Improvement of quasi-optimum quantum receiver for M-ary PSK coherent-state signals

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1 Intoroduction

The optimum quantum receiver is defined as a quantum receiver minimizing the average error probability[1]. To study realization of optimum quantum receivers is one of important subjects in quantum information theory.

There are many attempts of this subject. As a pioneering work, Kennedy proposed a quasi-optimum quantum receiver (so-called Kennedy receiver) for binary coherent-state signals in 1973[2]. After that, Dolinar proposed an optimum quantum receiver (so-called Dolinar receiver) for binary coherent-state signals[3]. Another realization method of binary quantum-state signals were proposed by Sasaki et al.[4] developping an idea of received quantum state control[5, 6, 7] and were generalized to codeword-states[8] and M-ary coherent-state signals[9]. However, realization of the optimum quantum receiver for M-ary signals based on Ref.[9] is extremely difficult for current technology. As for quasi-optimum quantum receivers, Kennedy receiver was generalized to QPSK coherent-state signals[10] and to M-ary coherent-state signals[11]. Although the quasi-optimum quantum receivers shown in Refs.[2, 10, 11] have closer performance to the optimum quantum receivers when the average number of photons of the signals is large, they are worse than Homodyne or Heterodyne receivers when the signals are very weak.

To cope with this problem, Tamori showed that Kennedy receiver is improved by regulating the amplitude of local oscillator light and the improved receiver always outperforms Homodyne receiver[12]. Takeoka et al. rediscovered Tamori's result and showed that the receiver is further improved by using squeezing[13].

In this paper, we improve quasi-optimum quantum receivers for M-ary signals by applying the method in Refs.[12, 13] and show that the improved receivers always outperform Heterodyne receiver[1].

2 Quasi-optimum quantum receiver

Figure 1 shows the quasi-optimum quantum receiver for M-ary PSK coherent state signals[10, 11]. For explanation, we express a process of the local oscillator by

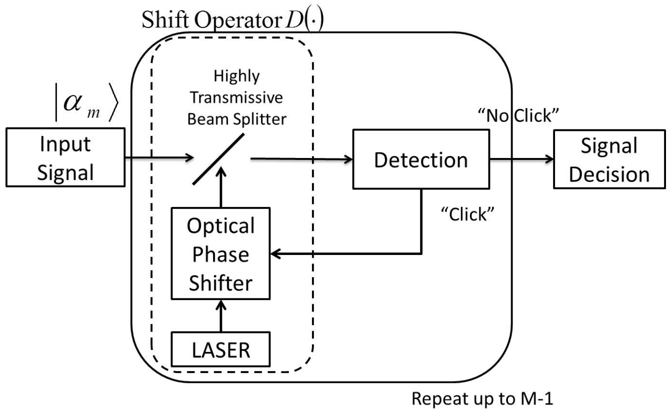

Figure 1: Block diagram of the quasi-optimum quantum receiver.

using a shift operator $\hat{D}(\cdot)$ on the Hilbert space which corresponds to whole signal duration. Note that certain conversion is necessary for exact treatment. Let $|\alpha_m\rangle$ $(m = 0, 1, \cdots, M-1)$ be the input signal state to the receiver. Here, $\alpha_m = \alpha e^{2\mathbf{i}m\pi/M}$ is the complex amplitude of the coherent state and we assume that α is a positive real number. The input state is first applied the shift operator $\hat{D}(-\alpha)$, by which the signal $|\alpha_0\rangle$ is transformed into vacuum, and is input to the detector. If a photon is detected, the result is fed back to the phase shifter and the phase of the local oscillator is shifted. If no photon is detected, we decide that the signal is i when feedback is applied i times. The feedback is applied at most $M-1$ times.

3 Improvement of quantum receiver

3.1 Improvement by regulating amplitude and phase of local oscillator

Following Refs.[12, 13], we optimize amplitude and phase of local oscillator of the quantum receiver for M-ary signals. Then we can expect improvement of the average error probability.

3.2 Improvement by increasing feedback

In order to consider further improvement of the quantum receiver for M-ary signals, we increase the number of times of feedback more than $M-1$ times. In Kennedy

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[1]We partly reported the results at a domestic conference and a journal[14, 15].

receiver[2] and the improved receiver[12, 13], no feedback is used. However, it is well known that feedback is applied infinite times in Dolinar receiver. So we can expect further improvement of the average error probability.

4 Performance

Here we show error performances of various quantum receivers for 3-ary and 4-ary PSK coherent-state signals.

4.1 3PSK

Figure 2 shows the average error probabilities of the classical optimum receiver (Heterodyne receiver), the optimum quantum receiver, the quasi-optimum receiver [11] and one of improved receivers in which the amplitude of the shift operator is optimized for 3-ary PSK coherent-state signals. Note that a recently analyzed *displacement receiver* with infinite-step feedforward [16] has exactly the same performance as that of the quasi-optimum receiver without improvement [11]. We see that the improved quasi-optimum quantum receiver always outperforms the classical optimum receiver. The quasi-optimum quantum receiver is further improved by optimizing the phase of the shift operator and by increasing the number of feedback. Figure 3 shows the average error probabilities of various improved quasi-optimum quantum receivers. Here, feedback is increased from two times to three times. From Fig.3, feedback is more effective than the phase optimization for 3-ary PSK signals.

4.2 4PSK

Figure 4 shows the comparison of one of the improved receivers with existing receivers and Fig.5 shows the comparison of the improved receivers. As in the case of 3-PSK signals, the improved quasi-optimum quantum receivers always outperform the classical optimum receiver. For 4-PSK signals, we increase feedback from three to four times. In the case of 4-PSK signals, if the increase of feedback is only one time, the improvement of the error performance is very small. However, the phase optimization is effective. Actually, the effect of the phase optimization for 4-PSK signals is greater than that for 3-PSK signals (Fig.6).

5 Conclusion

We have shown the error performances of the quasi-optimum quantum receivers are improved by optimizing a shift operator and by increasing feedback. As a result, the improved receivers always outperform the classical optimum receiver. It is expected that optimization of the phase of a shift operator is more effective when the number of signals increases. On the other hand, if the number of signals increases, further increase of feedback is desired.

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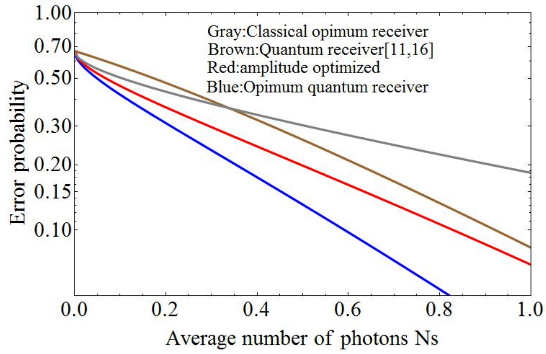

Figure 2: The average error probabilities of the classical optimum receiver (Heterodyne receiver), the optimum quantum receiver, the quasi-optimum receiver [11] and one of improved receivers in which the amplitude of the shift operator is optimized for 3-ary PSK coherent-state signals.

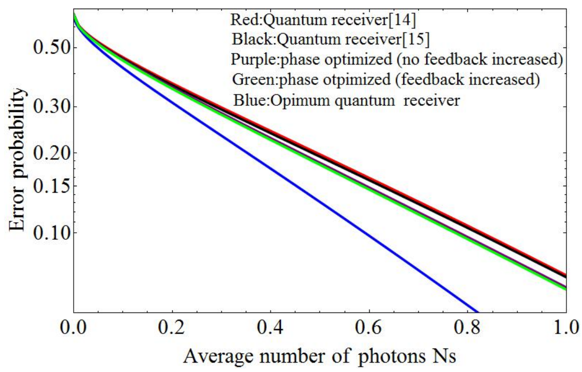

Figure 3: The average error probabilities of various improved quasi-optimum quantum receivers for 3-ary PSK coherent-state signals.

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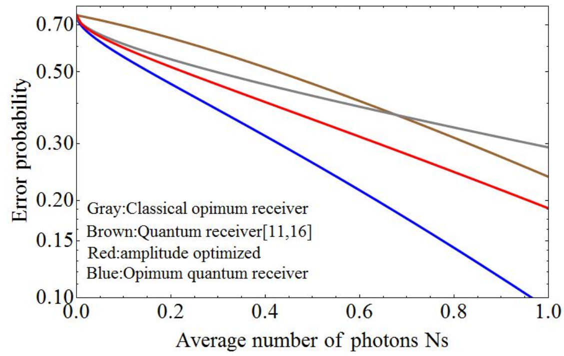

Figure 4: The average error probabilities of the classical optimum receiver (Heterodyne receiver), the optimum quantum receiver, the quasi-optimum receiver [10] and one of improved receivers in which the amplitude of the shift operator is optimized for 4-ary PSK coherent-state signals.

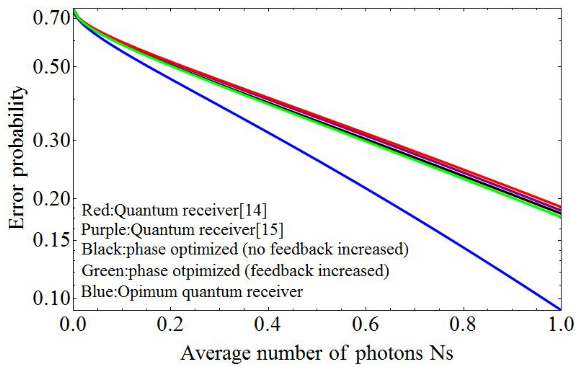

Figure 5: The average error probabilities of various improved quasi-optimum quantum receivers for 4-ary PSK coherent-state signals.

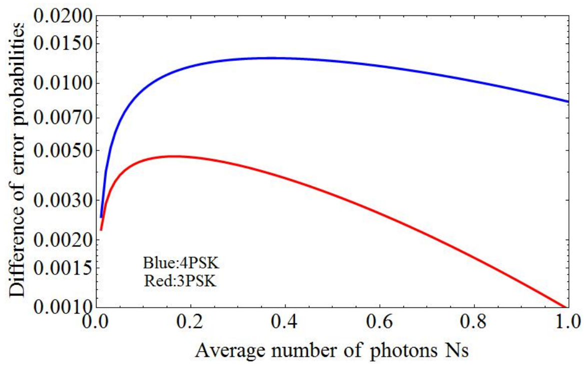

Figure 6: Difference of average error probabilies of improved quasi-optimum quantum receivers with and without phase optimization.