Constructing mehod of 2-EPP with different quantum error correcting codes

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1 Introduction

Entanglement purification protocols(EPPs) are important to share entanglement in a noisy quantum channel[1]. EPPs consist of fundamental procedures "LOCC(Local Operation and Classical Communication)". In Ref.[2], conversion method from an arbitary $[n, k]$stabilizer code to a 2-EPP was shown. In the present paper, we consider EPPs constructed from two (or more) quantum error correcting codes and show that our method has higher performance in comparison with using codes individually.

2 Preliminaries

In this study, we consider the following problem. Alice prepares n Bell states

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \tag{1}$$

and she sends half of each to Bob over a noisy quantum channel. Then, they apply an EPP to them. If they keep their pairs, they share k entangled states.

In Ref.[3], superiority of 2-EPPs to 1-EPPs with finite entangled states was shown for the phase-damping channel, which is represented as

$$\mathcal{E}_{PD}(\rho) = (1-p)\rho + pZ\rho Z^\dagger, \tag{2}$$

where $0 \le p \le 1$. Therefore, better 2-EPPs are desired for phase-damping channels. For this reason and also for simplicity, we assume a phase-damping channel to examine the performance of our construction method.

3 Method of constructing EPP

3.1 Construction method

In this section, we show a constructing method of an EPP, which uses two (or more) quantum error correcting codes. We consider two stabilizer codes; $C_1 : [n, k_1]$code and $C_2 : [n, k_2]$code. Let $G^{(1)}$ and $G^{(2)}$ be generators of C_1 and C_2, respectively:

$$\begin{aligned} G^{(1)} &= \{g_1^{(1)}, g_2^{(1)}, \cdots, g_{n-k_1}^{(1)}\}, \\ G^{(2)} &= \{g_1^{(2)}, g_2^{(2)}, \cdots, g_{n-k_2}^{(2)}\}. \end{aligned} \tag{3}$$

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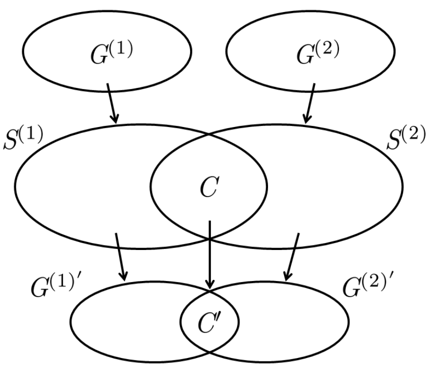

Figure 1: Relations of sets

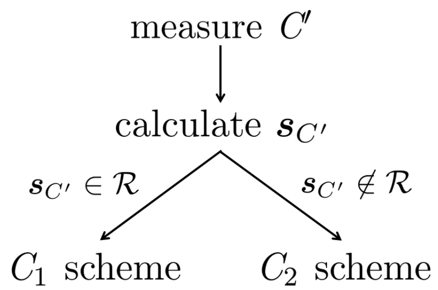

Figure 2: Diagram of protocol

$$\begin{aligned} S^{(1)} &= \langle g_1^{(1)}, g_2^{(1)}, \cdots, g_{n-k_1}^{(1)}\rangle, \\ S^{(2)} &= \langle g_1^{(2)}, g_2^{(2)}, \cdots, g_{n-k_2}^{(2)}\rangle, \end{aligned} \tag{4}$$

are stabilizers of C_1 and C_2. Define the set C from two stabilizers as

$$C = S^{(1)} \cap S^{(2)}. \tag{5}$$

Let $C' = \{c'_1, c'_2, \cdots, c'_l\}$ be a set of generators of C. Since $C' \subset C \subset S^{(1)}, S^{(2)}$, we have the following generators of C_1 and C_2:

$$\begin{aligned} G^{(1)'} &= \{g_1^{(1)'}, g_2^{(1)'}, \cdots, g_{n-k_1}^{(1)'}\}, \\ G^{(2)'} &= \{g_1^{(2)'}, g_2^{(2)'}, \cdots, g_{n-k_2}^{(2)'}\}, \end{aligned} \tag{6}$$

where

$$g_i^{(1)'} = g_i^{(2)'} = c'_i \quad (i = 1, \cdots, l). \tag{7}$$

Figure 1 shows the relation of sets.

The proposed protocol using $G^{(1)'}$ and $G^{(2)'}$ is performed as follows (Fig.2).

- Alice measures $c'_1, \cdots, c'_l$ on her own quantum states and gets a measurement outcome $\boldsymbol{a}_{C'} = (a_{c'_1}, \cdots, a_{c'_l})$.
- Bob measures $c'_1, \cdots, c'_l$ on her own quantum states and gets a measurement outcome $\boldsymbol{b}_{C'} = (b_{c'_1}, \cdots, b_{c'_l})$.
- Alice and Bob send their measurement outcomes to each other and they perform the following two processes according to error syndromes

$$\begin{aligned} \boldsymbol{s}_{C'} &= \boldsymbol{a}_{C'} \oplus \boldsymbol{b}_{C'} \\ &= (a_{c'_1} \oplus b_{c'_1}, \cdots, a_{c'_l} \oplus b_{c'_l}). \end{aligned} \tag{8}$$

- If $\boldsymbol{s}_{C'} \in \mathcal{R}$, Alice and Bob measure remaining operator $g_{l+1}^{(1)'}, \cdots, g_{k_1}^{(1)'}$. Then they calculate syndromes and they perform some processings depending on all error syndromes.
- If $\boldsymbol{s}_{C'} \notin \mathcal{R}$, Alice and Bob measure remaining operator $g_{l+1}^{(2)'}, \cdots, g_{k_2}^{(2)'}$. Then they calculate syndromes and they perform some processings depending on all error syndromes.

Here, $\mathcal{R} \subset \mathbb{F}_2^l$ is a subset of all syndromes which are obtained by measuring C'. Therefore, a procedure in the protocol is changed whether $\boldsymbol{s}_{C'} \notin \mathcal{R}$ or $\boldsymbol{s}_{C'} \in \mathcal{R}$.

3.2 Performance

In this section, we evaluate the performance of 2-EPPs consisting of difference quantum error correcting codes with simulation. In this paper, we use $[31, 21]$code and $[31, 16]$code. Since we consider the phase-damping channel, each generator consists of I or X. The parameters of the protocol are $n = 31$, $k_1 = 21$, $k_2 = 16$ and $l = 10$.

We employ the fidelity F between k Bell state $|\Phi^+\rangle^{\otimes k}$ and shared states ρ_{out} after purification and a purification rate which is defined as

$$R_P = \frac{k}{n} P_S, \tag{9}$$

where P_S is the success probability of the EPP.

In the following, we consider the 2-EPP from $[31, 21]$ code as the 'standard' protocol and compare it with the other protocols.

First, we compare the 2-EPP from $[31, 16]$ code with the standard protocol (Fig.3). From Fig.3, $[31, 21]$2-EPP is superior to $[31, 16]$2-EPP in purification rate, whereas $[31, 16]$2-EPP has higher performance in fidelity. Therefore, there is the relationship of 'trade-off' between fidelity and purification rate.

Next, we compare the proposed protocol with the standard protocol (Fig.4). We can see from Fig.4 that the proposed protocol is superior to $[31, 21]$2-EPP both in fidelity and in purification rate.

4 Conclusion

In this paper, we propose the method of constructing 2-EPP which consists of different quantum error correcting codes and show performance of the 2-EPPs for a phase-damping channel with simulation. As a result, the proposed protocol improves both fidelity and purification rate compared with a EPP from a single code when the number of initial shared entanglement is 31.

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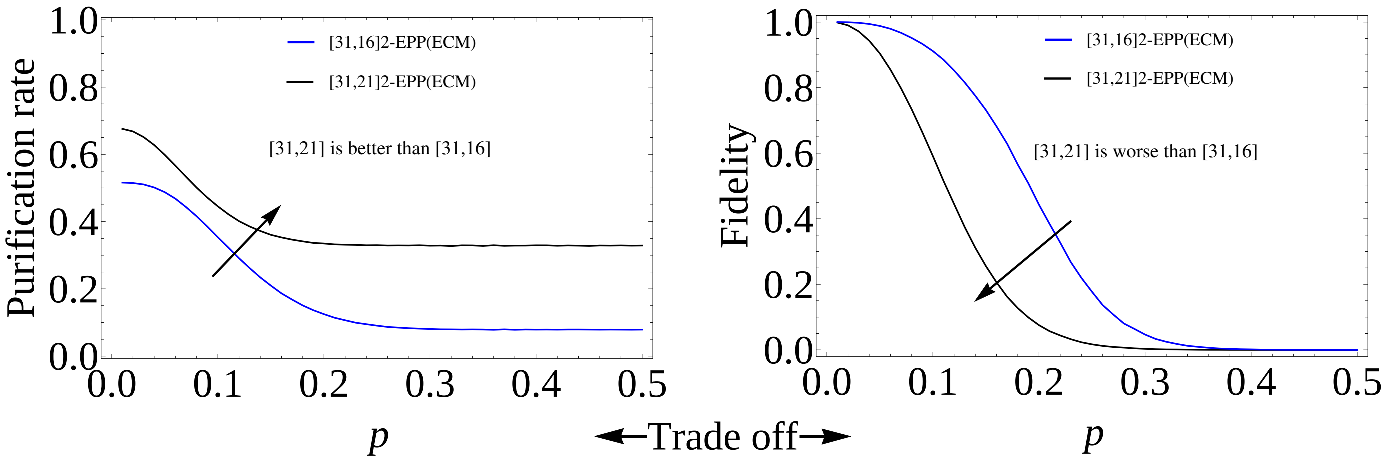

Figure 3: conventional method

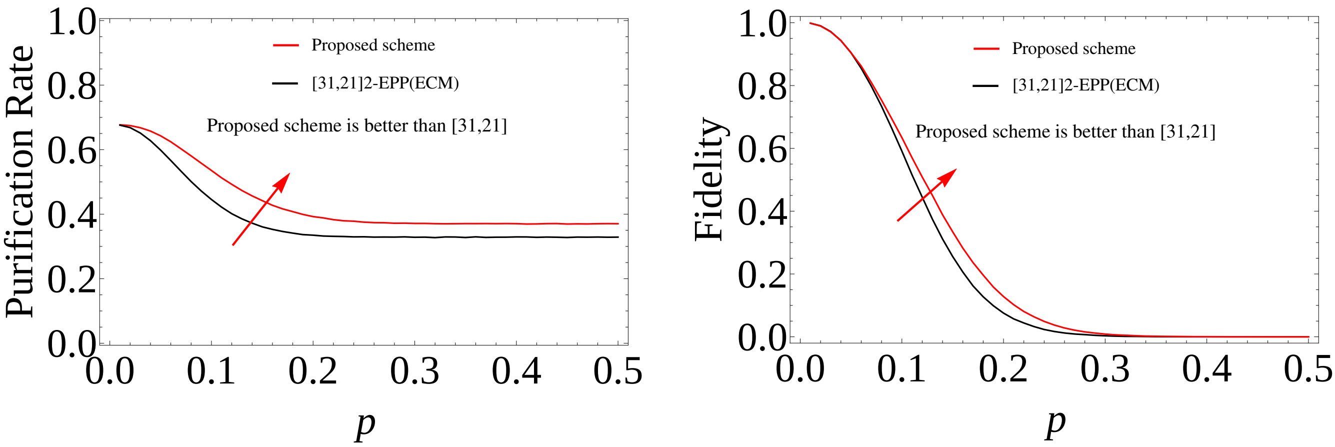

Figure 4: proposed method