Gate teleportation using Series entanglement distributions on arbitrary remote states

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INTRODUCTION

The counter-intuitive and one of the most striking features of the quantum world is entanglement, which has found practical use in the field of communication and cryptography. Various quantum communication protocols, like qubit teleportation, superdense coding, quantum information splitting, secret sharing, remote state preparation have been theoretically and experimentally demonstrated through entangled channels. Gate teleportation is another important protocol implementing a multi-partite quantum gates between qubits, which are spatially distributed. The protocol requires application of a unitary gate, that can not be decomposed into individual local operations i.e., $|\psi\rangle_{\mathcal{AB}} \longrightarrow \mathcal{U}|\psi\rangle_{\mathcal{AB}}$, where $\mathcal{U} \neq \mathcal{U}_{\mathcal{A}} \otimes \mathcal{U}_{\mathcal{B}}$. This can be achieved using entangled channels shared by the remote parties. In the context of distributed quantum computing, it is necessary to produce controlled operations between remote parties, which may be more than two in number. Together with the local single particle operations, these can produce the necessary entanglement between remote parties for implementation of quantum tasks.

In the familiar network [1], each of the control parties shares one entangled state with the target party and none of the control parties shares entanglement between them whereas in series network, the entanglement distribution forms a series, such that the control parties share entanglement with the adjacent control parties and only one of them shares entanglement with the target party. As this network is linear, the target party has to maintain only one entangled channel. It is particularly useful when the entanglement sharing is difficult between each controlling agent with the target party.

The paper is organized as follows. We start with a three party scenario, where Alice and Bob simultaneously teleport controlled-Hermitian gate to Charlie in series entangled network, which is then generalized for arbitrary multi-partite state. Sect. II deals with the teleportation of controlled-controlled-Unitary gate and the generalization to n-controlled Unitary gate teleportation. Sect. III is devoted to conclusion and directions for future work.

1 SIMULTANEOUS TELEPORTATION OF CONTROLLED-HERMITIAN GATE USING SERIES ENTANGLEMENT NETWORK

Here, we illustrate the scheme for the simultaneous teleportation of controlled-Hermitian (as well as unitary) gates from two parties to one, where the entangled network is in series connection. Suppose Alice, Bob and Charlie possess qubits 1,4 and 7 respectively of the arbitrary state,

$$|\psi\rangle_{147} = (d_0|000\rangle + d_1|001\rangle + d_2|010\rangle + d_3|011\rangle + d_4|100\rangle + d_5|101\rangle + d_6|110\rangle + d_7|111\rangle) \tag{1}$$

with $\Sigma_{i=0}^{7}|d_i|^2 = 1$, on which Alice and Bob want to implement simultaneous controlled-Hermitian gate on Charlie's qubit. Alice and Bob share a Bell state between their respective qubits 2 and 3; Bob and Charlie share a Bell state

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between their respective qubits 5 and 6 :

$$|\Phi\rangle_{23} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \tag{2}$$

and

$$|\Phi\rangle_{56} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \tag{3}$$

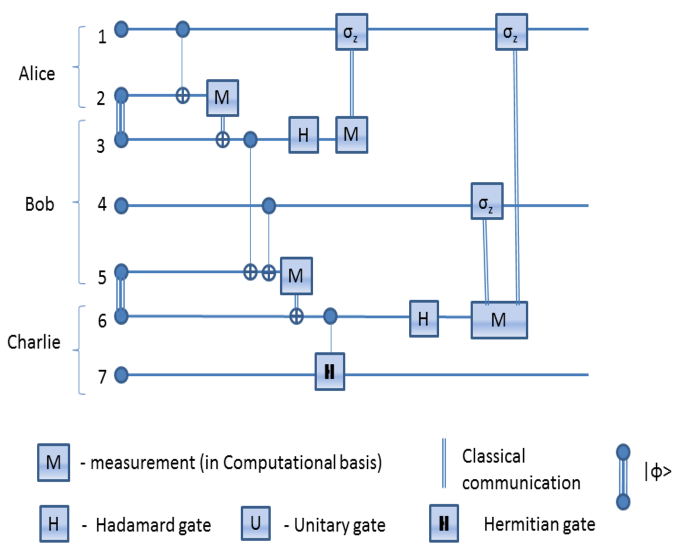

Figure 1: Simultaneous $C^{\mathcal{H}}$ teleportation through series network

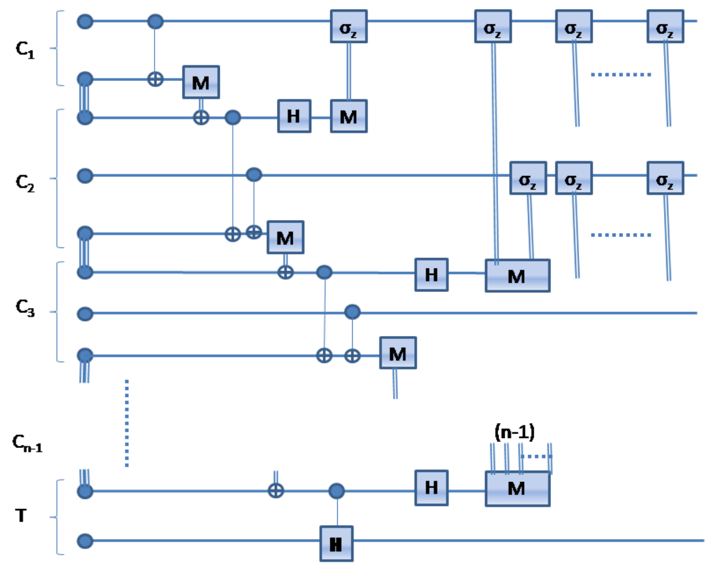

Figure 2: Generalized simultaneous $C^{\mathcal{H}}$ teleportation using the series network

Here, Alice shares entanglement with another control party, Bob. And Bob shares entanglement with the target party, Charlie. Thus there is no direct entanglement between Alice and Charlie, which makes a series entanglement connection.

The pictorial representation of local operations and measurements for simultaneous $C^{\mathcal{H}}$ teleportation through series network of Bell states has been depicted in figure1. The simultaneous remote teleportation of controlled-Hermitian gate from two parties to one consumes 2 ebits and total 5 cbits to communicate the measurement outcomes.

n-party generalized gate teleportation :

The generalized protocol of simultaneous $C^{\mathcal{H}}$ gate teleportation using the above series network is given in figure 2, where the unknown state given by,

$$|\chi\rangle = \Sigma_{m=0}^{2^n-1} a_m |i\rangle, \tag{4}$$

where ' i ' is the binary representation of decimal number ' m '. For n parties the communication cost is $(n-1)$ ebits and $(n^2+n-2)/2$ cbits.

From the above protocol, it can be inferred that the Unitary as well as Hermitian operators have significance in series entangled network. This operator has the additional property of involution (i.e., the operator is same as its inverse), which is responsible for making this protocol deterministic. Most of the important gates like controlled-Pauli gates, controlled-Hadamard gate etc., belong to this category, making this implementation powerful.

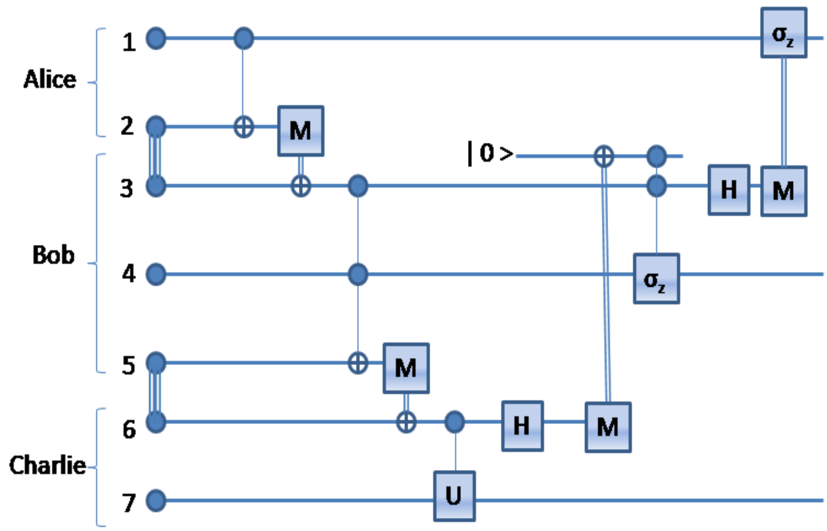

Figure 3: Controlled-controlled-Unitary gate teleportation through series network.

2 TOFFOLI GATE TELEPORTATION USING SERIES ENTANGLEMENT NETWORK

It has been shown that a more general form of Toffoli gate, i.e., controlled-controlled-Unitary gate can be deterministically teleported using two Bell pairs (2 ebits of entanglement) and 4 cbits to communicate the measurement outcomes [1]. In this present protocol, we implement that same non-local gate with the same communication cost using series entanglement distribution considered in the above section The details of the protocol is given in figure 3.

n-party generalized gate teleportation :

The above procedure can be generalized to teleport a 'n'-qubit gate, where $(n-1)$ qubits are controls and the unitary operator acts on the target qubit, only if, all the control qubits are $|1\rangle$s and the communication cost is $(n-1)$ ebits and $2(n-1)$ cbits which is same as for the network given in Ref.[1]. The details have been domonstrated in Ref. [2].

3 CONCLUSION

In conclusion, we have demonstrated gate teleportation protocols in series entanglement network. We show that in this configuration, classical communication cost for implementing this simultaneous gate teleportation is more as compared to the Eisert et. al. distribution. However, the series network is linear and also advantageous when the entanglement sharing is difficult between each controlling side with the target party. The fact that, our network comprises of Bell states, which are realized in laboratory conditions, makes our protocol experimentally achievable. Optimal protocol may be a subject of further investigation for the simultaneous controlled-Unitary gate teleportation in series entangled channels. In future, we would also like to study the teleportation protocols with minimum communication costs of other important gates, which do not belong to this category.

References:

[1] J. Eisert, K. Jacobs, P. Papadopoulos and M. B. Plenio, 'Optimal local implementation of nonlocal quantum gates', Phys. Rev. A 62, 052317 (2000). [2] D. Saha, S. Nandan and P. K. Panigrahi, 'Gate teleportation using Parallel and Series entanglement distributions on arbitrary remote states', arXiv:1206.6323v2 [quant-ph] (2012) and the references there.