Maximization of Holevo information for entanglement-assisted classical communication using quasi-Bell states

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1 Introduction

Entanglement is known as an important resource in quantum information systems. Entangled states by nonorthogonal states called "quasi-Bell states"[1, 2], such as coherent states of light, have been shown to be capable of "perfect entanglement" and are expected to be robust against attenuation in quantum channel.

In this study, we consider the entanglement-assisted classical communication[3] using quasi-Bell states. As previous studies, a classical information transmission assisted by a quasi-Bell state was considered for an ideal channel[4] and a lossy channel[5]. In these papers, an approximate encoding which works for sufficiently large coherent amplitude was assumed. Recently, we showed the capacity of a classical communication assisted by a degraded quasi-Bell state using rigorously realizable encodings[6]. In Ref.[6], the capacity was computed by optimizing *a priori* distribution, whereas a fixed encoding function was used. Therefore, simultaneous optimization of *a priori* distribution and an encoding function is desired. However, to optimize directly the both quantities is computationally hard, so that reduction of computational complexity is necessary.

In the present paper, we apply a quantum version[7] of the Arimoto-Blahut algorithms[8, 9] to compute the capacity. As a result, simultaneous optimization of *a priori* distribution and an encoding function is achieved.

2 Preliminary

2.1 Quantum Arimoto-Blahut algorithm[7]

Quantum Arimoto-Blahut algorithm is known for computing the capacity of quantum channel or the Holevo capacity[7]. There are some examples of applications of the algorithm (e.g. [10, 11]). We apply it to our problem partially. Figure 1 describes a diagram of the algorithm.

Let $\Pi_n = \{(p_1, \cdots, p_n; \sigma_1, \cdots, \sigma_n)\}$ be the set of possible input. Here, $\{p_i\}$ is *a priori* probability distribution and $\{\sigma_i\}$ is a set of input quantum states. Let $I(\pi)$ be the Holevo information or the quantum mutual information for an input $\pi \in \Pi_n$. Then the Holevo capacity is defined as

$$C = \sup_{\pi \in \Pi_n} I(\pi). \tag{1}$$

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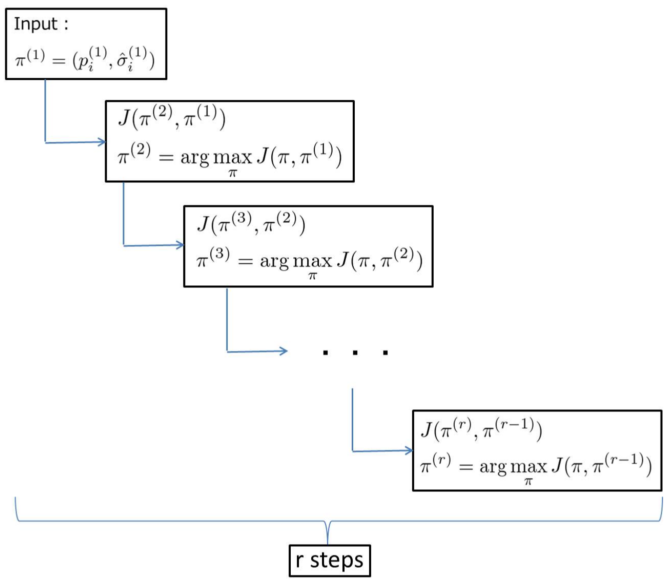

Figure 1: Quantum Arimoto-Blahut algorithm.

In Ref.[7], a two-variable extension $J(\pi, \pi')$ of $I(\pi)$ was introduced. It holds that

$$I(\pi) = J(\pi, \pi) = \max_{\pi' \in \Pi_n} J(\pi, \pi'). \tag{2}$$

Let the sequence $\{\pi^{(k)}\}_{k=1}^{\infty}$ be defined by

$$\pi^{(k+1)} = \operatorname*{argmax}_{\pi} J(\pi, \pi^{(k)}). \tag{3}$$

Then

$$I(\pi^{(k)}) \leq J(\pi^{(k+1)}, \pi^{(k)}) \leq I(\pi^{(k+1)}). \tag{4}$$

We can recursively compute $I(\pi^{(k)})$ and the limit value $I(\pi^{(\infty)})$ is expected as the Holevo capacity if the algorithm works well.

2.2 Quasi-Bell state by coherent states

Quasi-Bell states are based on nonorthogonal states[1]. A coherent state whose amplitude α is expressed as $|\alpha\rangle$. We use one of these, which is $|\Psi_4\rangle = h_4(|0\rangle|0\rangle - |\alpha\rangle|\beta\rangle)$.

2.3 Entanglement-assisted classical communication

The entanglement-assisted classical communication[3], which is also called the quantum superdense coding, had been shown that the use of entanglement enhances classical communications(Fig.2). In the ideal qubit channel, two bits can be obtained by transmitting 1-qubit.

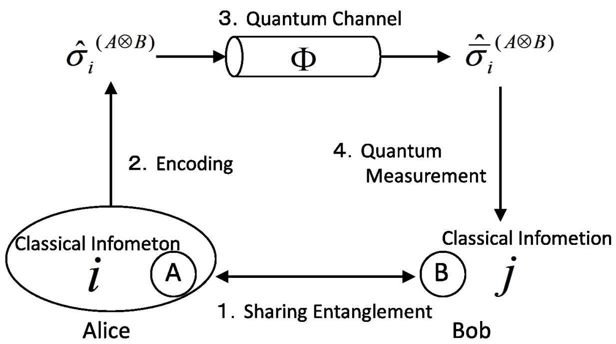

Figure 2: Schematic diagram of a classical communication assisted by an entangled state.

3 Computing the capacity

3.1 Case of fixed input states

First, we consider the case that the input states are fixed and *a priori* probability distribution will be optimized. Figure 3 shows the Holevo information $I(\pi^{(r)}) = J(\pi^{(r)}, \pi^{(r)})$ in the process of the recursion $\pi^{(r)} \to \pi^{(r+1)}$. Figure 4 shows the variation of the Holevo information $J(\pi^{(r+1)}, \pi^{(r+1)}) - J(\pi^{(r)}, \pi^{(r)})$. We can see that it approaches 0 when $r \gtrsim 10$. In this case, $J(\pi^{(r)}, \pi^{(r)})$ is monotonically nondecreasing. There is some possibility that the limit value is a local maximum because the algorithm does not assure us of the global maximum. Therefore we executed the algorithm many times by changing the initial inputs. Since the Holevo informations converge at the same value for every initial inputs, the obtained value is expected as the global maximum.

3.2 Case of searched input states

Next, we consider the full optimization. Figure 5 shows the Holevo information $J(\pi^{(r)}, \pi^{(r)})$ in the process of the recursion $\pi^{(r)} \to \pi^{(r+1)}$ with searched input states. Here, we use the quantum Arimoto-Blahut algorithm only for optimization of the probability distribution and the input quantum states are optimized using another algorithm. Because, in our problem, input quantum states are restricted to the states obtained by local operations and the quantum Arimoto-Blahut algorithm can not be directly applied for optimizing input quantum states. However, since the input quantum states can be optimized by one parameter [6], optimization of states only are computationally easy problem and any algorithm is allowable. Figure 6 shows the variation of the Holevo information $J(\pi^{(r+1)}, \pi^{(r+1)}) - J(\pi^{(r)}, \pi^{(r)})$. We see that it

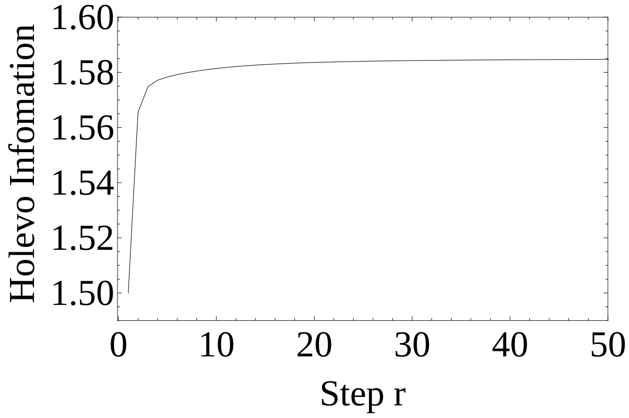

Figure 3: The Holevo information $J(\pi^{(r)}, \pi^{(r)})$ in the process of the recursion $\pi^{(r)} \to \pi^{(r+1)}$ with fixed input states.

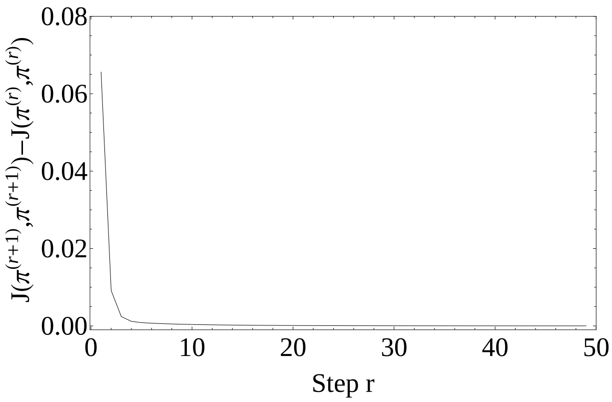

Figure 4: $J(\pi^{(r+1)}, \pi^{(r+1)}) - J(\pi^{(r)}, \pi^{(r)})$ with fixed input states.

approaches 0 when $r \gtrsim 10$. Since the limit value might be a local maximum, we executed the algorithm many times by changing the initial input distributions as in the previous section. Figure 5 is an example. Although the values of $J(\pi^{(r)}, \pi^{(r)})$ are different depending on the initial inputs for small r and they are not monotonically nondecreasing, they converge the same value. Therefore, the obtained value is expected as the global maximum and the full optimization is achieved.

4 Conclusion

We apply the quantum Arimoto-Blahut algorithm to computation of the Holevo capacity for the entanglement-assisted classical communication when quasi-Bell state is used as the shared initial entanglement.

The capacity is computed sufficiently fast with high precision. As a typical example, comparing with a random search which was used in the previous study, precision is 10000 times higher and computation time is more than 100 times faster.

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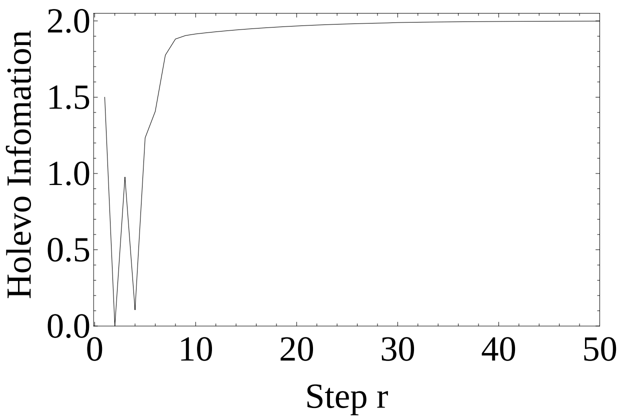

Figure 5: The Holevo information $J(\pi^{(r)}, \pi^{(r)})$ in the process of the recursion $\pi^{(r)} \to \pi^{(r+1)}$ with searched input states.

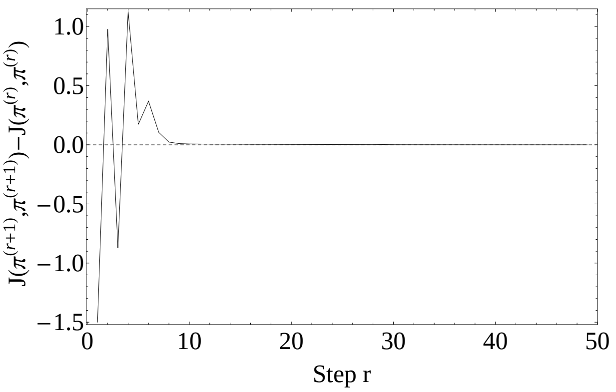

Figure 6: $J(\pi^{(r+1)}, \pi^{(r+1)}) - J(\pi^{(r)}, \pi^{(r)})$ with seacrhed input states.

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