# Broadcasting of correlation and its use in quantum information processing

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#### Abstract

It often happens in quantum information processing that we are given an entangled pair and asked to create more entangled pairs by using local quantum operations. This need arises from the fact that sometimes there is more priority on increasing number of participants in information processing scheme rather than on the efficiency of it. To address this we consider a $2 \otimes 2$ dimensional system and apply optimal universal quantum cloner to create more entangled pairs. Then we analyze the dependence of maximal fidelity of teleportation and super dense coding capacity on broadcasting and cloning fidelities. Since the amount of quantum correlation, namely entanglement and discord, between the participating subsystems play a vital role in the efficiency of information processing schemes, we further study the dependence of these quantum correlations, on broadcasting and cloning fidelities.

## 1 Introduction

The importance of the work stems from the fact that in today's perspective improving the efficiency of quantum information processing as well as simultaneously extending it by creating more entangled pairs is a topic of great interest and importance. The two very essential and smart ways of information processing in quantum world are teleportation and super dense coding schemes. Both of them are fundamentally dependent on the phenomena of entanglement (or in a greater sense quantum correlation) between the participating subsystems. Thus the enhancement and extension of quantum correlations to more pairs given one, can be of great utility and hence the need arises to broadcast entanglement. Fortunately, the method to efficiently disperse entanglement from between a pair to two, has already been successfully addressed earlier by Adhikari et. al.(2006) depending on the method of the use of local quantum copier. The efficiency of teleportation between two subsystems is portrayed by the measure of maximal fidelity of teleportation and that of dense coding is portrayed by the measure of super dense coding capacity. Here we look forward to extend this study so as to analyze how does the maximal fidelities of teleportation and capacities of dense coding schemes and measures of quantum correlations, namely entanglement and discord vary with broadcasting and cloning fidelities. This will enlighten us with insights for enhancing quantum correlations between the participating subsystems and for simultaneously extending it to other subsystems. Thus in turn the insight will fertilize the efficiency of quantum information processing between the participating subsystems.

We start with a 2 qubit non-maximally entangled state of a systems made of two subsystems A and B and study the maximal teleportation fidelity and super dense coding capacity of that state. We also calculate the entanglement measure and discord measure for analyzing the magnitude of quantum correlation present between the two subsystems. Next we apply local state independent B-H cloning transformation to generate local cloned copies, namely C and D, from A and B respectively. Proceeding in this way, we intend to broadcast the quantum correlation, namely entanglement, as was addressed by Adhikari et. al.(2006). This is done to create two entangled pairs from one. Following the methodology of Adhikari et. al.(2006), i.e. next using Peres-Horodecki criterion of inseparability, we also check for which values of the input state parameter, say $\alpha$, the states are entangled and for which separable. This provides us the insight to choose appropriate states which will increase the correlation between the nonlocal copies, say A and D or B and C and decrease the same between the local ones A and C or B and D and the nonlocal copies, say A and B or C and D. Lastly in this section, we analytically calculate the respective broadcasting and cloning fidelities and study the dependence of the fidelities of information processing schemes on them. Further we also study another measure of correlation, namely discord, of the states participating in the information processing tasks, since sometimes discord gives us a measure of quantum correlation beyond entanglement. We similarly analyze the results starting with the Werner state to generalize our scheme in case of mixed states as well.

## 2 Analytical Expressions and Results

### 2.1 Non-maximally entangled state

The Schimdt decomposition form of the non-maximally entangled state:$|\psi\rangle_{12}^{in} = \alpha|00\rangle + \beta|11\rangle$

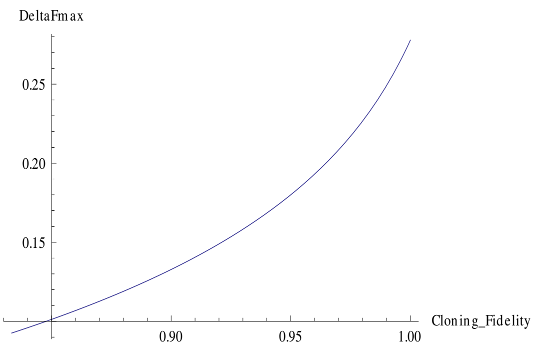
where $\alpha$, $\beta$ are the probability amplitudes and are defined as $\alpha = \frac{1}{\sqrt{1+n^2}}$ and $\beta = \frac{n}{\sqrt{1+n^2}}$; $\alpha^2 + \beta^2 = 1$

#### 2.1.1 Teleportation

We find the analytical expression to study the dependence of its maximal fidelity of teleportation on cloning fidelity $(FC)$ and broadcasting fidelity $(FB)$ applying the cloning transformation. Here the difference of maximal fidelities of teleportation before cloning $(F_{max}^{in})$ and after cloning $(F_{max}^{c})$ is given by $\triangle F_{max} = F_{max}^{in} - F_{max}^{c}$.

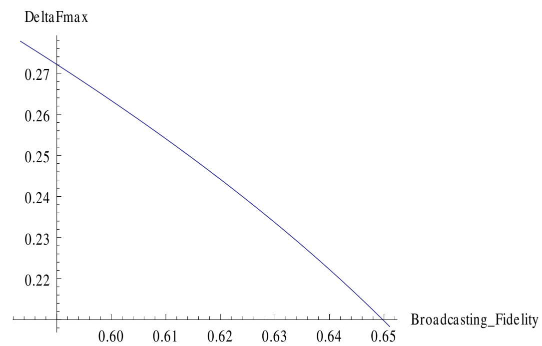
⋆ With Cloning Fidelity:

**Plot of $\triangle F_{max}$ versus $FC$ :**




⋆ With Broadcasting Fidelity:

**Plot of $\triangle F_{max}$ versus $FB$ :**



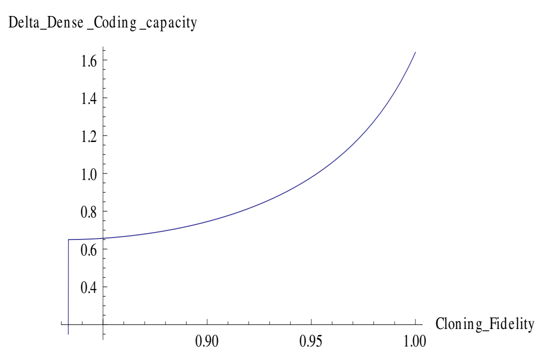

FC values : min $= 5/6$ and max $= 1$

#### 2.1.2 Super Dense Coding

We find the analytical expression to study the dependence of its dense coding capacity on cloning fidelity $(FC)$ and broadcasting fidelity $(FB)$ applying the cloning transformation. Here the difference in dense coding capacities before cloning $(C_{12})$ and after cloning $(C_{14})$ is given by $\triangle C = C_{12} - C_{14}$.

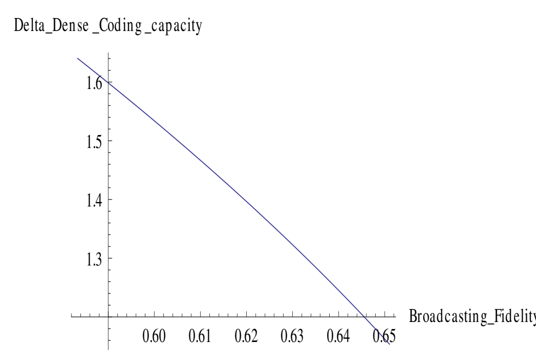
⋆ With Cloning Fidelity:

**Plot of $\triangle C$ versus $FC$ :**




⋆ With Broadcasting Fidelity:

**Plot of $\triangle C$ versus $FB$ :**




FB values : min $= \frac{7}{12}$ and max $= \frac{125}{192}$; after applying the Peres-Horodecki criterion on $\alpha^2$.

### 2.2 Werner State

Definition of the Werner state:

$$\rho_{12}^{in} = \left(\frac{1-p}{4}\right)\{|00\rangle_{12}\langle 00| + |01\rangle_{12}\langle 01| + |10\rangle_{12}\langle 10| + |11\rangle_{12}\langle 11|\} + p(|\psi\rangle_{12}\langle\psi|)$$

where $|\psi\rangle_{12}^{in} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, maximally entangled state and $p$ is the classically mixing parameter.

### 2.2.1 Dependence on Cloning Fidelity

In our case, we want to study the fidelity of cloning from initial state $\rho_1^{in}$ to say final state $\rho_3$.

Thus we find after substitution of values in the above expression,

$FC = \left(Tr\left(\sqrt{\sqrt{\rho_1^{in}} * \rho_3 * \sqrt{\rho_1^{in}}}\right)\right)^2 = 1$, as per Uhlmann's definition.

where $\rho_1^{in}$ and $\rho_3$ as discussed above.

Looking deeply into the perspective of the Werner state suggests that,

$\rho_{in}^1\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{I}{2}$, the variable $P$ dependence is absent and so the fidelity of cloning comes to be 1.

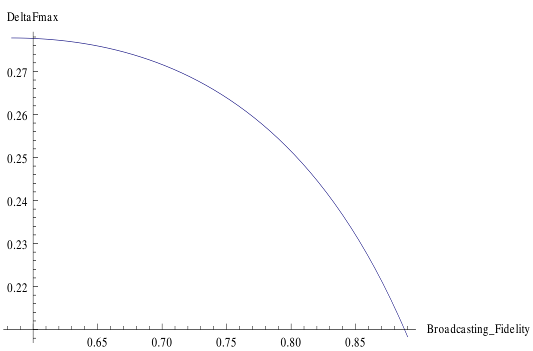
Similar reasons follows from above and holds to be true. So, for this reason I guess its physically relevant and contextual to the cause.

### 2.2.2 Dependence on Broadcasting Fidelity

We find the analytical expression to study the dependence of its maximal teleportation fidelity and super dense coding capacity on broadcasting fidelity $(FB)$ after applying the cloning transformation. Here the difference in dense coding capacities before cloning $(C_{12})$ and after cloning $(C_{14})$ is given by $\triangle C = C_{12} - C_{14}$.

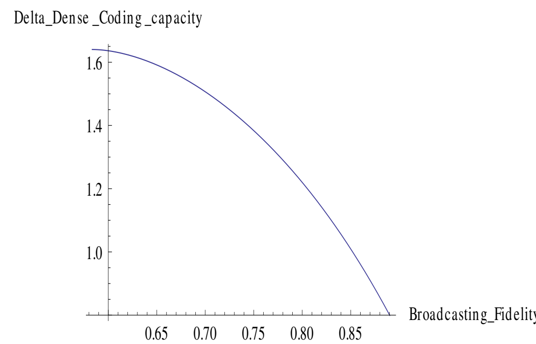
- ⋆ Maximal Fidelity of Teleportation:

**Plot of $\triangle F_{max}$ versus $FB$ :**




- ⋆ Super Dense Coding Capacity:

**Plot of $\triangle C$ versus $FB$ :**




FB values : min $= \frac{7}{12}$ and max $= \frac{1}{16}\left(8 + \sqrt{39}\right)$; after applying the Peres-Horodecki criterion on $p$.

Note: The dependence of analytical expression for the measure of entanglement and discord for initial and final states on cloning and broadcasting fidelities in both cases (namely non-maximally entangled state and Werner state) have also been studied and deduced in the paper. They haven't been included here since the expressions are too big to be included. The resultant expression can be sent later, if asked for.

## References

[1] S. Adhikari, B.S. Choudhary, and I. Chakrabarty, *J. Phys. A*, **39**, 8439 (2006).

[2] S. Sazim, I. Chakrabarty, arxiv:quant-ph/**1210.1312v2** (2012)

[3] R. Horodecki, M. Horodecki, and P. Horodecki, arxiv:quant-ph/**9606027v1** (1996).