiqubit states with $N > 3$. We hope that this work will shed more light on the significance of local filtering action on quantum states. It is an open question to study the filtering action upon higher dimensional states and utilize the enhanced entanglement coherences and purity to the desired applications.

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